Problem 2.1. (15 points) Let $\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$. The alphabet Σ_3 contains all size 3 columns of 0s and 1s. A string of symbols in Σ_3 gives three rows of 0s and 1s. Consider each row to be a binary number and let

$$G = \{w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top two rows}\}.$$

For example, $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in G$, but $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \notin G$. Draw the state diagram of a finite automaton that accepts the language G. Is your automaton deterministic?

Hint: An input word is read bit-triple by bit-triple, and the most significant bit first. You have to take care whether a possible carry-over from the *next* bit has occurred or not.

Problem 2.2. (15 points) Let $\Sigma = \{0, 1\}$. Let $L \subseteq \Sigma^*$ be the language of all words that contain at least one letter, and that begin and end with the same letter. For example, $0, 00, 1011 \in L$ and $\varepsilon, 01, 1100 \notin L$.

a. Draw the transition diagram of a nondeterministic finite automaton N that accepts L and has 4 states.

b. Turn the automaton N into a deterministic finite automaton M using the subset construction. Label each state of the DFA M with a set of states of the NFA N.