
Problem 1.1. (15 points) Draw state diagrams for DFAs that accept the following languages.

a.

$$\begin{aligned} A &= \{w \in \{\texttt{0},\texttt{1}\}^* \mid \text{in } w \text{ every } \texttt{0} \text{ is immediately preceded by } \texttt{1}\}. \\ B &= \{w \in \{\texttt{1},\texttt{2}\}^* \mid w \text{ contains the subword } \texttt{212}\}. \end{aligned}$$

b.

$$\begin{aligned} C &= \{w \in \{\texttt{0},\texttt{1},\texttt{2}\}^* \mid w \text{ contains the subword } \texttt{212}, \\ &\qquad\qquad \text{and in } w \text{ every } \texttt{0} \text{ is immediately preceded by } \texttt{1}\}. \\ D &= A \cap B. \end{aligned}$$

Note that $\texttt{21210} \notin D$.

Problem 1.2. (15 points) For two languages A and B, let

$$A \mid B = \{a_1a_2a_3 \dots a_{2k} \mid a_1a_3a_5 \dots a_{2k-1} \in A \text{ and } a_2a_4a_6 \dots a_{2k} \in B\}.$$

For example, if $\texttt{ab} \in A$ and $\texttt{cd} \in B$, then $\texttt{acbd} \in A \mid B$.

a. Given a DFA $M_A = (Q_A, \Sigma_A, \delta_A, q_A, F_A)$ that accepts A, and a DFA $M_B = (Q_B, \Sigma_B, \delta_B, q_B, F_B)$ that accepts B, define a generic construction of a finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ that accepts $A \mid B$ (for arbitrary M_A and M_B).

Hint: Use a similar construction to the product construction from Handout 1, but instead of moving both automata simultaneously the two automata should move one after the other.

b. Use your construction to draw the state diagram of a DFA that accepts the language $A \mid B$ for the two languages A and B from Problem 1.1(a).