Problem 7.1. (30 points) Consider the following four languages:

- \( L_1 = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) \subseteq \{00, 11\} \} \)
- \( L_2 = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) = \{00, 11\} \} \)
- \( L_3 = \{ \langle M, q \rangle \mid \text{TM } M \text{ visits state } q \text{ on some input} \} \)
- \( L_4 = \{ \langle M, q \rangle \mid \text{TM } M \text{ visits state } q \text{ on some input within 1000 steps} \} \)

Consider the following four mutually exclusive statements about a language \( L \):

- **S1** The language \( L \) is recursive
- **S2** The language \( L \) is r.e., but not recursive.
- **S3** The language \( L \) is co-r.e., but not recursive.
- **S4** The language \( L \) is not r.e., nor co-r.e.

You are asked to determine for each language \( L_1 \) to \( L_4 \) which one of the statements **S1** to **S4** is true. You need to justify your answers as follows:

- To justify **S1**, give a high-level description of a Turing decider that accepts \( L \).
- To justify **S2**, give (i) a high-level description of a Turing recognizer that accepts \( L \), and (ii) a mapping reduction from either \( A_{TM} \) or \( \overline{E}_{TM} \) to \( L \).
- To justify **S3**, give (i) a high-level description of a Turing recognizer that accepts the complement of \( L \) and (ii) a mapping reduction from \( \overline{A}_{TM} \) or \( E_{TM} \) to \( L \).
- To justify **S4**, give (i) a mapping reduction from either \( A_{TM} \) or \( \overline{E}_{TM} \) to \( L \) and (ii) a mapping reduction from \( \overline{A}_{TM} \) or \( E_{TM} \) to \( L \).