
Problem 4.1. (15 points) For each of the following three languages B_i over the alphabet $\{\mathtt{0}, \mathtt{1}\}$, give either a PDA that accepts B_i or a CFG that generates B_i.

a. B_1 is the set of palindromes, i.e., the set of words that read the same forwards and backwards. For eample, 010 is a palindrome; 0100 is not.

b. $B_2 = \{\mathtt{1}^{2k}\mathtt{0}\mathtt{1}^{3k} \mid k \geq 0\}$.

c. B_3 is the set of words that contain more 1's than 0's.

Problem 4.2. (15 points) For a word x of even length, let $\mathit{half}(x)$ be the first half of x. For a language C, let

$$\mathit{half}(C) \; = \; \{\mathit{half}(x) \mid x \in C \text{ and } |x| \text{ is even}\}.$$

a. If C is a regular language, is $\mathit{half}(C)$ necessarily regular as well? If so, then given a finite automaton for C, construct a finite automaton for $\mathit{half}(C)$. If not, then prove that $\mathit{half}(C)$ is not regular for some regular language C of your choice.

b. Show that if C is regular, then $\mathit{half}(C)$ is context-free, e.g., given a finite automaton for C, define a generic construction of a PDA that accepts $\mathit{half}(C)$.