
Problem 8.1. (30 points) In the following, let G be an undirected graph, let s and t be two nodes of G, and let k be a natural number. A path of G is *simple* if no node occurs more than once along the path.

$$\textsc{Spath} = \{\langle G, s, t, k\rangle \mid G \text{ contains a simple path of length at } \textit{most } k \text{ from } s \text{ to } t\}$$
$$\textsc{Lpath} = \{\langle G, s, t, k\rangle \mid G \text{ contains a simple path of length at } \textit{least } k \text{ from } s \text{ to } t\}$$

One of the two problems is in P, the other one is NP-complete. Which is which?

- For the problem in P, give a polynomial-time algorithm.
- For the NP-complete problem, give a certificate that can be verified in polynomial time and a reduction from a known NP-hard problem.

Hint. For a known NP-hard problem, use either CLIQUE or the undirected graph version of HAMPATH. That is, make use of the fact that the following two problems are NP-hard:

$\textsc{UHamPath} = \{\langle G, s, t\rangle \mid$ the undirected graph G has a Hamiltonian path from vertex s to vertex $t\}$.
$\textsc{Clique} = \{\langle G, k\rangle \mid$ the undirected graph G has a clique with k vertices$\}$.