Problem 1.1. (15 points) Draw state diagrams for DFAs that accept the following languages.

a. 
\[
A = \{w \in \{0,1\}^* \mid \text{in } w \text{ every 0 is immediately preceded by 1}\}.
\]
\[
B = \{w \in \{1,2\}^* \mid \text{w contains the subword 212}\}.
\]

b. 
\[
C = \{w \in \{0,1,2\}^* \mid \text{w contains the subword 212, and in w every 0 is immediately preceded by 1}\}.
\]
\[
D = A \cap B.
\]

Note that 21210 \notin D.

Problem 1.2. (15 points) For two languages A and B, let
\[
A \mid B = \{a_1a_2a_3\ldots a_{2k} \mid a_1a_3a_5\ldots a_{2k-1} \in A \text{ and } a_2a_4a_6\ldots a_{2k} \in B\}.
\]
For example, if ab \in A and cd \in B, then acbd \in A \mid B.

a. Given a DFA $M_A = (Q_A, \Sigma_A, \delta_A, q_A, F_A)$ that accepts A, and a DFA $M_B = (Q_B, \Sigma_B, \delta_B, q_B, F_B)$ that accepts B, define a generic construction of a finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ that accepts $A \mid B$ (for arbitrary $M_A$ and $M_B$).

**Hint:** Use a similar construction to the product construction from Handout 1, but instead of moving both automata simultaneously the two automata should move one after the other.

b. Use your construction to draw the state diagram of a DFA that accepts the language $A \mid B$ for the two languages A and B from Problem 1.1(a).