Advanced Topics in Theoretical Computer Science **Problem Set 6**
Due December 20, 2012

Problem 6.1. (15 points)
Consider the following language.

A is the set of all words over the alphabet $\Sigma = \{\mathtt{0}, \mathtt{1}, \#\}$ that have the form

$$u_1 u_2 \dots u_k \# v_1 v_2 \dots v_m, \quad \text{where } k, m \geq 1,$$

for binary numbers $u = u_1 \dots u_k$ and $v = v_1 \dots v_m$ such that $u \leq v$.

Prove that A is recursive. For this purpose, give both a high-level description and a state-transition diagram of a Turing decider for A.

Hint: The machine should look at v and u bit by bit starting from the least significant bits. The machine should figure out cases when it has reached the leftmost position of its tape.

Problem 6.2. (15 points) Assume that C and D are r.e. languages, and $C \cap D$ and $C \cup D$ are both recursive. Prove that both C and D are recursive. (Given Turing recognizers for C and D, and Turing deciders for $C \cap D$ and $C \cup D$, construct Turing deciders for C and D. Give high-level descriptions for the constructed Turing deciders for C and D.)