

# Automated Reasoning and Program Verification

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# Outline

Randomised Algorithms for Satisfiability-Checking

# Randomised Algorithms for SAT (continued)

- ▶ Choose a **random interpretation**.

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The flipped variables are chosen using heuristics or randomly, or both.

$$\text{flip}(I, p)(q) = \begin{cases} I(q), & \text{if } p \neq q; \\ 1, & \text{if } p = q \text{ and } I(p) = 0; \\ 0, & \text{if } p = q \text{ and } I(p) = 1. \end{cases}$$

## Randomised Algorithms for SAT (continued)

- ▶ Choose a **random interpretation**.
- ▶ If this interpretation is not a model, repeatedly choose a variable and **change its value in the interpretation** (**flip** the variable).

The flipped variables are chosen using heuristics or randomly, or both.

$$\text{flip}(I, p)(q) = \begin{cases} I(q), & \text{if } p \neq q; \\ 1, & \text{if } p = q \text{ and } I(p) = 0; \\ 0, & \text{if } p = q \text{ and } I(p) = 1. \end{cases}$$

In other words, the interpretation  $\text{flip}(I, p)$  is obtained from  $I$  by changing its value on  $p$ .

# GSAT

**procedure**  $GSAT(S)$

**input:** set of clauses  $S$

**output:** interpretation  $I$  such that  $I \models S$  or *don't know*



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# GSAT

procedure *GSAT*(*S*)

input: set of clauses *S*

output: interpretation *I* such that  $I \models S$  or *don't know*

parameters: integers *MAX-TRIES*, *MAX-FLIPS*

begin

repeat *MAX-TRIES* times

*I* := random interpretation

if  $I \models S$  then return *I*

end

# GSAT

**procedure** *GSAT*(*S*)

**input:** set of clauses *S*

**output:** interpretation *I* such that  $I \models S$  or *don't know*

**parameters:** integers *MAX-TRIES*, *MAX-FLIPS*

**begin**

**repeat** *MAX-TRIES* times

*I* := random interpretation

**if**  $I \models S$  **then return** *I*

**repeat** *MAX-FLIPS* times

*p* := a variable such that *flip*(*I*, *p*) satisfies  
the maximal number of clauses in *S*

*I* = *flip*(*I*, *p*)

**if**  $I \models S$  **then return** *I*

**return** *don't know*

**end**

# GSAT example

0		0		1
$p_1$	$\vee$	$\neg p_2$	$\vee$	$p_3$
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	$p_2$		
$p_1$	$\vee$	$p_2$		

# GSAT example

	0		0		1
	$p_1$	∨	$\neg p_2$	∨	$p_3$
			$\neg p_2$	∨	$\neg p_3$
	$\neg p_1$			∨	$\neg p_3$
	$\neg p_1$	∨	$p_2$		
	$p_1$	∨	$p_2$		

flip no.	interpretation			satisfied clauses			candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$	$p_1$	$p_2$	$p_3$		
1	0	0	1	4				

# GSAT example

0		0		1
$p_1$	$\vee$	$\neg p_2$	$\vee$	$p_3$
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	$p_2$		
$p_1$	$\vee$	$p_2$		

flip no.	interpretation			satisfied clauses			candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$	$p_1$	$p_2$	$p_3$		
1	0	0	1	4	3	4	4	

# GSAT example

0		1		1
$p_1$	$\vee$	$\neg p_2$	$\vee$	$p_3$
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	$p_2$		
$p_1$	$\vee$	$p_2$		

flip no.	interpretation			satisfied clauses			candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$	$p_1$	$p_2$	$p_3$		
1	0	0	1	4	3	4	$p_2, p_3$	$p_2$
2	0	1	1					

# GSAT example

$$\begin{array}{r}
 \begin{array}{c} 0 \\ \hline p_1 \vee \neg p_2 \vee p_3 \\ \neg p_1 \vee p_2 \\ p_1 \vee p_2 \end{array}
 \quad
 \begin{array}{c} 1 \\ \hline \neg p_2 \vee \neg p_3 \\ \vee \neg p_3 \end{array}
 \quad
 \begin{array}{c} 1 \\ \hline p_3 \\ \neg p_3 \end{array}
 \end{array}$$

flip no.	interpretation			satisfied clauses			candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$	$p_1$	$p_2$	$p_3$		
1	0	0	1	4	3	4	$p_2, p_3$	$p_2$
2	0	1	1	4				



# GSAT example

$$\begin{array}{r}
 \begin{array}{c} 0 \qquad \qquad 1 \qquad \qquad 1 \\ \hline p_1 \vee \neg p_2 \vee p_3 \\ \qquad \qquad \neg p_2 \vee \neg p_3 \\ \neg p_1 \qquad \qquad \vee \neg p_3 \\ \neg p_1 \vee p_2 \\ p_1 \vee p_2 \end{array}
 \end{array}$$

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$		$p_1$	$p_2$	$p_3$		
1	0	0	1	4	3	4	4	$p_2, p_3$	$p_2$
2	0	1	1	4	3	4	4		

# GSAT example

0		1		0
$p_1$	$\vee$	$\neg p_2$	$\vee$	$p_3$
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	$p_2$		
$p_1$	$\vee$	$p_2$		

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$	$p_1$	$p_2$	$p_3$			
1	0	0	1	4	3	4	4	$p_2, p_3$	$p_2$
2	0	1	1	4	3	4	4	$p_2, p_3$	$p_3$
3	0	1	0						

# GSAT example

$$\begin{array}{r}
 \begin{array}{c} 0 \\ \hline p_1 \vee \neg p_2 \vee p_3 \\ \neg p_1 \vee \neg p_2 \vee \neg p_3 \\ \neg p_1 \vee p_2 \\ p_1 \vee p_2 \end{array}
 \end{array}$$

flip no.	interpretation			satisfied clauses			candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$	$p_1$	$p_2$	$p_3$		
1	0	0	1	4	3	4	$p_2, p_3$	$p_2$
2	0	1	1	4	3	4	$p_2, p_3$	$p_3$
3	0	1	0	4				

# GSAT example

$$\begin{array}{r}
 \begin{array}{c} 0 \\ \hline p_1 \vee \neg p_2 \vee p_3 \\ \neg p_1 \vee \neg p_2 \vee \neg p_3 \\ \neg p_1 \vee p_2 \\ p_1 \vee p_2 \end{array}
 \end{array}$$

flip no.	interpretation			satisfied clauses			candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$	$p_1$	$p_2$	$p_3$		
1	0	0	1	4	3	4	$p_2, p_3$	$p_2$
2	0	1	1	4	3	4	$p_2, p_3$	$p_3$
3	0	1	0	4	5	4		

# GSAT example

$$\begin{array}{r}
 \color{red}{1} \qquad \qquad 1 \qquad \qquad 0 \\
 \hline
 p_1 \vee \neg p_2 \vee p_3 \\
 \qquad \qquad \neg p_2 \vee \neg p_3 \\
 \neg p_1 \qquad \qquad \qquad \vee \neg p_3 \\
 \neg p_1 \vee p_2 \\
 p_1 \vee p_2
 \end{array}$$

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$	$p_1$	$p_2$	$p_3$			
1	0	0	1	4	3	4	4	$p_2, p_3$	$p_2$
2	0	1	1	4	3	4	4	$p_2, p_3$	$p_3$
3	0	1	0	4	5	4	4	$p_1$	$p_1$
	1	1	0						

# GSAT example

$$\begin{array}{r}
 1 \qquad \qquad 1 \qquad \qquad 0 \\
 \hline
 p_1 \vee \neg p_2 \vee p_3 \\
 \qquad \qquad \neg p_2 \vee \neg p_3 \\
 \neg p_1 \qquad \qquad \vee \neg p_3 \\
 \neg p_1 \vee p_2 \\
 p_1 \vee p_2
 \end{array}$$

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$		$p_1$	$p_2$	$p_3$		
1	0	0	1	4	3	4	4	$p_2, p_3$	$p_2$
2	0	1	1	4	3	4	4	$p_2, p_3$	$p_3$
3	0	1	0	4	5	4	4	$p_1$	$p_1$
	1	1	0	5					

# WSAT

**procedure** *WSAT*(*S*)

**input:** set of clauses *S*

**output:** interpretation *I* such that  $I \models S$  or *don't know*

**parameters:** integers *MAX-TRIES*, *MAX-FLIPS*

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procedure *WSAT*(*S*)

input: set of clauses *S*

output: interpretation *I* such that  $I \models S$  or *don't know*

parameters: integers *MAX-TRIES*, *MAX-FLIPS*

begin

repeat *MAX-TRIES* times

*I* := random interpretation

if  $I \models S$  then return *I*

end



# WSAT

**procedure** *WSAT*(*S*)

**input:** set of clauses *S*

**output:** interpretation *I* such that  $I \models S$  or *don't know*

**parameters:** integers *MAX-TRIES*, *MAX-FLIPS*

**begin**

**repeat** *MAX-TRIES* times

*I* := random interpretation

**if**  $I \models S$  **then return** *I*

**repeat** *MAX-FLIPS* times

randomly select a clause  $C \in S$  such that  $I \not\models C$

randomly select a variable *p* in *C*

*I* = *flip*(*I*, *p*)

**if**  $I \models S$  **then return** *I*

**return** *don't know*

**end**

# WSAT example

0		0		1
$p_1$	$\vee$	$\neg p_2$	$\vee$	$p_3$
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	$p_2$		
$p_1$	$\vee$	$p_2$		

# WSAT example

0		0		1
$p_1$	$\vee$	$\neg p_2$	$\vee$	$p_3$
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	$p_2$		
$p_1$	$\vee$	$p_2$		

flip no.	interpretation			unsatisfied clauses	candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$			
1	0	0	1			

# WSAT example

$$\begin{array}{r}
 \begin{array}{c} 0 \\ \hline p_1 \vee \neg p_2 \\ \neg p_1 \vee p_2 \\ p_1 \vee p_2 \end{array}
 \quad
 \begin{array}{c} 0 \\ \hline \neg p_2 \vee \neg p_3 \\ \neg p_2 \vee \neg p_3 \\ \vee \\ p_2 \end{array}
 \quad
 \begin{array}{c} 1 \\ \hline p_3 \\ \neg p_3 \\ \vee \\ \neg p_3 \end{array}
 \end{array}$$

flip no.	interpretation			unsatisfied clauses	candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$			
1	0	0	1	$p_1 \vee p_2$	$p_1, p_2$	

# WSAT example

$$\begin{array}{rcc}
 1 & & 0 & & 1 \\
 \hline
 p_1 & \vee & \neg p_2 & \vee & p_3 \\
 & & \neg p_2 & \vee & \neg p_3 \\
 & & & \vee & \neg p_3 \\
 \neg p_1 & & & & \\
 \neg p_1 & \vee & p_2 & & \\
 p_1 & \vee & p_2 & & 
 \end{array}$$

flip no.	interpretation			unsatisfied clauses	candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$			
1	0	0	1	$p_1 \vee p_2$	$p_1, p_2$	$p_1$
2	1	0	1			

# WSAT example

1		0		1
$p_1$	$\vee$	$\neg p_2$	$\vee$	$p_3$
		$\neg p_2$	$\vee$	$\neg p_3$
$\neg p_1$			$\vee$	$\neg p_3$
$\neg p_1$	$\vee$	$p_2$		
$p_1$	$\vee$	$p_2$		

flip no.	interpretation			unsatisfied clauses	candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$			
1	0	0	1	$p_1 \vee p_2$	$p_1, p_2$	$p_1$
2	1	0	1	$\neg p_1 \vee \neg p_3$ $\neg p_1 \vee p_2$	$p_1, p_2, p_3$	

# WSAT example

$$\begin{array}{r}
 1 \qquad \qquad 1 \qquad \qquad 1 \\
 \hline
 p_1 \vee \neg p_2 \vee p_3 \\
 \qquad \qquad \neg p_2 \vee \neg p_3 \\
 \neg p_1 \qquad \qquad \vee \neg p_3 \\
 \neg p_1 \vee p_2 \\
 p_1 \vee p_2
 \end{array}$$

flip no.	interpretation			unsatisfied clauses	candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$			
1	0	0	1	$p_1 \vee p_2$	$p_1, p_2$	$p_1$
2	1	0	1	$\neg p_1 \vee \neg p_3$ $\neg p_1 \vee p_2$	$p_1, p_2, p_3$	$p_2$
3	1	1	1			

# WSAT example

$$\begin{array}{r}
 \begin{array}{c} 1 \\ \hline p_1 \vee \neg p_2 \vee p_3 \\ \neg p_1 \vee p_2 \\ p_1 \vee p_2 \end{array}
 \end{array}$$

flip no.	interpretation			unsatisfied clauses	candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$			
1	0	0	1	$p_1 \vee p_2$	$p_1, p_2$	$p_1$
2	1	0	1	$\neg p_1 \vee \neg p_3$ $\neg p_1 \vee p_2$	$p_1, p_2, p_3$	$p_2$
3	1	1	1	$\neg p_2 \vee \neg p_3$ $\neg p_1 \vee \neg p_3$	$p_1, p_2, p_3$	



# WSAT example

$$\begin{array}{r}
 1 \qquad \qquad 1 \qquad \qquad 0 \\
 \hline
 p_1 \vee \neg p_2 \vee p_3 \\
 \qquad \qquad \neg p_2 \vee \neg p_3 \\
 \neg p_1 \qquad \qquad \vee \neg p_3 \\
 \neg p_1 \vee p_2 \\
 p_1 \vee p_2
 \end{array}$$

flip no.	interpretation			unsatisfied clauses	candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$			
1	0	0	1	$p_1 \vee p_2$	$p_1, p_2$	$p_1$
2	1	0	1	$\neg p_1 \vee \neg p_3$ $\neg p_1 \vee p_2$	$p_1, p_2, p_3$	$p_2$
3	1	1	1	$\neg p_2 \vee \neg p_3$ $\neg p_1 \vee \neg p_3$	$p_1, p_2, p_3$	$p_3$
	1	1	0			

# WSAT example

$$\begin{array}{r}
 \begin{array}{c} 1 \\ \hline p_1 \end{array} \vee \begin{array}{c} 1 \\ \hline \neg p_2 \end{array} \vee \begin{array}{c} 0 \\ \hline p_3 \\ \hline \neg p_3 \end{array} \\
 \neg p_1 \vee \begin{array}{c} 1 \\ \hline p_2 \\ \hline p_2 \end{array}
 \end{array}$$

flip no.	interpretation			unsatisfied clauses	candidates for flipping	flipped variable
	$p_1$	$p_2$	$p_3$			
1	0	0	1	$p_1 \vee p_2$	$p_1, p_2$	$p_1$
2	1	0	1	$\neg p_1 \vee \neg p_3$ $\neg p_1 \vee p_2$	$p_1, p_2, p_3$	$p_2$
3	1	1	1	$\neg p_2 \vee \neg p_3$ $\neg p_1 \vee \neg p_3$	$p_1, p_2, p_3$	$p_3$
	1	1	0			