Automated Reasoning and Program Verification

Midterm Assignment
Due March 17, 2015

Problem 1. (20 points) Find a model of the formula $((\neg p \to q) \to p) \to \neg p$ using only the pure atom rule.

Problem 2. (20 points) Consider the following puzzle.
Isaac and Albert were excitedly describing the result of the Third Annual International Science Fair Extravaganza in Sweden. There were three contestants, Louis, Rene, and Johannes. Isaac reported that Louis won the fair, while Rene came in second. Albert, on the other hand, reported that Johannes won the fair, while Louis came in second.
In fact, neither Isaac nor Albert had given a correct report of the results of the science fair. Each of them had given one true statement and one false statement.
What was the actual placing of the three contestants? For answering this question of the puzzle, represent the puzzle as a set P of propositional formulas and, using DPLL, find a model of P. How many models it has?

Problem 3. (20 points) Let A be a formula. Let S be the set of clauses obtained from A by applying the standard CNF transformation, and let p an atom occurring in A.
Show that p is pure in A if and only if one of the literals p, $\neg p$ is pure in S.

Problem 4. (20 points) Apply the DPLL algorithm to the following sets of clauses:

$$
\begin{array}{l}
p \lor q \lor r \lor s \\
p \lor r \lor \neg s \\
\neg p \lor q \lor r \\
\neg q \lor r \\
\neg r \lor s \\
p \lor \neg q \lor \neg s \\
\neg p \lor \neg q \\
p \lor q \lor \neg r \lor \neg s \\
\neg p \lor q \lor \neg s
\end{array}
$$

Is this set satisfiable? If yes, find a model of this set.

Problem 5. (20 points)
Consider the set consisting of the following two clauses:

$$p \qquad \neg p \lor q$$

Suppose that the initial random interpretation is $\{p \mapsto 0, q \mapsto 0\}$ and $k \geq 1$ is a positive integer.

(a) What is the probability that GSAT will find a model of this set after $2k$ flips?

(b) What is the probability that WSAT will find a model of this set after $2k$ flips?