Advanced Topics in Theoretical Computer Science **Problem Set 3**
Due November 8, 2012

Problem 3.1. (10 points) Give a regular expression for the complement of the language defined by $\mathtt{0}^*(\mathtt{01} \cup \mathtt{1})^*$.

Hint: Translate the regular expression into/from a finite automata and use operations on finite automata.

Problem 3.2. (20 points) For each of the following three languages, either show that the language is regular by giving a finite automaton (or a regular expression), or prove that the language is not regular using the pumping lemma. In both cases you have the freedom to use the closure properties of regular languages. You may assume that the language of all words over the alphabet $\Sigma = \{\mathtt{0}, \mathtt{1}\}$ that contain the same number of 0's and 1's is nonregular.

a. The set A_1 of all words over the alphabet $\Sigma = \{\mathtt{0}, \mathtt{1}\}$ that contain the same number of the substrings 00 and 11. For example, $\mathtt{1001110000111} \in A_1$.

b. The set A_2 of all words over the alphabet $\Sigma = \{\mathtt{0}, \mathtt{1}\}$ that contain the same number of the substrings 01 and 10. For example, $\mathtt{110101} \in A_2$.

c. The set A_3 of all words over the alphabet $\Sigma = \{\mathtt{0}, \mathtt{1}, \#\}$ that have the form

$$u_1 u_2 \ldots u_k \# v_1 v_2 \ldots v_m \# w_1 w_2 \ldots w_n,$$

for binary numbers $u = u_1 \ldots u_k$ and $v = v_1 \ldots v_m$ and $w = w_1 \ldots w_n$ such that $w = u + v$. For example, $\mathtt{010\#0011\#101} \in A_3$.