Intra-Procedural Dataflow Analysis

Forward Analyses

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Formalising the Development

- the programming language of interest
  - abstract syntax
  - labelled program fragments
- abstract flow graphs
  - control and data flow between labelled program fragments
- extract equations from the program
  - specify the information to be compuated at entry and exit of labeled fragments
- compute the solution to the equations
  - work list algorithms
  - compute entry and exit information at entry and exit of labeled fragments

WHILE Language

- Syntactic categories

a	$\in$	AExp	artithmetic expressions
b	$\in$	BExp	boolean expressions
S	$\in$	Stmt	statements
x, y	$\in$	Var	variables
n	$\in$	Num	numerals
ℓ	$\in$	Lab	labels
op_a	$\in$	Op_a	arithmetic operators
op_b	$\in$	Op_b	boolean operators
op_r	$\in$	Op_r	relational operators

Abstract Syntax

$$
\begin{array}{lcl}
a & ::= & x \mid n \mid a_1\ op_a\ a_2 \\
b & ::= & \text{true} \mid \text{false} \mid \text{not}\ b \mid b_1\ op_b\ b_2 \mid a_1\ op_r\ a_2 \\
S & ::= & [\text{x:=a}]^\ell \mid [\text{skip}]^\ell \\
 & & \mid \text{if}\ [b]^\ell\ \text{then}\ S_1\ \text{else}\ S_2 \\
 & & \mid \text{while}[b]^\ell\ \text{do}\ S\ \text{od} \\
 & & \mid S_1; S_2
\end{array}
$$

Assignments and tests are (uniquely) labelled to allow analyses to refer to these program fragments – the labels correspond to pointers into the syntax tree. We use abstract syntax and insert paranthesis to disambiguate syntax.

We will often refer to labelled fragments as *elementary blocks*.

Auxiliary Functions for Flow Graphs

labels(S)	set of nodes of flow graphs of S
init(S)	initial node of flow graph of S; the unique node where execution of program starts
final(S)	final nodes of flow graph for S; set of nodes where program execution may terminate
flow(S)	edges of flow graphs for S (used for forward analyses)
flow^R(S)	reverse edges of flow graphs for S (used for backward analyses)
blocks(S)	set of elementary blocks in a flow graph

Computing the Information (1)

S	labels(S)	init(S)	final(S)
$[x := a]^\ell$	$\{\ell\}$	ℓ	$\{\ell\}$
$[\text{skip}]^\ell$	$\{\ell\}$	ℓ	$\{\ell\}$
$S_1; S_2$	labels(S_1) $\cup$ labels(S_2)	init(S_1)	final(S_2)
if $[b]^\ell$ then (S_1) else (S_2)	$\{\ell\}$ $\cup$ labels(S_1) $\cup$ labels(S_2)	ℓ	final(S_1) $\cup$ final(S_2)
while $[b]^\ell$ do S od	$\{\ell\} \cup$ labels(S)	ℓ	$\{\ell\}$

Computing the Information (2)

S	$\mathsf{flow}(S)$	$\mathsf{blocks}(S)$
$[x := a]^\ell$	$\emptyset$	$\{[x := a]^\ell\}$
$[\mathsf{skip}]^\ell$	$\emptyset$	$\{[\mathsf{skip}]^\ell\}$
$S_1; S_2$	$\mathsf{flow}(S_1) \cup \mathsf{flow}(S_2) \cup \{(\ell, \mathsf{init}(S_2)) \mid \ell \in \mathsf{final}(S_1)\}$	$\mathsf{blocks}(S_1) \cup \mathsf{blocks}(S_2)$
if $[b]^\ell$ then (S_1) else (S_2)	$\mathsf{flow}(S_1) \cup \mathsf{flow}(S_2) \cup \{(\ell, \mathsf{init}(S_1)), (\ell, \mathsf{init}(S_2))\}$	$\{[b]^\ell\} \cup \mathsf{blocks}(S_1) \cup \mathsf{blocks}(S_2)$
while $[b]^\ell$ do S od	$\{(\ell, \mathsf{init}(S))\} \cup \mathsf{flow}(S) \cup \{(\ell', \ell) \mid \ell' \in \mathsf{final}(S)\}$	$\{[b]^\ell\} \cup \mathsf{blocks}(S)$

$$\mathsf{flow}^R(S) = \{(\ell, \ell') \mid (\ell', \ell) \in \mathsf{flow}(S)\}$$

Program of Interest

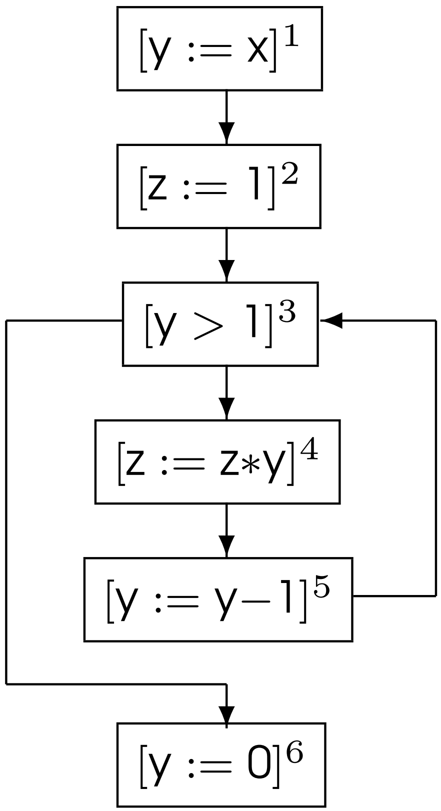
We shall use the notation

- $S_\star$ to represent the program being analyzed (the "top level" statement)
- Lab$_\star$ to represent the labels (labels$(S_\star)$) appearing in $S_\star$
- Var$_\star$ to represent the variables (FV$(S_\star)$) appearing in $S_\star$
- Blocks$_\star$ to represent the elementary blocks (blocks$(S_\star)$) occuring in $S_\star$
- AExp$_\star$ to represent the set of *non-trivial* arithmetic subexpressions in $S_\star$; an expression is trivial if it is a single variable or constant
- AExp(a), AExp(b) to refer to the set of non-trivial arithmetic subexpressions of a given arithmetic, respectively boolean, expression

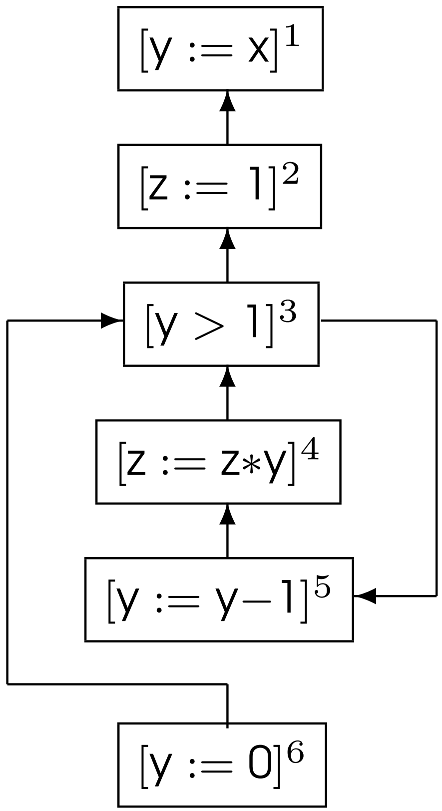
Example Flow Graphs

Example:

$[\mathsf{y} := \mathsf{x}]^1; [\mathsf{z} := 1]^2; \mathsf{while}\ [\mathsf{y} > 1]^3\ \mathsf{do}\ [\mathsf{z} := \mathsf{z} * \mathsf{y}]^4; [\mathsf{y} := \mathsf{y} - 1]^5\ \mathsf{od}; [\mathsf{y} := 0]^6$



$\mathsf{flow}(S_\star) = \{(1,2),(2,3),(3,4),(4,5),(5,3),(3,6)\}$



$\mathsf{flow}^R(S_\star) = \{(6,3),(3,5),(5,4),(4,3),(3,2),(2,1)\}$

Example

Example:

$[y := x]^1; [z := 1]^2; \text{while } [y > 1]^3 \text{ do } [z := z * y]^4; [y := y - 1]^5 \text{ od}; [y := 0]^6$

$$
\begin{aligned}
\text{labels}(S_\star) &= \{1, 2, 3, 4, 5, 6\} \\
\text{init}(S_\star) &= 1 \\
\text{final}(S_\star) &= \{6\} \\
\text{flow}(S_\star) &= \{(1,2), (2,3), (3,4), (4,5), (5,3), (3,6)\} \\
\text{flow}^R(S_\star) &= \{(6,3), (3,5), (5,4), (4,3), (3,2), (2,1)\} \\
\text{blocks}(S_\star) &= \{[y := x]^1, [z := 1]^2, [y > 1]^3, \\
&\quad [z := z * y]^4, [y := y - 1]^5, [y := 0]^6\}
\end{aligned}
$$

Simplifying Assumptions

The program of interest $S_\star$ is often assumed to satisfy:

- $S_\star$ has isolated entries if there are no edges leading into init$(S_\star)$:

$$\forall \ell : (\ell, \mathsf{init}(S_\star)) \notin \mathsf{flow}(S_\star)$$

- $S_\star$ has isolated exits if there are no edges leading out of labels in final$(S_\star)$:

$$\forall \ell \in \mathsf{final}(S_\star), \forall \ell' : (\ell, \ell') \notin \mathsf{flow}(S_\star)$$

- $S_\star$ is label consistent if

$$\forall B_1^{\ell_1}, B_2^{\ell_2} \in \mathsf{blocks}(S_\star) : \ell_1 = \ell_2 \rightarrow B_1 = B_2$$

  This holds if $S_\star$ is uniquely labelled.

Reaching Definitions Analysis

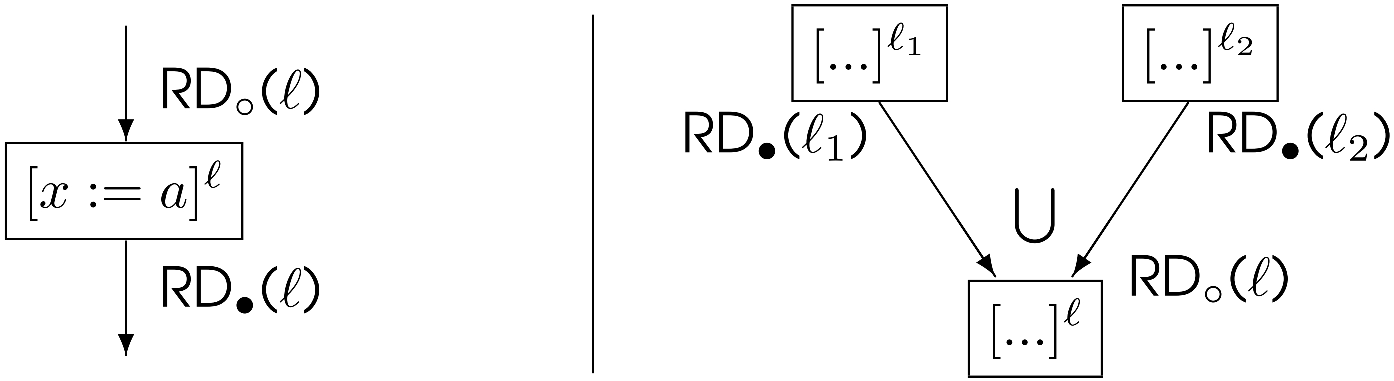
The aim of the Reaching Definitions Analysis is to determine

For each program point, which assignments *may* have been made and not overwritten, when program execution reaches this point along some path.

Example:
$[y := x]^1; [z := 1]^2;$ while $[y > 1]^3$ do $[z := z * y]^4; [y := y - 1]^5$ od; $[y := 0]^6$

- The assignments labelled 1,2,4,5 reach the entry at 4.
- Only the assignments labelled 1,4,5 reach the entry at 5.

Basic Idea



Analysis information: $RD_\circ(\ell)$,$RD_\bullet(\ell)$: $Lab_\star \rightarrow \mathcal{P}(Var_\star \times Lab_\star^?)$

- $RD_\circ(\ell)$: the definitions that reach entry of block ℓ.
- $RD_\bullet(\ell)$: the definitions that reach exit of block ℓ.

Analysis properties:

- Direction: forward
- May analysis with combination operator $\bigcup$

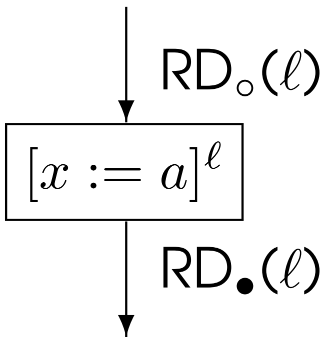
Analysis of Elementary Blocks

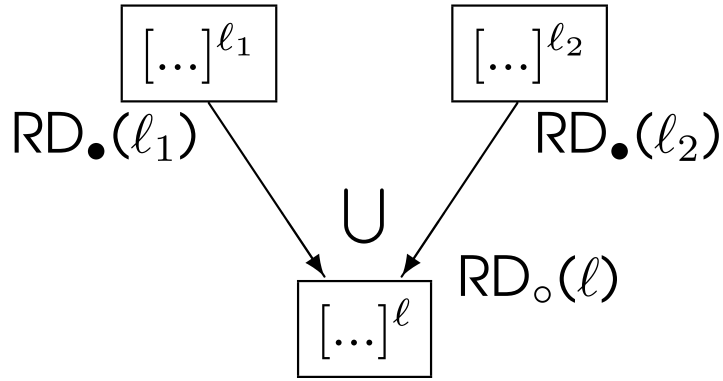
$$\begin{array}{rcl}
\mathsf{kill}_{\mathsf{RD}}([x := a]^\ell) & = & \{(x,?)\} \cup \{(x,\ell') \mid B^{\ell'} \text{is an assignment to } x\} \\
\mathsf{kill}_{\mathsf{RD}}([\mathsf{skip}]^\ell) & = & \emptyset \\
\mathsf{kill}_{\mathsf{RD}}([b]^\ell) & = & \emptyset \\
\mathsf{gen}_{\mathsf{RD}}([x := a]^\ell) & = & \{(x,\ell)\} \\
\mathsf{gen}_{\mathsf{RD}}([\mathsf{skip}]^\ell) & = & \emptyset \\
\mathsf{gen}_{\mathsf{RD}}([b]^\ell) & = & \emptyset
\end{array}$$

Example:
$[\mathsf{x} := \mathsf{y}]^1; [\mathsf{x} := \mathsf{x} + 3]^2;$

- $\mathsf{kill}_{\mathsf{RD}}([\mathsf{x} := \mathsf{y}]^1) = \{(x,?)\} \cup \{(x,1),(x,2)\}$
- $\mathsf{gen}_{\mathsf{RD}}([\mathsf{x} := \mathsf{y}]^1) = \{(x,1)\}$

Analysis of the Program





$$RD_\circ(\ell) = \begin{cases} \{(x,?) \mid x \in FV(S_\star)\} & : \text{ if } \ell = \text{init}(S_\star) \\ \bigcup\{RD_\bullet(\ell') \mid (\ell',\ell) \in \text{flow}(S_\star)\} & : \text{ otherwise} \end{cases}$$

$$RD_\bullet(\ell) = (RD_\circ(\ell) \backslash \text{kill}_{RD}(B^\ell)) \cup \text{gen}_{RD}(B^\ell) \quad \text{where } B^\ell \in \text{blocks}(S_\star)$$

Example

Example:

[y := x]^1; [z := 1]^2; while [y > 1]^3 do [z := z $*$ y]^4; [y := y $-$ 1]^5 od; [y := 0]^6

Equations: Let $S_1 = \{(y,?),(y,1),(y,5),(y,6)\}$, $S_2 = \{(z,?),(z,2),(z,4)\}$

$$
\begin{array}{lcllcl}
\text{RD}_\circ(1) & = & \{(x,?),(y,?),(z,?)\} & \text{RD}_\bullet(1) & = & \text{RD}_\circ(1) \setminus S_1 \cup \{(y,1)\} \\
\text{RD}_\circ(2) & = & \text{RD}_\bullet(1) & \text{RD}_\bullet(2) & = & \text{RD}_\circ(2) \setminus S_2 \cup \{(z,2)\} \\
\text{RD}_\circ(3) & = & \text{RD}_\bullet(2) \cup \text{RD}_\bullet(5) & \text{RD}_\bullet(3) & = & \text{RD}_\circ(3) \\
\text{RD}_\circ(4) & = & \text{RD}_\bullet(3) & \text{RD}_\bullet(4) & = & \text{RD}_\circ(4) \setminus S_2 \cup \{(z,4)\} \\
\text{RD}_\circ(5) & = & \text{RD}_\bullet(4) & \text{RD}_\bullet(5) & = & \text{RD}_\circ(5) \setminus S_1 \cup \{(y,5)\} \\
\text{RD}_\circ(6) & = & \text{RD}_\bullet(3) & \text{RD}_\bullet(6) & = & \text{RD}_\circ(6) \setminus S_1 \cup \{(y,6)\}
\end{array}
$$

ℓ	$\text{RD}_\circ(\ell)$	$\text{RD}_\bullet(\ell)$
1	{(x,?),(y,?),(z,?)}	{(x,?),(y,1),(z,?)}
2	{(x,?),(y,1),(z,?)}	{(x,?),(z,2),(y,1)}
3	{(x,?),(z,4),(z,2),(y,5),(y,1)}	{(x,?),(z,4),(z,2),(y,5),(y,1)}
4	{(x,?),(z,4),(z,2),(y,5),(y,1)}	{(z,4),(x,?),(y,5),(y,1)}
5	{(z,4),(x,?),(y,5),(y,1)}	{(z,4),(x,?),(y,5)}
6	{(x,?),(z,4),(z,2),(y,5),(y,1)}	{(z,4),(x,?),(z,2),(y,6)}

Solving RD Equations

Input

- a set of reaching definitions equations

Output

- the *least solution* to the equations: $\mathrm{RD}_\circ$

Data structures

- The current analysis result for block entries: $\mathrm{RD}_\circ$
- The worklist W: a list of pairs (ℓ, ℓ') indicating that the current analysis result has changed at the entry to the block ℓ and hence the information must be recomputed for ℓ'.

Solving RD Equations - Algorithm

```
W:=nil;
foreach (ℓ,ℓ') ∈ flow(S⋆) do W := cons((ℓ,ℓ'),W); od;
foreach ℓ ∈ labels(S⋆) do
   if ℓ ∈ init(S⋆) then
      RD∘(ℓ) := {(x,?) | x ∈ FV(S⋆)}
   else
      RD∘(ℓ) := ∅
   fi
od
while W ≠ nil do
   (ℓ,ℓ') := head(W);
   W := tail(W);
   if (RD∘(ℓ)\kill_RD(B^ℓ)) ∪ gen_RD(B^ℓ) ⊈ RD∘(ℓ') then
      RD∘(ℓ') := RD∘(ℓ') ∪ (RD∘(ℓ)\kill_RD(B^ℓ)) ∪ gen_RD(B^ℓ);
      foreach ℓ'' with (ℓ',ℓ'') in flow(S⋆) do
         W := cons((ℓ',ℓ''),W);
      od
   fi
od
```

Use-Definition and Definition-Use Chains

- Use-Definition chains or *ud* chains

  each use of a variable is linked to all assignments that *reach* it

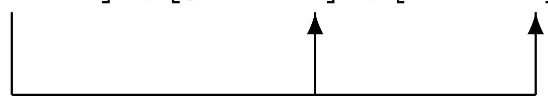
  $[\mathsf{x} := 0]^1; [\mathsf{x} := 5]^2; [\mathsf{y} := \mathsf{x}]^3; [\mathsf{z} := \mathsf{x}]^4$

- Definition-Use chains or *du* chains

  each assignment of a variable is linked to all uses of it

  $[\mathsf{x} := 0]^1; [\mathsf{x} := 5]^2; [\mathsf{y} := \mathsf{x}]^3; [\mathsf{z} := \mathsf{x}]^4$

UD/DU Chains - Defined via RDs

$$\text{UD,DU} : \text{Var}_\star \times \text{Lab}_\star \rightarrow \mathcal{P}(\text{Lab}_\star)$$

are defined by

$$\text{UD}(x,\ell) = \begin{cases} \{\ell' \mid (x,\ell') \in \text{RD}_\circ(\ell)\} & : \text{ if } x \in \text{used}(B^\ell) \\ \emptyset & : \text{ otherwise} \end{cases}$$

where used($[x := a]^\ell$) = FV(a), used($[b]^\ell$) = FV(b), used($[\text{skip}]^\ell$) = $\emptyset$

and

$$\text{DU}(x,\ell) = \{\ell' \mid \ell \in \text{UD}(x,\ell')\}$$

Available Expressions Analysis

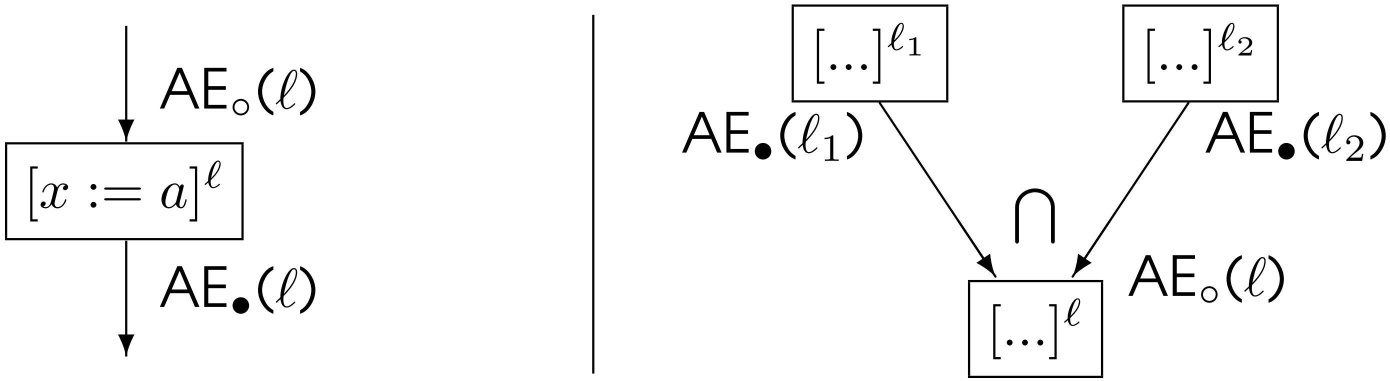
The aim of the Available Expressions Analysis is to determine

For each program point, which expressions *must* have already been computed, and not later modified, on all paths to the program point.

Example:
$[x := a{+}b]^1; [y := a{*}x]^2;$ while $[y > a{+}b]^3$ do $[a := a + 1]^4; [x := a + b]^5$ od

- No expression is available at the start of the program
- An expression is considered available if no path kills it
- The expression a+b is available every time execution reaches the test in the loop at 3.

Basic Idea



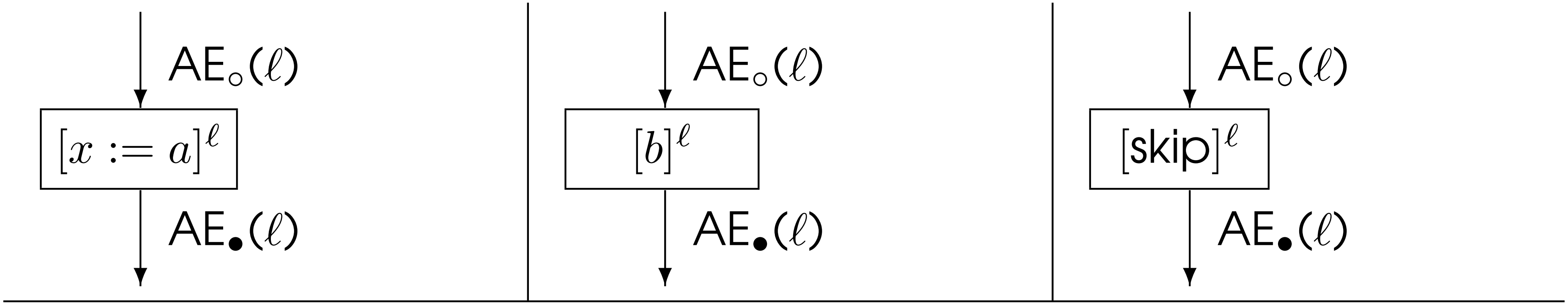
Analysis information: $\text{AE}_\circ(\ell)$,$\text{AE}_\bullet(\ell)$: $\text{Lab}_\star \rightarrow \mathcal{P}(\text{AExp}_\star)$

- $\text{AE}_\circ(\ell)$: the expressions that have been comp. at entry of block ℓ.
- $\text{AE}_\bullet(\ell)$: the expressions that have been comp. at exit of block ℓ.

Analysis properties:

- Direction: forward
- Must analysis with combination operator $\bigcap$

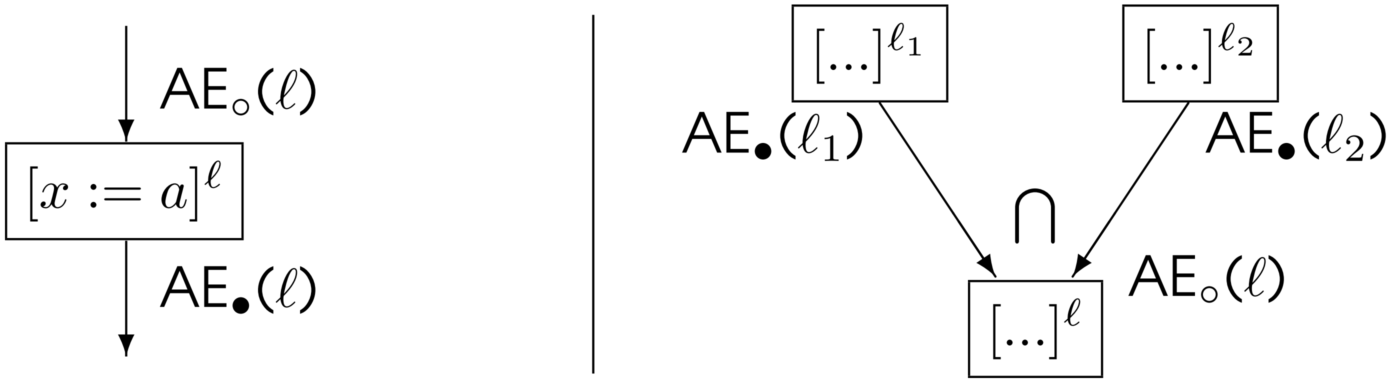
Analysis of Elementary Blocks



$$
\begin{aligned}
\mathsf{kill}_{\mathsf{AE}}([x := a]^\ell) &= \{a' \in \mathsf{AExp}_\star \mid x \in FV(a')\}\} \\
\mathsf{kill}_{\mathsf{AE}}([\mathsf{skip}]^\ell) &= \emptyset \\
\mathsf{kill}_{\mathsf{AE}}([b]^\ell) &= \emptyset \\
\mathsf{gen}_{\mathsf{AE}}([x := a]^\ell) &= \{a' \in \mathsf{AExp}(a) \mid x \notin FV(a')\} \\
\mathsf{gen}_{\mathsf{AE}}([\mathsf{skip}]^\ell) &= \emptyset \\
\mathsf{gen}_{\mathsf{AE}}([b]^\ell) &= \mathsf{AExp}(b)
\end{aligned}
$$

$$
\mathsf{AE}_\bullet(\ell) \quad = \quad (\mathsf{AE}_\circ(\ell) \backslash \mathsf{kill}_{\mathsf{AE}}(B^\ell)) \cup \mathsf{gen}_{\mathsf{AE}}(B^\ell) \quad \text{where } B^\ell \in \mathsf{blocks}(S_\star)
$$

Analysis of the Program



$$AE_\circ(\ell) = \begin{cases} \emptyset & : \text{ if } \ell = \mathsf{init}(S_\star) \\ \bigcap\{AE_\bullet(\ell') \mid (\ell', \ell) \in \mathsf{flow}(S_\star)\} & : \text{ otherwise} \end{cases}$$

$$AE_\bullet(\ell) = (AE_\circ(\ell) \backslash \mathsf{kill}_{AE}(B^\ell)) \cup \mathsf{gen}_{AE}(B^\ell) \quad \text{where } B^\ell \in \mathsf{blocks}(S_\star)$$

Example

Example:

[x := a+b]^1; [y := a∗x]^2; while [y > a+b]^3 do [a := a + 1]^4; [x := a + b]^5 od

Equations:

$$
\begin{array}{lcl}
AE_\circ(1) & = & \emptyset \\
AE_\circ(2) & = & AE_\bullet(1) \\
AE_\circ(3) & = & AE_\bullet(2) \cap AE_\bullet(5) \\
AE_\circ(4) & = & AE_\bullet(3) \\
AE_\circ(5) & = & AE_\bullet(4)
\end{array}
\qquad
\begin{array}{lcl}
AE_\bullet(1) & = & AE_\circ(1) \setminus \{a*x\} \cup \{a+b\} \\
AE_\bullet(2) & = & AE_\circ(2) \setminus \emptyset \cup \{a*x\} \\
AE_\bullet(3) & = & AE_\circ(3) \setminus \emptyset \cup \{a+b\} \\
AE_\bullet(4) & = & AE_\circ(4) \setminus \{a+b, a*x, a+1\} \cup \emptyset \\
AE_\bullet(5) & = & AE_\circ(5) \setminus \{a*x\} \cup \{a+b\}
\end{array}
$$

ℓ	$AE_\circ(\ell)$	$AE_\bullet(\ell)$
1	∅	{a+b}
2	{a+b}	{a+b,a*x}
3	{a+b}	{a+b }
4	{a+b}	∅
5	∅	{a+b}

Solving AE Equations

Input

- a set of available expressions equations

Output

- the *largest solution* to the equations: $\mathsf{AE}_\circ$

Data structures

- The current analysis result for block entries: $\mathsf{AE}_\circ$
- The worklist W: a list of pairs (ℓ, ℓ') indicating that the current analysis result has changed at the entry to the block ℓ and hence the information must be recomputed for ℓ'.

Solving AE Equations - Algorithm

```
W:=nil;
foreach (ℓ,ℓ') ∈ flow(S⋆) do W := cons((ℓ,ℓ'),W); od;
foreach ℓ ∈ labels(S⋆) do
   if ℓ ∈ init(S⋆) then
      AE∘(ℓ) := ∅
   else
      AE∘(ℓ) := AExp⋆
   fi
od
while W ≠ nil do
   (ℓ,ℓ') := head(W);
   W := tail(W);
   if (AE∘(ℓ)\kill_AE(B^ℓ)) ∪ gen_AE(B^ℓ) ⊉ AE∘(ℓ') then
      AE∘(ℓ') := AE∘(ℓ') ∩ (AE∘(ℓ)\kill_AE(B^ℓ)) ∪ gen_AE(B^ℓ);
      foreach ℓ'' with (ℓ',ℓ'') in flow(S⋆) do
         W := cons((ℓ',ℓ''),W);
      od
   fi
od
```

Common Subexpression Elimination (CSE)

The aim is to find computations that are always performed at least twice on a given execution path and to eliminate the second and later occurrences; it uses Available Expressions Analysis to determine the redundant computations.

Example:
$[\mathsf{x} := \mathsf{a}{+}\mathsf{b}]^1; [\mathsf{y} := \mathsf{a}{*}\mathsf{x}]^2;$ while $[\mathsf{y} > \mathsf{a}{+}\mathsf{b}]^3$ do $[\mathsf{a} := \mathsf{a} + 1]^4; [\mathsf{x} := \mathsf{a} + \mathsf{b}]^5$ od

- Expression a+b is computed at 1 and 5 and recomputation can be eliminated at 3.

The Optimization - CSE

Let $S_\star^N$ be the normalized form of $S_\star$ such that there is at most one operator on the right hand side of an assignment.

For each $[...a...]^\ell$ in $S_\star^N$ with $a \in \mathsf{AE}_\circ(\ell)$ do

- determine the set $\{[y_1 := a]^{\ell_1}, \ldots, [y_k := a]^{\ell_k}\}$ of elementary blocks in $S_\star^N$ "defining" a that reaches $[...a...]^\ell$
- create a fresh variable u and
  - replace each occurrence of $[y_i := a]^{\ell_i}$ with $[u := a]^{\ell_i};[y_i := u]^{\ell_i'}$ for $1 \leq i \leq k$
  - replace $[...a...]^\ell$ with $[...u...]^\ell$

$[x := a]^{\ell'}$ reaches $[...a...]^\ell$ if there is a path in flow$(S_\star^N)$ from ℓ' to ℓ that does not contain *any* assignments with expression a on the right hand side and no variable of a is modified.

Computing the "reaches" Information

$[x := a]^{\ell'}$ reaches $[...a...]^{\ell}$ if there is a path in flow$(S_\star^N)$ from ℓ' to ℓ that does not contain *any* assignments with expression a on the right hand side and no variable of a is modified.

The set of elementary blocks that reaches $[...a...]^{\ell}$ can be computed as reaches$_\circ(a, \ell)$ where

$$\text{reaches}_\circ(a,\ell) = \begin{cases} \emptyset & : \text{ if } \ell = \text{init}(S_\star) \\ \bigcup \text{reaches}_\bullet(a,\ell') & : \text{ otherwise} \end{cases}$$

$$\text{reaches}_\bullet(a,\ell) = \begin{cases} \{B^\ell\} & : \text{ if } B^\ell \text{ has the form} [x := a]^\ell \text{ and } x \notin \text{FV}(a) \\ \emptyset & : \text{ if } B^\ell \text{ has the form} [x := ...]^\ell \text{ and } x \in \text{FV}(a) \\ \text{reaches}_\circ(a,\ell) & : \text{ otherwise} \end{cases}$$

Example - CSE

Example:

[x := a+b]^1; [y := a$*$x]^2; while [y $>$ a+b]^3 do [a := a $+$ 1]^4; [x := a $+$ b]^5 od

ℓ	$AE_\circ(\ell)$
1	$\emptyset$
2	{a+b}
3	{a+b}
4	{a+b}
5	$\emptyset$

reaches(a+b,3)=$\{[x := a + b]^1, [x := a + b]^5\}$

Result of CSE optimization wrt. reaches(a+b,3)

[u := a+b]$^{1'}$; [x := u]^1; [y := a$*$x]^2; while [y $>$ u]^3 do [a := a $+$ 1]^4; [u := a $+$ b]$^{5'}$; [x := u]^5 od

Copy Analysis

The aim of Copy Analysis is to determine for each program point ℓ', which copy statements $[x := y]^\ell$ that still are relevant (i.e. neither x nor y have been redefined) when control reaches point ℓ'.

Example:
$[\text{a} := \text{b}]^1; \text{if } [\text{x} > \text{b}]^2 \text{ then } ([\text{y} := \text{a}]^3) \text{ else } ([\text{b} := \text{b} + 1]^4; [\text{y} := \text{a}]^5); [\text{skip}]^6$

ℓ	$C_\circ(\ell)$	$C_\bullet(\ell)$
1	$\emptyset$	{(a,b)}
2	{(a,b)}	{(a,b)}
3	{(a,b)}	{(y,a),(a,b)}
4	{(a,b)}	$\emptyset$
5	$\emptyset$	{(y,a)}
6	{(y,a)}	{(y,a)}

Copy Propagation (CP)

The aim is to find copy statements $[x := y]^{\ell_j}$ and eliminate them if possible

If x is used in $B^{\ell'}$ then x can be replaced by y in $B^{\ell'}$ provided that

- $[x := y]^{\ell_j}$ is the only kind of definition of x that reaches $B^{\ell'}$ – this information can be obtained from the def-use chain.
- on every path from ℓ_j to ℓ' (including paths going through ℓ' several times but only once through ℓ_j) there are no redefinitions of y; this can be detected by Copy Analysis.

Example 1

$[\mathsf{u} := \mathsf{a{+}b}]^{1'}; [\mathsf{x} := \mathsf{u}]^{1}; [\mathsf{y} := \mathsf{a{*}x}]^{2};$ while $[\mathsf{y} > \mathsf{u}]^{3}$ do $[\mathsf{a} := \mathsf{a} + 1]^{4}; [\mathsf{u} := \mathsf{a} + \mathsf{b}]^{5'}; [\mathsf{x} := \mathsf{u}]^{5}$ od

becomes after CP

$[\mathsf{u} := \mathsf{a{+}b}]^{1'}; [\mathsf{y} := \mathsf{a{*}u}]^{2};$ while $[\mathsf{y} > \mathsf{u}]^{3}$ do $[\mathsf{a} := \mathsf{a} + 1]^{4}; [\mathsf{u} := \mathsf{a} + \mathsf{b}]^{5'}; [\mathsf{x} := \mathsf{u}]^{5}$ od

The Optimization - CP

For each copy statement $[x := y]^{\ell_j}$ in $S_\star$ do

- determine the set $\{[...x...]^{\ell_1}, ..., [...x...]^{\ell_i}\}$,$1 \leq i \leq k$, of elementary blocks in $S_\star$ that uses $[x := y]^{\ell_j}$ – this can be computed from DU(x,ℓ_j)
- for each $[...x...]^{\ell_i}$ in this set determine whether $\{(x', y') \in \mathsf{C}_\circ(\ell_i) \mid x' = x\} = \{(x, y)\}$; if so then $[x := y]$ is the only kind of definition of x that reaches ℓ_i from all ℓ_j.
- if this holds for all i ($1 \leq i \leq k$) then
  - remove $[x := y]^{\ell_j}$
  - replace $[...x...]^{\ell_i}$ with $[...y...]^{\ell_i}$ for $1 \leq i \leq k$.

Examples - CP

Example 2

$[a := 2]^1; \text{if } [y > u]^2 \text{ then } ([a := a + 1]^3; [x := a]^4;) \text{ else } ([a := a * 2]^5; [x := a]^6;) [y := y * x]^7;$

becomes after CP

$[a := 2]^1; \text{if } [y > u]^2 \text{ then } ([a := a + 1]^3; \quad ;) \text{ else } ([a := a * 2]^5; \quad ;) [y := y * a]^7;$

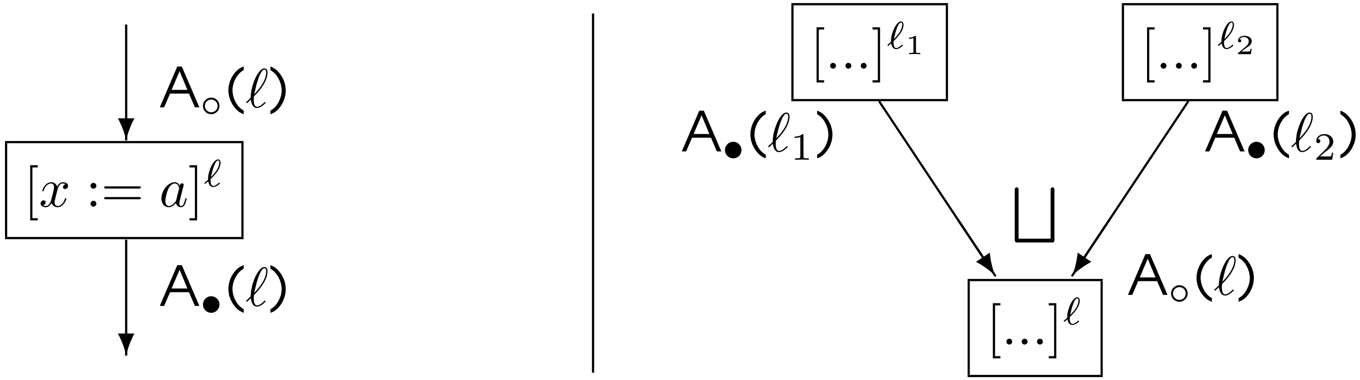
Example 3

$[a := 10]^1; [b := a]^2; \text{while } [a > 1]^3 \text{ do } [a := a - 1]^4; [b := a]^5; \text{ od } [y := y * b]^6;$

becomes after CP

$[a := 10]^1; \quad ; \text{while } [a > 1]^3 \text{ do } [a := a - 1]^4; \quad ; \text{ od } [y := y * a]^6;$

Summary: Forward Analyses



$$A_\circ(\ell) = \begin{cases} \iota_A & : \text{ if } \ell = \mathsf{init}(S_\star) \\ \bigsqcup_A \{A_\bullet(\ell') \mid (\ell', \ell) \in \mathsf{flow}(S_\star)\} & : \text{ otherwise} \end{cases}$$

$$A_\bullet(\ell) = (A_\circ(\ell) \setminus \mathsf{kill}_A(B^\ell)) \cup \mathsf{gen}_A(B^\ell) \quad \text{where } B^\ell \in \mathsf{blocks}(S_\star)$$

where

Analysis	RD	AE
ι_A	$\{(x, ?) \mid x \in FV(S_\star)\}$	$\emptyset$
$\bigsqcup_A$	$\cup$	$\cap$

References

- Material for this 2nd lecture

  `www.complang.tuwien.ac.at/markus/optub.html`

- Book

  Flemming Nielson, Hanne Riis Nielson, Chris Hankin:

  Principles of Program Analysis.

  Springer, (2nd edition, 452 pages, ISBN 3-540-65410-0), 2005.
  - Chapter 1 (Introduction)
  - Chapter 2 (Data Flow Analysis)