Problem 1.
For two words y and w, we say that y is a *subword* of w, if $w = xyz$ for some words x and z. For example, the word 011 has the following subwords: ε, 0, 1, 01, 11, and 011.

For a language L, the set of its subwords, denoted by $subwords(L)$, contains all the subwords of the words in L. That is,

$$subwords(L) \; = \; \{y \mid \text{ there exists } w \in L \text{ such that } y \text{ is a subword of } w\}.$$

Show that if L is regular, $subwords(L)$ is also a regular language. For this purpose, given a finite automaton for L, construct a finite automaton for $subwords(L)$. Your construction should be generic.

Problem 2. Which of the following four languages L_i over the alphabet $\{\mathtt{0}, \mathtt{1}\}$ is context-free? For each i, give a CFG that generates L_i, or use the pumping lemma for the context-free languages to prove that L_i is not context-free.

a. $L_1 = \{\mathtt{0}^i\mathtt{1}^j\mathtt{0}^k \mid j = i^2 \cdot k\}$.

b. $L_2 = \{\mathtt{0}^i\mathtt{1}^j\mathtt{0}^k \mid 8 * i = 2 * j + k \text{ and } i, j, k \geq 0\}$.

c. $L_3 = \{\mathtt{0}^i\mathtt{0}^i\mathtt{1}^i\mathtt{1}^i \mid i \geq 0\}$.

d. $L_4 \; = \; \{\mathtt{0}^i\mathtt{1}^i\mathtt{1}^i\mathtt{0}^i \mid i \geq 0\}$