Optimizing Compilers

Inter-Procedural Dataflow Analysis

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Syntax

$$
\begin{array}{lll}
P_\star & ::= & \text{begin } D_\star\ S_\star \text{ end} \\
D & ::= & D; D \mid \text{proc } p(\text{val } x;\ \text{res } y) \text{ is}^{\ell_n}\ S \text{ end}^{\ell_x} \\
S & ::= & \ldots \mid [\text{call } p(a, z)]^{\ell_c}_{\ell_r}
\end{array}
$$

Labeling scheme

- procedure declarations

  ℓ_n: for entering the body

  ℓ_x: for exiting the body

- procedure calls

  ℓ_c: for the call

  ℓ_r: for the return

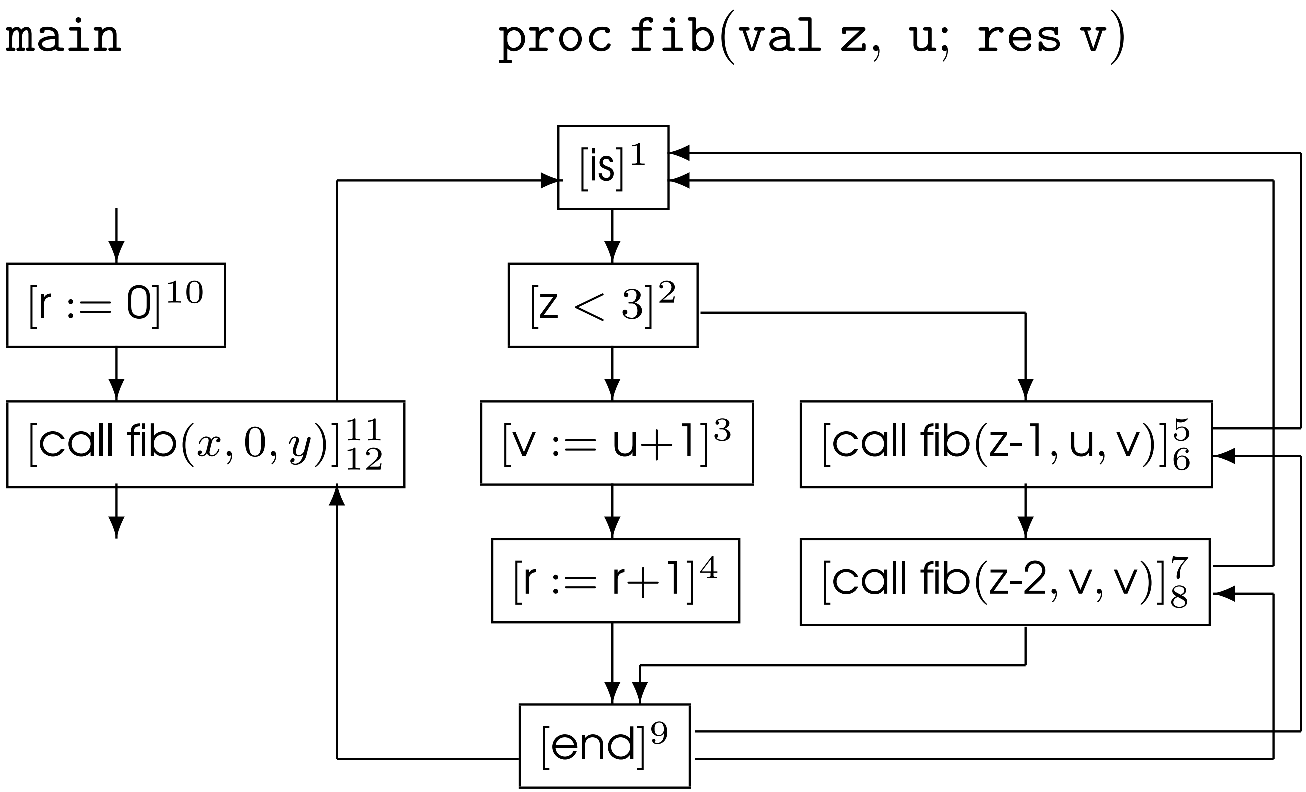
Analysing Procedures

- We consider procedures with call-by-value and call-by-result parameters.

Example:

```
begin
   proc fib(val z,u; res v) is
      if z<3 then
         (v:=u+1; r:=r+1)
      else (
         call fib (z-1,u,v);
         call fib (z-2,v,v)
         )
   end;
r:=0;
call fib(x,0,y)
end
```

Example Flow Graph



Flow Graph for Procedures

	$[\text{call } p(a,z)]^{\ell_c}_{\ell_r}$	proc p(val x; res y) is$^{\ell_n}$ S end$^{\ell_x}$
init	ℓ_c	ℓ_n
final	$\{\ell_r\}$	$\{\ell_x\}$
blocks	$\{[\text{call } p(a,z)]^{\ell_c}_{\ell_r}\}$	$\{\text{is}^{\ell_n}\} \cup \text{blocks}(S) \cup \{\text{end}^{\ell_x}\}$
labels	$\{\ell_c, \ell_r\}$	$\{\ell_c, \ell_r\} \cup \text{labels}(S)$
flow	$\{(\ell_c; \ell_n), (\ell_x; \ell_r)\}$	$\{(\ell_n, \text{init}(S))\} \cup \text{flow}(S) \cup \{\ell, \ell_x) \mid \ell \in \text{final}(S))\}$

- $(\ell_c; \ell_n)$ is the flow corresponding to calling a procedure at ℓ_c and entering the procedure body at ℓ_n and
- $(\ell_x; \ell_r)$ is the flow corresponding to exiting a procedure body at ℓ_x and returning to the call at ℓ_r.

Naive Formulation

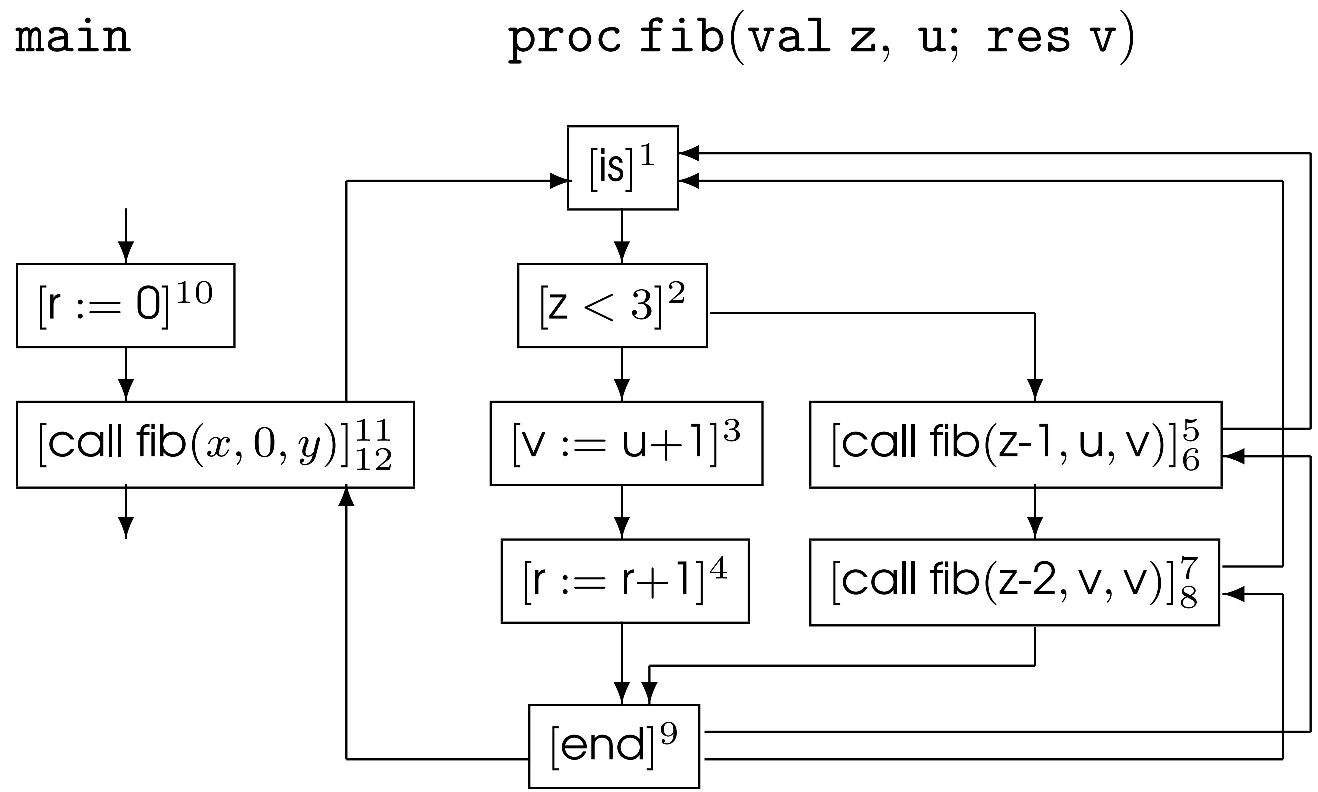
Treat the three kinds of flow, (ℓ_1, ℓ_2), $(\ell_c; \ell_n)$, $(\ell_x; \ell_r)$ in the same way.

Equation system:

$$
\begin{array}{rcl}
A_\circ(\ell) & = & \bigsqcup\{A_\bullet(\ell') \mid (\ell', \ell) \in F \vee (\ell'; \ell) \in F\} \sqcup \iota_E^\ell \\
A_\bullet(\ell) & = & f_\ell^A(A_\circ(\ell))
\end{array}
$$

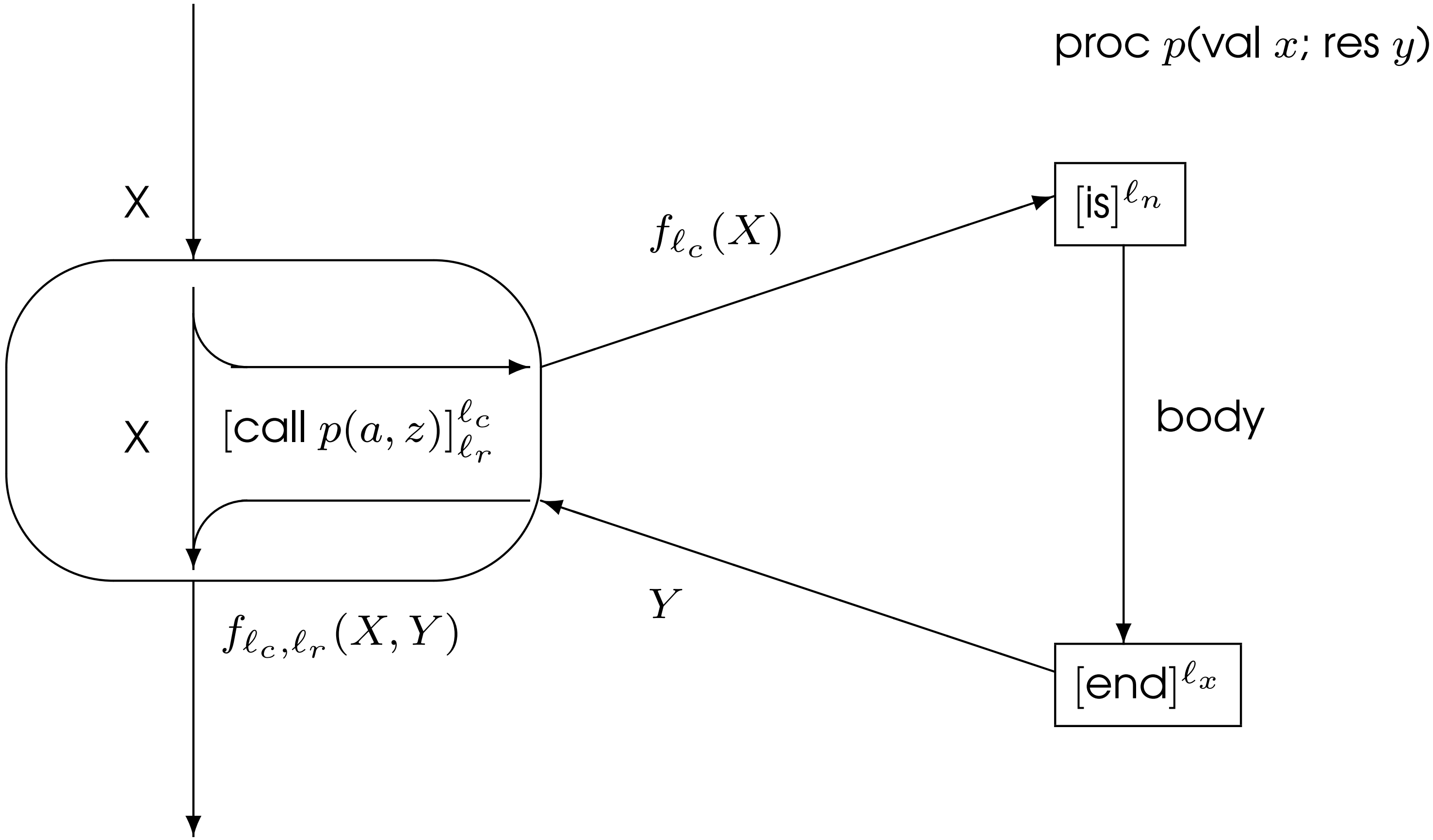
- both procedure calls $(\ell_c; \ell_n)$ and procedure returns $(\ell_x; \ell_r)$ are treated like "goto's".
- there is no mechanism for ensuring that information flowing along $(\ell_c; \ell_n)$ flows back along $(\ell_x; \ell_r)$ to the *same* call
- intuitively, the equation system considers a much too large set of "paths" through the program and hence will be grossly imprecise (although formally on the safe side)

Matching Procedure Entries and Exits



We want to overcome the shortcoming of the naive formulation by restricting attention to paths that have the proper nesting of procedure calls and exits.

General Formulation: Calls and Returns



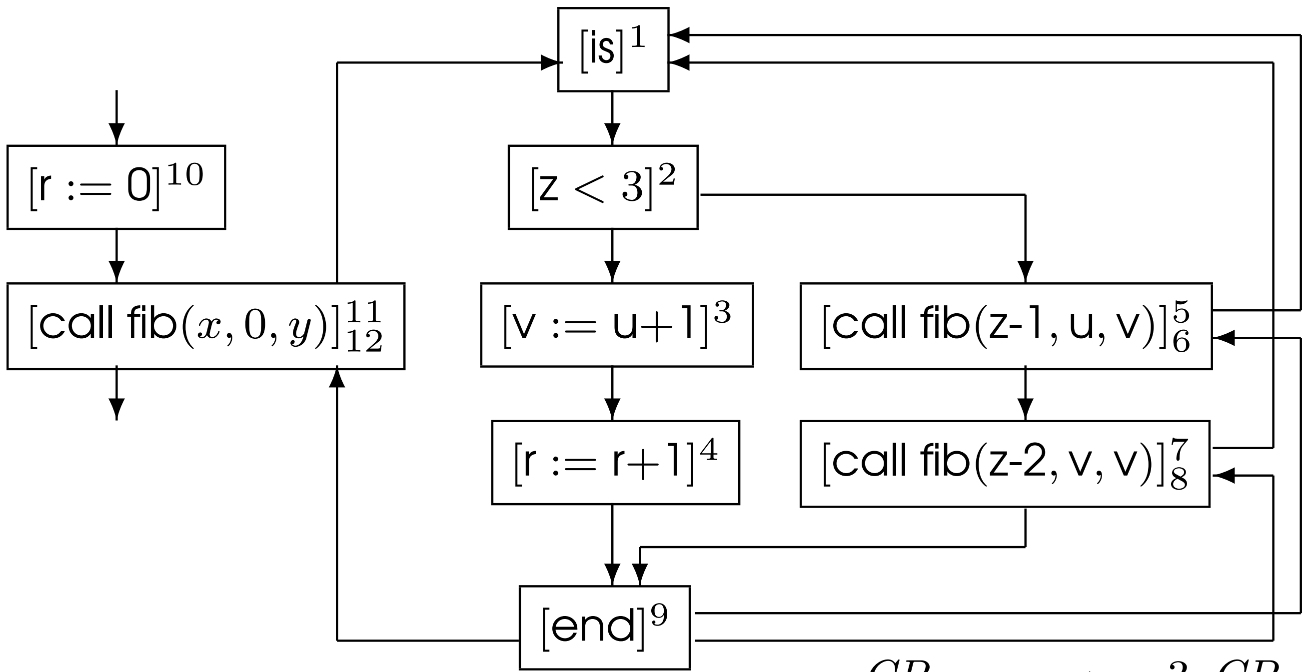
"Meet" over Valid Paths (MVP)

A complete path from ℓ_1 to ℓ_2 in $P_\star$ has proper nesting of procedure entries and exits; and a procedure returns to the point where it was called:

$CP_{\ell_1,\ell_2} \longrightarrow \ell_1$	whenever $\ell_1 = \ell_2$
$CP_{\ell_1,\ell_3} \longrightarrow \ell_1, CP_{\ell_2,\ell_3}$	whenever $(\ell_1, \ell_2) \in$ flow$_\star$
$CP_{\ell_c,\ell} \longrightarrow \ell_c, CP_{\ell_n,\ell_x}, CP_{\ell_r,\ell}$	whenever$P_\star$ contains [call $p(a, z)]^{\ell_c}_{\ell_r}$ andproc $p($val $x;$ res $y)$ is$^{\ell_n}$ S end$^{\ell_x}$

Definition: $(\ell_c, \ell_n, \ell_r, \ell_x) \in$ interflow$_\star$ if $P_\star$ contains [call $p(a, z)]^{\ell_c}_{\ell_r}$ as well as proc $p($val $x;$ res $y)$ is$^{\ell_n}$ S end$^{\ell_x}$

Example



$$
\begin{array}{lcl}
CP_{10,12} & \rightarrow & 10,\ CP_{11,12} \\
CP_{11,12} & \rightarrow & 11,\ CP_{1,9},\ CP_{12,12} \\
CP_{1,9} & \rightarrow & 1,\ CP_{2,9} \\
CP_{2,9} & \rightarrow & 2,\ CP_{3,9} \\
CP_{2,9} & \rightarrow & 2,\ CP_{5,9}
\end{array}
\qquad
\begin{array}{lcl}
CP_{3,9} & \rightarrow & 3,\ CP_{4,9} \\
CP_{4,9} & \rightarrow & 4,\ CP_{9,9} \\
CP_{5,9} & \rightarrow & 5,\ CP_{1,9},\ CP_{6,9} \\
CP_{6,9} & \rightarrow & 6,\ CP_{7,9} \\
CP_{7,9} & \rightarrow & 7,\ CP_{1,9},\ CP_{8,9} \\
CP_{8,9} & \rightarrow & 8,\ CP_{9,9}
\end{array}
\qquad
\begin{array}{lcl}
CP_{12,12} & \rightarrow & 12 \\
CP_{9,9} & \rightarrow & 9
\end{array}
$$

Some valid paths: (10,11,1,2,3,4,9,12) and (10,11,1,2,5,1,2,3,4,9,6,7,1,2,3,4,9,8,9,12)

A non-valid path: (10,11,1,2,5,1,2,3,4,9,12)

Valid Paths

A valid path starts at the entry node init$_\star$ of $P_\star$, all the procedure exits match the procedure entries but some procedures might be entered but not yet exited:

$VP_\star \longrightarrow VP_{\mathsf{init}_\star,\ell}$	whenever $\ell \in \mathsf{Lab}_\star$
$VP_{\ell_1,\ell_2} \longrightarrow \ell_1$	whenever $\ell_1 = \ell_2$
$VP_{\ell_1,\ell_3} \longrightarrow \ell_1, VP_{\ell_2,\ell_3}$	whenever $(\ell_1, \ell_2) \in \mathsf{flow}_\star$
$VP_{\ell_c,\ell} \longrightarrow \ell_c, CP_{\ell_n,\ell_x}, VP_{\ell_r,\ell}$	whenever$P_\star$ contains $[\mathsf{call}\ p(a, z)]^{\ell_c}_{\ell_r}$ andproc $p(\mathsf{val}\ x;\ \mathsf{res}\ y)$ is$^{\ell_n}$ S end$^{\ell_x}$
$VP_{\ell_c,\ell} \longrightarrow \ell_c, VP_{\ell_n,\ell}$	whenever$P_\star$ contains $[\mathsf{call}\ p(a, z)]^{\ell_c}_{\ell_r}$ andproc $p(\mathsf{val}\ x;\ \mathsf{res}\ y)$ is$^{\ell_n}$ S end$^{\ell_x}$

MVP Solution

$$MVP_\circ(\ell) = \bigsqcup\{f_{\vec{\ell}}(\iota) | \vec{\ell} \in vpath_\circ(\ell)\}$$

$$MVP_\bullet(\ell) = \bigsqcup\{f_{\vec{\ell}}(\iota) | \vec{\ell} \in vpath_\bullet(\ell)\}$$

where

$vpath_\circ(\ell) = \{[\ell_1, \ldots, \ell_{n-1}] \mid n \geq 1 \land \ell_n = \ell \land [\ell_1, \ldots, \ell_n]$ is valid path$\}$

$vpath_\bullet(\ell) = \{[\ell_1, \ldots, \ell_n] \mid n \geq 1 \land \ell_n = \ell \land [\ell_1, \ldots, \ell_n]$ is valid path$\}$

The MVP solution may be undecidable for lattices satisfying the Ascending Chain Condition, just as was the case for the MOP solution.

Making Context Explicit

- The MVP solution may be undecidable for lattices of finite height (as was the case for the MOP solution)
- We have to reconsider the MFP solution and avoid taking too many invalid paths into account
- Encode information about the paths taken into data flow properties themselves
- Introduce context information

MFP Counterpart

Context sensitive analysis: add context information

- call strings:
  - an abstraction of the sequences of procedure calls that have been performed so far
  - example: the program point where the call was initiated
- assumption sets:
  - an abstraction of the states in which previous calls have been performed
  - example: an abstraction of the actual parameters of the call

Context insensitive analysis: take no context information into account.

Call Strings as Context

- Encode the path taken
- Only record flows of the form (ℓ_c, ℓ_n) corresponding to a procedure call
- we take as context $\Delta = \mathrm{Lab}^*$ where the most recent label ℓ_c of a procedure call is at the right end
- Elements of Δ are called call strings
- The sequence of labels $\ell_c^1, \ell_c^2, \ldots, \ell_c^n$ is the call string leading to the current call which happened at ℓ_c^1; the previous calls where at $\ell_c^2 \ldots \ell_c^n$. If $n = 0$ then no calls have been performed so far.

For the example program the following call strings are of interest:

$\Lambda, [11], [11,5], [11,7], [11,5,5], [11,5,7].[11,7,5], [11,7,7], ...$

Abstracting Call Strings

Problem: call strings can be arbitrarily long (recursive calls)

Solution: truncate the call strings to have length of at most k for some fixed number k

- $\Delta = \text{Lab}^{\leq k}$
- $k = 0$: context insensitive analysis
  - Λ (the call string is the empty string)
- $k = 1$: remember the last procedure call
  - $\Lambda, [11], [5], [7]$
- $k = 2$: remember the last two procedure calls
  - $\Lambda, [11], [11, 5], [11, 7], [5, 5], [5, 7], [7, 5], [7, 7]$

References

- Material for this 4th lecture (part 2)

  `www.complang.tuwien.ac.at/markus/optub.html`

- Book

  Flemming Nielson, Hanne Riis Nielson, Chris Hankin:

  Principles of Program Analysis.

  Springer, (450 pages, ISBN 3-540-65410-0), 1999.

  – Chapter 2 (Data Flow Analysis)