Advanced Topics in Theoretical Computer Science **Problem Set 7**
Due January 10, 2013

Problem 7.1. (30 points) Consider the following four languages:

- $L_1 = \{\langle M \rangle \mid M \text{ is a TM and } L(M) \subseteq \{00, 11\}\}$
- $L_2 = \{\langle M \rangle \mid M \text{ is a TM and } L(M) = \{00, 11\}\}$
- $L_3 = \{\langle M, q \rangle \mid \text{TM } M \text{ visits state } q \text{ on some input } \}$
- $L_4 = \{\langle M, q \rangle \mid \text{TM } M \text{ visits state } q \text{ on some input within 1000 steps } \}$

Consider the following four mutually exclusive statements about a language L:

S1 The language L is recursive

S2 The language L is r.e., but not recursive.

S3 The language L is co-r.e., but not recursive.

S4 The language L is not r.e., nor co-r.e.

You are asked to determine for each language L_1 to L_4 which one of the statements **S1** to **S4** is true. You need to justify your answers as follows:

- To justify **S1**, give a high-level description of a Turing decider that accepts L.
- To justify **S2**, give (i) a high-level description of a Turing recognizer that accepts L, and (ii) a mapping reduction from either A_{TM} or $\overline{E}_{\mathsf{TM}}$ to L.
- To justify **S3**, give (i) a high-level description of a Turing recognizer that accepts the complement of L and (ii) a mapping reduction from $\overline{A}_{\mathsf{TM}}$ or E_{TM} to L.
- To justify **S4**, give (i) a mapping reduction from either A_{TM} or $\overline{E}_{\mathsf{TM}}$ to L and (ii) a mapping reduction from $\overline{A}_{\mathsf{TM}}$ or E_{TM} to L.