Intra-Procedural Dataflow Analysis

Forward Analyses

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Formalising the Development

- the programming language of interest
  - abstract syntax
  - labelled program fragments
- abstract flow graphs
  - control and data flow between labelled program fragments
- extract equations from the program
  - specify the information to be compuated at entry and exit of labeled fragments
- compute the solution to the equations
  - work list algorithms
  - compute entry and exit information at entry and exit of labeled fragments

WHILE Language

Syntactic categories

a	$\in$ AExp	artithmetic expressions
b	$\in$ BExp	boolean expressions
S	$\in$ Stmt	statements
x, y	$\in$ Var	variables
n	$\in$ Num	numerals
ℓ	$\in$ Lab	labels
op_a	$\in$ Op$_a$	arithmetic operators
op_b	$\in$ Op$_b$	boolean operators
op_r	$\in$ Op$_r$	relational operators

Abstract Syntax

a ::= $x \mid n \mid a_1\ op_a\ a_2$

b ::= true | false | not b | $b_1\ op_b\ b_2 \mid a_1\ op_r\ a_2$

S ::= [x:=a]$^\ell$ | [skip]$^\ell$
| if $[b]^\ell$ then S_1 else S_2
| while$[b]^\ell$ do S od
| $S_1; S_2$

Assignments and tests are (uniquely) labelled to allow analyses to refer to these program fragments – the labels correspond to pointers into the syntax tree. We use abstract syntax and insert paranthesis to disambiguate syntax.

We will often refer to labelled fragments as *elementary blocks.*

Auxiliary Functions for Flow Graphs

labels(S)	set of nodes of flow graphs of S
init(S)	initial node of flow graph of S; the unique node where execution of program starts
final(S)	final nodes of flow graph for S; set of nodes where program execution may terminate
flow(S)	edges of flow graphs for S (used for forward analyses)
flow^R(S)	reverse edges of flow graphs for S (used for backward analyses)
blocks(S)	set of elementary blocks in a flow graph

Computing the Information (1)

S	labels(S)	init(S)	final(S)
$[x := a]^\ell$	$\{\ell\}$	ℓ	$\{\ell\}$
[skip]$^\ell$	$\{\ell\}$	ℓ	$\{\ell\}$
$S_1; S_2$	labels(S_1) $\cup$ labels(S_2)	init(S_1)	final(S_2)
if $[b]^\ell$ then (S_1) else (S_2)	$\{\ell\}$ $\cup$ labels(S_1) $\cup$ labels(S_2)	ℓ	final(S_1) $\cup$ final(S_2)
while $[b]^\ell$ do S od	$\{\ell\} \cup$ labels(S)	ℓ	$\{\ell\}$

Computing the Information (2)

S	flow(S)	blocks(S)
$[x := a]^\ell$	$\emptyset$	$\{[x := a]^\ell\}$
$[\text{skip}]^\ell$	$\emptyset$	$\{[\text{skip}]^\ell\}$
$S_1; S_2$	flow(S_1) $\cup$ flow(S_2) $\cup$ $\{(\ell, \text{init}(S_2)) \mid \ell \in \text{final}(S_1)\}$	blocks(S_1) $\cup$ blocks(S_2)
if $[b]^\ell$ then (S_1) else (S_2)	flow(S_1) $\cup$ flow(S_2) $\cup$ $\{(\ell, \text{init}(S_1)), (\ell, \text{init}(S_2))\}$	$\{[b]^\ell\}$ $\cup$ blocks(S_1) $\cup$ blocks(S_2)
while $[b]^\ell$ do S od	$\{(\ell, \text{init}(S))\}$ $\cup$ flow(S) $\cup$ $\{(\ell', \ell) \mid \ell' \in \text{final}(S)\}$	$\{[b]^\ell\}$ $\cup$ blocks(S)

$$\text{flow}^R(S) = \{(\ell, \ell') \mid (\ell', \ell) \in \text{flow}(S)\}$$

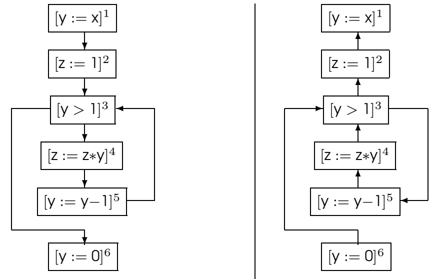
Program of Interest

We shall use the notation

- $S_\star$ to represent the program being analyzed (the "top level" statement)
- Lab$_\star$ to represent the labels (labels($S_\star$)) appearing in $S_\star$
- Var$_\star$ to represent the variables (FV($S_\star$)) appearing in $S_\star$
- Blocks$_\star$ to represent the elementary blocks (blocks($S_\star$)) occuring in $S_\star$
- AExp$_\star$ to represent the set of *non-trivial* arithmetic subexpressions in $S_\star$; an expression is trivial if it is a single variable or constant
- AExp(a), AExp(b) to refer to the set of non-trivial arithmetic subexpressions of a given arithmetic, respectively boolean, expression

Example Flow Graphs

Example:
[y := x]^1; [z := 1]^2; while [y > 1]^3 do [z := z $*$ y]^4; [y := y $-$ 1]^5 od; [y := 0]^6



flow$(S_\star) = \{(1,2),(2,3),(3,4),(4,5),(5,3),(3,6)\}$

flow$^R(S_\star) = \{(6,3),(3,5),(5,4),(4,3),(3,2),(2,1)\}$

Example

Example:
[y := x]^1; [z := 1]^2; while [y > 1]^3 do [z := z $*$ y]^4; [y := y $-$ 1]^5 od; [y := 0]^6

$$\begin{aligned}
\text{labels}(S_\star) &= \{1,2,3,4,5,6\} \\
\text{init}(S_\star) &= 1 \\
\text{final}(S_\star) &= \{6\} \\
\text{flow}(S_\star) &= \{(1,2),(2,3),(3,4),(4,5),(5,3),(3,6)\} \\
\text{flow}^R(S_\star) &= \{(6,3),(3,5),(5,4),(4,3),(3,2),(2,1)\} \\
\text{blocks}(S_\star) &= \{[\text{y} := \text{x}]^1, [\text{z} := 1]^2, [\text{y} > 1]^3, \\
&\quad [\text{z} := \text{z} * \text{y}]^4, [\text{y} := \text{y} - 1]^5, [\text{y} := 0]^6\}
\end{aligned}$$

Simplifying Assumptions

The program of interest $S_\star$ is often assumed to satisfy:

- $S_\star$ has isolated entries if there are no edges leading into init($S_\star$):
$$\forall \ell : (\ell, \text{init}(S_\star)) \notin \text{flow}(S_\star)$$
- $S_\star$ has isolated exits if there are no edges leading out of labels in final($S_\star$):
$$\forall \ell \in \text{final}(S_\star), \forall \ell' : (\ell, \ell') \notin \text{flow}(S_\star)$$
- $S_\star$ is label consistent if
$$\forall B_1^{\ell_1}, B_2^{\ell_2} \in \text{blocks}(S_\star) : \ell_1 = \ell_2 \rightarrow B_1 = B_2$$
This holds if $S_\star$ is uniquely labelled.

Reaching Definitions Analysis

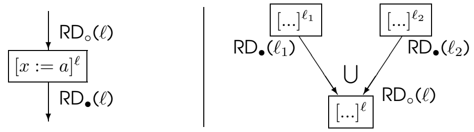
The aim of the Reaching Definitions Analysis is to determine

For each program point, which assignments *may* have been made and not overwritten, when program execution reaches this point along some path.

Example:
[y := x]^1; [z := 1]^2; while [y > 1]^3 do [z := z $*$ y]^4; [y := y $-$ 1]^5 od; [y := 0]^6

- The assignments labelled 1,2,4,5 reach the entry at 4.
- Only the assignments labelled 1,4,5 reach the entry at 5.

Basic Idea



Analysis information: $RD_\circ(\ell)$,$RD_\bullet(\ell)$: $Lab_\star \to \mathcal{P}(Var_\star \times Lab_\star^?)$

- $RD_\circ(\ell)$: the definitions that reach entry of block ℓ.
- $RD_\bullet(\ell)$: the definitions that reach exit of block ℓ.

Analysis properties:

- Direction: forward
- May analysis with combination operator $\bigcup$

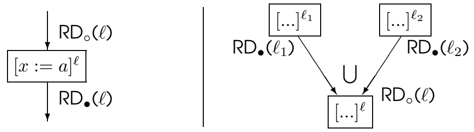
Analysis of Elementary Blocks

$$
\begin{aligned}
kill_{RD}([x := a]^\ell) &= \{(x,?)\} \cup \{(x,\ell') \mid B^{\ell'} \text{ is an assignment to } x\}\\
kill_{RD}([skip]^\ell) &= \emptyset\\
kill_{RD}([b]^\ell) &= \emptyset\\
gen_{RD}([x := a]^\ell) &= \{(x,\ell)\}\\
gen_{RD}([skip]^\ell) &= \emptyset\\
gen_{RD}([b]^\ell) &= \emptyset
\end{aligned}
$$

Example:
$[x := y]^1; [x := x + 3]^2;$

- $kill_{RD}([x := y]^1) = \{(x,?)\} \cup \{(x,1),(x,2)\}$
- $gen_{RD}([x := y]^1) = \{(x,1)\}$

Analysis of the Program



$$
\begin{aligned}
RD_\circ(\ell) &= \begin{cases} \{(x,?) \mid x \in FV(S_\star)\} & : \text{ if } \ell = init(S_\star)\\ \bigcup\{RD_\bullet(\ell') \mid (\ell',\ell) \in flow(S_\star)\} & : \text{ otherwise}\end{cases}\\
RD_\bullet(\ell) &= (RD_\circ(\ell) \backslash kill_{RD}(B^\ell)) \cup gen_{RD}(B^\ell) \quad \text{where } B^\ell \in blocks(S_\star)
\end{aligned}
$$

Example

Example:
$[y := x]^1; [z := 1]^2;$ while $[y > 1]^3$ do $[z := z * y]^4; [y := y - 1]^5$ od; $[y := 0]^6$

Equations: Let $S_1 = \{(y,?),(y,1),(y,5),(y,6)\}, S_2 = \{(z,?),(z,2),(z,4)\}$

$$
\begin{aligned}
RD_\circ(1) &= \{(x,?),(y,?),(z,?)\} & RD_\bullet(1) &= RD_\circ(1) \setminus S_1 \cup \{(y,1)\}\\
RD_\circ(2) &= RD_\bullet(1) & RD_\bullet(2) &= RD_\circ(2) \setminus S_2 \cup \{(z,2)\}\\
RD_\circ(3) &= RD_\bullet(2) \cup RD_\bullet(5) & RD_\bullet(3) &= RD_\circ(3)\\
RD_\circ(4) &= RD_\bullet(3) & RD_\bullet(4) &= RD_\circ(4) \setminus S_2 \cup \{(z,4)\}\\
RD_\circ(5) &= RD_\bullet(4) & RD_\bullet(5) &= RD_\circ(5) \setminus S_1 \cup \{(y,5)\}\\
RD_\circ(6) &= RD_\bullet(3) & RD_\bullet(6) &= RD_\circ(6) \setminus S_1 \cup \{(y,6)\}
\end{aligned}
$$

ℓ	$RD_\circ(\ell)$	$RD_\bullet(\ell)$
1	{(x,?),(y,?),(z,?)}	{(x,?),(y,1),(z,?)}
2	{(x,?),(y,1),(z,?)}	{(x,?),(z,2),(y,1)}
3	{(x,?),(z,4),(z,2),(y,5),(y,1)}	{(x,?),(z,4),(z,2),(y,5),(y,1)}
4	{(x,?),(z,4),(z,2),(y,5),(y,1)}	{(z,4),(x,?),(y,5),(y,1)}
5	{(z,4),(x,?),(y,5),(y,1)}	{(z,4),(x,?),(y,5)}
6	{(x,?),(z,4),(z,2),(y,5),(y,1)}	{(z,4),(x,?),(z,2),(y,6)}

Solving RD Equations

Input

- a set of reaching definitions equations

Output

- the *least solution* to the equations: $RD_\circ$

Data structures

- The current analysis result for block entries: $RD_\circ$
- The worklist W: a list of pairs (ℓ,ℓ') indicating that the current analysis result has changed at the entry to the block ℓ and hence the information must be recomputed for ℓ'.

Solving RD Equations - Algorithm

```
W:=nil;
foreach (ℓ,ℓ') ∈ flow(S⋆) do W := cons((ℓ,ℓ'),W); od;
foreach ℓ ∈ labels(S⋆) do
  if ℓ ∈ init(S⋆) then
    RD∘(ℓ) := {(x,?) | x ∈ FV(S⋆)}
  else
    RD∘(ℓ) := ∅
  fi
od
while W ≠ nil do
  (ℓ,ℓ') := head(W);
  W := tail(W);
  if (RD∘(ℓ)\killRD(B^ℓ)) ∪ genRD(B^ℓ) ⊈ RD∘(ℓ') then
    RD∘(ℓ') := RD∘(ℓ') ∪ (RD∘(ℓ)\killRD(B^ℓ)) ∪ genRD(B^ℓ);
    foreach ℓ'' with (ℓ',ℓ'') in flow(S⋆) do
      W := cons((ℓ',ℓ''),W);
    od
  fi
od
```

Use-Definition and Definition-Use Chains

- Use-Definition chains or *ud* chains

  each use of a variable is linked to all assignments that *reach* it

  [x := 0]^1; [x := 5]^2; [y := x]^3; [z := x]^4

- Definition-Use chains or *du* chains

  each assignment of a variable is linked to all uses of it

  [x := 0]^1; [x := 5]^2; [y := x]^3; [z := x]^4

UD/DU Chains - Defined via RDs

$$\text{UD,DU} : \text{Var}_\star \times \text{Lab}_\star \to \mathcal{P}(\text{Lab}_\star)$$

are defined by

$$\text{UD}(x,\ell) = \begin{cases} \{\ell' \mid (x,\ell') \in \text{RD}_\circ(\ell)\} & : \text{ if } x \in \text{used}(B^\ell) \\ \emptyset & : \text{ otherwise} \end{cases}$$

where used($[x := a]^\ell$) = FV(a), used($[b]^\ell$) = FV(b), used($[\text{skip}]^\ell$) = $\emptyset$

and

$$\text{DU}(x,\ell) = \{\ell' \mid \ell \in \text{UD}(x,\ell')\}$$

Available Expressions Analysis

The aim of the Available Expressions Analysis is to determine

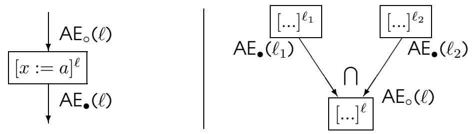
For each program point, which expressions *must* have already been computed, and not later modified, on all paths to the program point.

Example:

[x := a+b]^1; [y := a*x]^2; while [y > a+b]^3 do [a := a + 1]^4; [x := a + b]^5 od

- No expression is available at the start of the program
- An expression is considered available if no path kills it
- The expression a+b is available every time execution reaches the test in the loop at 3.

Basic Idea



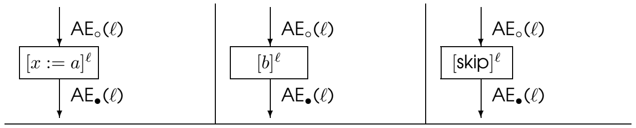
Analysis information: $\text{AE}_\circ(\ell)$,$\text{AE}_\bullet(\ell)$: $\text{Lab}_\star \to \mathcal{P}(\text{AExp}_\star)$

- $\text{AE}_\circ(\ell)$: the expressions that have been comp. at entry of block ℓ.
- $\text{AE}_\bullet(\ell)$: the expressions that have been comp. at exit of block ℓ.

Analysis properties:

- Direction: forward
- Must analysis with combination operator $\bigcap$

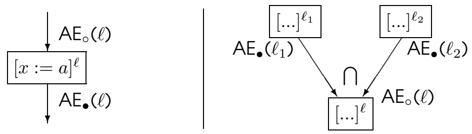
Analysis of Elementary Blocks



$$\begin{aligned}
\text{kill}_{\text{AE}}([x := a]^\ell) &= \{a' \in \text{AExp}_\star \mid x \in FV(a')\}\} \\
\text{kill}_{\text{AE}}([\text{skip}]^\ell) &= \emptyset \\
\text{kill}_{\text{AE}}([b]^\ell) &= \emptyset \\
\text{gen}_{\text{AE}}([x := a]^\ell) &= \{a' \in \text{AExp}(a) \mid x \notin FV(a')\} \\
\text{gen}_{\text{AE}}([\text{skip}]^\ell) &= \emptyset \\
\text{gen}_{\text{AE}}([b]^\ell) &= \text{AExp}(b)
\end{aligned}$$

$$\text{AE}_\bullet(\ell) = (\text{AE}_\circ(\ell) \backslash \text{kill}_{\text{AE}}(B^\ell)) \cup \text{gen}_{\text{AE}}(B^\ell) \quad \text{where } B^\ell \in \text{blocks}(S_\star)$$

Analysis of the Program



$$\begin{aligned}
\text{AE}_\circ(\ell) &= \begin{cases} \emptyset & : \text{ if } \ell = \text{init}(S_\star) \\ \bigcap\{\text{AE}_\bullet(\ell') \mid (\ell',\ell) \in \text{flow}(S_\star)\} & : \text{ otherwise} \end{cases} \\
\text{AE}_\bullet(\ell) &= (\text{AE}_\circ(\ell) \backslash \text{kill}_{\text{AE}}(B^\ell)) \cup \text{gen}_{\text{AE}}(B^\ell) \quad \text{where } B^\ell \in \text{blocks}(S_\star)
\end{aligned}$$

Example

Example:

[x := a+b]^1; [y := a∗x]^2; while [y > a+b]^3 do [a := a + 1]^4; [x := a + b]^5 od

Equations:

$$
\begin{aligned}
\mathsf{AE}_\circ(1) &= \emptyset & \mathsf{AE}_\bullet(1) &= \mathsf{AE}_\circ(1) \setminus \{a*x\} \cup \{a+b\}\\
\mathsf{AE}_\circ(2) &= \mathsf{AE}_\bullet(1) & \mathsf{AE}_\bullet(2) &= \mathsf{AE}_\circ(2) \setminus \emptyset \cup \{a*x\}\\
\mathsf{AE}_\circ(3) &= \mathsf{AE}_\bullet(2) \cap \mathsf{AE}_\bullet(5) & \mathsf{AE}_\bullet(3) &= \mathsf{AE}_\circ(3) \setminus \emptyset \cup \{a+b\}\\
\mathsf{AE}_\circ(4) &= \mathsf{AE}_\bullet(3) & \mathsf{AE}_\bullet(4) &= \mathsf{AE}_\circ(4) \setminus \{a+b, a*x, a+1\} \cup \emptyset\\
\mathsf{AE}_\circ(5) &= \mathsf{AE}_\bullet(4) & \mathsf{AE}_\bullet(5) &= \mathsf{AE}_\circ(5) \setminus \{a*x\} \cup \{a+b\}
\end{aligned}
$$

ℓ	$\mathsf{AE}_\circ(\ell)$	$\mathsf{AE}_\bullet(\ell)$
1	$\emptyset$	{a+b}
2	{a+b}	{a+b,a*x}
3	{a+b}	{a+b }
4	{a+b}	$\emptyset$
5	$\emptyset$	{a+b}

Solving AE Equations

Input

- a set of available expressions equations

Output

- the *largest solution* to the equations: $\mathsf{AE}_\circ$

Data structures

- The current analysis result for block entries: $\mathsf{AE}_\circ$
- The worklist W: a list of pairs (ℓ, ℓ') indicating that the current analysis result has changed at the entry to the block ℓ and hence the information must be recomputed for ℓ'.

Solving AE Equations - Algorithm

```
W:=nil;
foreach (ℓ,ℓ') ∈ flow(S⋆) do W := cons((ℓ,ℓ'),W); od;
foreach ℓ ∈ labels(S⋆) do
  if ℓ ∈ init(S⋆) then
    AE∘(ℓ) := ∅
  else
    AE∘(ℓ) := AExp⋆
  fi
od
while W ≠ nil do
  (ℓ,ℓ') := head(W);
  W := tail(W);
  if (AE∘(ℓ)\kill_AE(B^ℓ)) ∪ gen_AE(B^ℓ) ⊉ AE∘(ℓ') then
    AE∘(ℓ') := AE∘(ℓ') ∩ (AE∘(ℓ)\kill_AE(B^ℓ)) ∪ gen_AE(B^ℓ);
    foreach ℓ'' with (ℓ',ℓ'') in flow(S⋆) do
      W := cons((ℓ',ℓ''),W);
    od
  fi
od
```


Common Subexpression Elimination (CSE)

The aim is to find computations that are always performed at least twice on a given execution path and to eliminate the second and later occurrences; it uses Available Expressions Analysis to determine the redundant computations.

Example:

[x := a+b]^1; [y := a∗x]^2; while [y > a+b]^3 do [a := a + 1]^4; [x := a + b]^5 od

- Expression a+b is computed at 1 and 5 and recomputation can be eliminated at 3.

The Optimization - CSE

Let $S_\star^N$ be the normalized form of $S_\star$ such that there is at most one operator on the right hand side of an assignment.

For each $[...a...]^\ell$ in $S_\star^N$ with $a \in \mathsf{AE}_\circ(\ell)$ do

- determine the set $\{[y_1 := a]^{\ell_1}, \ldots, [y_k := a]^{\ell_k}\}$ of elementary blocks in $S_\star^N$ "defining" a that reaches $[...a...]^\ell$
- create a fresh variable u and
  - replace each occurrence of $[y_i := a]^{\ell_i}$ with $[u := a]^{\ell_i}$;$[y_i := u]^{\ell'_i}$ for $1 \le i \le k$
  - replace $[...a...]^\ell$ with $[...u...]^\ell$

$[x := a]^{\ell'}$ reaches $[...a...]^\ell$ if there is a path in flow($S_\star^N$) from ℓ' to ℓ that does not contain *any* assignments with expression a on the right hand side and no variable of a is modified.

Computing the "reaches" Information

$[x := a]^{\ell'}$ reaches $[...a...]^\ell$ if there is a path in flow($S_\star^N$) from ℓ' to ℓ that does not contain *any* assignments with expression a on the right hand side and no variable of a is modified.

The set of elementary blocks that reaches $[...a...]^\ell$ can be computed as $\mathsf{reaches}_\circ(a, \ell)$ where

$$
\mathsf{reaches}_\circ(a,\ell) = \begin{cases} \emptyset & : \text{if } \ell = \mathsf{init}(S_\star)\\ \bigcup \mathsf{reaches}_\bullet(a,\ell') & : \text{otherwise} \end{cases}
$$

$$
\mathsf{reaches}_\bullet(a,\ell) = \begin{cases} \{B^\ell\} & : \text{if } B^\ell \text{ has the form} [x := a]^\ell \text{ and } x \notin \mathsf{FV}(a)\\ \emptyset & : \text{if } B^\ell \text{ has the form} [x := ...]^\ell \text{ and } x \in \mathsf{FV}(a)\\ \mathsf{reaches}_\circ(a,\ell) & : \text{otherwise} \end{cases}
$$

Example - CSE

Example:
$[x := a+b]^1; [y := a*x]^2;$ while $[y > a+b]^3$ do $[a := a+1]^4; [x := a+b]^5$ od

ℓ	$AE_\circ(\ell)$
1	$\emptyset$
2	{a+b}
3	{a+b}
4	{a+b}
5	$\emptyset$

reaches(a+b,3)=$\{[x := a+b]^1, [x := a+b]^5\}$

Result of CSE optimization wrt. reaches(a+b,3)

$[u := a+b]^{1'}; [x := u]^1; [y := a*x]^2;$ while $[y > u]^3$ do $[a := a+1]^4; [u := a+b]^{5'}; [x := u]^5$ od

Copy Analysis

The aim of Copy Analysis is to determine for each program point ℓ', which copy statements $[x := y]^\ell$ that still are relevant (i.e. neither x nor y have been redefined) when control reaches point ℓ'.

Example:
$[a := b]^1;$ if $[x > b]^2$ then $([y := a]^3)$ else $([b := b+1]^4; [y := a]^5)$; $[\text{skip}]^6$

ℓ	$C_\circ(\ell)$	$C_\bullet(\ell)$
1	$\emptyset$	{(a,b)}
2	{(a,b)}	{(a,b)}
3	{(a,b)}	{(y,a),(a,b)}
4	{(a,b)}	$\emptyset$
5	$\emptyset$	{(y,a)}
6	{(y,a)}	{(y,a)}

Copy Propagation (CP)

The aim is to find copy statements $[x := y]^{\ell_j}$ and eliminate them if possible
If x is used in $B^{\ell'}$ then x can be replaced by y in $B^{\ell'}$ provided that

- $[x := y]^{\ell_j}$ is the only kind of definition of x that reaches $B^{\ell'}$ – this information can be obtained from the def-use chain.
- on every path from ℓ_j to ℓ' (including paths going through ℓ' several times but only once through ℓ_j) there are no redefinitions of y; this can be detected by Copy Analysis.

Example 1

$[u := a+b]^{1'}; [x := u]^1; [y := a*x]^2;$ while $[y > u]^3$ do $[a := a+1]^4; [u := a+b]^{5'}; [x := u]^5$ od

becomes after CP

$[u := a+b]^{1'}; [y := a*u]^2;$ while $[y > u]^3$ do $[a := a+1]^4; [u := a+b]^{5'}; [x := u]^5$ od

The Optimization - CP

For each copy statement $[x := y]^{\ell_j}$ in $S_\star$ do

- determine the set $\{[...x...]^{\ell_1}, ..., [...x...]^{\ell_i}\}$,$1 \le i \le k$, of elementary blocks in $S_\star$ that uses $[x := y]^{\ell_j}$ – this can be computed from DU(x,ℓ_j)
- for each $[...x...]^{\ell_i}$ in this set determine whether $\{(x', y') \in C_\circ(\ell_i) \mid x' = x\} = \{(x, y)\}$; if so then $[x := y]$ is the only kind of definition of x that reaches ℓ_i from all ℓ_j.
- if this holds for all i ($1 \le i \le k$) then
  - remove $[x := y]^{\ell_j}$
  - replace $[...x...]^{\ell_i}$ with $[...y...]^{\ell_i}$ for $1 \le i \le k$.

Examples - CP

Example 2

$[a := 2]^1;$ if $[y > u]^2$ then $([a := a+1]^3; [x := a]^4;)$ else $([a := a*2]^5; [x := a]^6;)[y := y*x]^7;$

becomes after CP

$[a := 2]^1;$ if $[y > u]^2$ then $([a := a+1]^3;$;) else $([a := a*2]^5;$;)$[y := y*a]^7;$

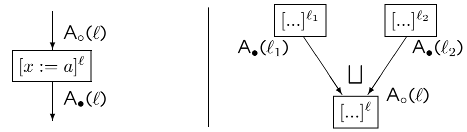
Example 3

$[a := 10]^1; [b := a]^2;$ while $[a > 1]^3$ do $[a := a-1]^4; [b := a]^5;$ od $[y := y*b]^6;$

becomes after CP

$[a := 10]^1;$; while $[a > 1]^3$ do $[a := a-1]^4;$; od $[y := y*a]^6;$

Summary: Forward Analyses



$$A_\circ(\ell) = \begin{cases} \iota_A & : \text{ if } \ell = \text{init}(S_\star) \\ \bigsqcup_A \{A_\bullet(\ell') \mid (\ell', \ell) \in \text{flow}(S_\star)\} & : \text{ otherwise} \end{cases}$$
$$A_\bullet(\ell) = (A_\circ(\ell) \backslash \text{kill}_A(B^\ell)) \cup \text{gen}_A(B^\ell) \quad \text{where } B^\ell \in \text{blocks}(S_\star)$$

where

Analysis	RD	AE
ι_A	$\{(x, ?) \mid x \in FV(S_\star)\}$	$\emptyset$
$\bigsqcup_A$	$\cup$	$\cap$

References

- Material for this 2nd lecture

  `www.complang.tuwien.ac.at/markus/optub.html`

- Book

  Flemming Nielson, Hanne Riis Nielson, Chris Hankin:

  Principles of Program Analysis.

  Springer, (2nd edition, 452 pages, ISBN 3-540-65410-0), 2005.
  - Chapter 1 (Introduction)
  - Chapter 2 (Data Flow Analysis)