

# On Undecidability Results of Real Programming Languages

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## Frequent Argument for Conservative Program Analysis

Property to be analyzed is undecidable

- live variables
- common sub-expressions
- worst case execution time

Main Argument: Undecidability of the halting problem

This Talk

Critical view on this argument



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# Roadmap

- Consider a representative subset of  $C$  ( $C-K$ )

- Loops, conditionals, assignments, blocks, arithmetic and Boolean expressions
- local and global variables
- Only basic datatypes
- Pointers, heap
- Procedures and functions (recursion is allowed)

- We show that the semantics can be modeled as a pushdown automaton

- Reachability on pushdown automata is decidable

⇒ Halting problem on  $C-K$  is decidable

⇔ Undefined behaviour is modeled

- Difference implementation semantics and language semantics
- Short discussion on other programming languages



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# Semantics of $C-K$ as a pushdown machine

## Key observation

All basic data types in  $C$  are finite

## (Global) States

- Store for global variables:  $\sigma : \text{GLOB} \rightarrow \text{BIT}^*$
- Heap:  $h : \text{BIT}^* \rightarrow \text{BIT}^*$

## Stack Frames (Stack Alphabet)



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- instruction  $ip : \text{LABEL}$  to be executed
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# Semantics of $C-K$ as a pushdown machine (C'ted)

## Transitions

- Calling a procedure  $p$  pushes a stack frame of  $p$  onto the runtime stack with the first instruction to be executed
- Assignments etc. change either global store, local store or registers on the top frame
- Returning from a procedure removes the top frame from the runtime stack

The semantics of a program is a pushdown-automaton





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**The semantics of a program is a pushdown-automaton**



# Example

```
int n;  
int fak(int n) {  
    if (n<=1) return 1;  
    return fak(n-1)*n;  
}  
void main() {  
    n=2;  
    int x=fak(n);  
    return;  
}
```

{Global) States:

• Local variables:  $arr(x) \rightarrow BIT^{2^{20}}$

• Program points:  $p \in \{0, \dots, 5\}$

• Registers:  $r \in \{0, 2, 3\} \rightarrow BIT^{2^2}$

Stack Frames for main:

• Local variables:  $arr(x) \rightarrow BIT^{2^{20}}$

• Program points:  $p \in \{0, \dots, 5\}$

• Registers:  $r \in \{0, 2, 3\} \rightarrow BIT^{2^2}$

$\rightarrow 6 \cdot 2^{220}$  stack frames

Stack Frames for fak:

• Local variables:  $arr(x) \rightarrow BIT^{2^{20}}$

• Program points:  $p \in \{5, \dots, 18\}$

• Registers:  $r \in \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow BIT^{2^2}$

$\rightarrow 13 \cdot 2^{220}$  stack frames



# Example

```
int n;  
int fak(int n) {  
  if9 (n6 ≤8 17) return11 110;  
  return18 fak15 (n12-14 113)*17 n16;  
}  
void main() {  
  n=120;  
  int x=4fak3(n2);  
  return5;  
}
```

(Global) States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{22}$
- No heap
- $\text{stack} = \emptyset$

Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{22}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{22}$
- $6 \cdot 2^{22}$  stack frames

Stack Frames for fak:

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int n;  
int fak(int n) {  
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}  
void main() {  
  n = 120;  
  int x = 44 fak3 (n2);  
  return5;  
}
```

## {Global} States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- $\Rightarrow 2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{96}$  stack frames

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  return5;  
}
```

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void main() {  
  n=120;  
  int x=4fak3(n2);  
  return5;  
}
```

## {Global} States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
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- $\Rightarrow 2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{96}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 19 \cdot 2^{96}$  stack frames



# Example

```
int n;  
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## {Global} States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
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## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
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- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{2048}$  stack frames



# Example

```
int n;  
int fak(int n) {  
  if9 (n6 <= 817) return11 110;  
  return18 fak15 (n12-1413)*17 n16;  
}  
void main() {  
  n=120;  
  int x=4fak3(n2);  
  return5;  
}
```

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- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
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## Stack Frames for main:

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  - Program points:  $p \in \{0, \dots, 5\}$
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- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

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  - Program points:  $p \in \{0, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{304}$  stack frames



# Example

```
int n;  
int fak(int n) {  
    if9 (n6 <= 817) return11 110;  
    return18 fak15 (n12 - 1413) *17 n16;  
}  
void main() {  
    n = 120;  
    int x = 45 fak3 (n2);  
    return5;  
}
```

## {Global} States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
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## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
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- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{304}$  stack frames



# Example

```
int n;  
int fak(int n) {  
  if9 (n6 <= 817) return11 110;  
  return18 fak15 (n12 - 1413) *17 n16;  
}  
void main() {  
  n = 120;  
  int x = 43 fak5 (n2);  
  return;  
}
```

## {Global} States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- $\Rightarrow 2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{304}$  stack frames



# Example

```
int n;  
int fak(int n) {  
    if (n9 ≤ 817) return11 110;  
    return18 fak15 (n12-14 113) *17 n16;  
}  
void main() {  
    n =1 20;  
    int x =4 fak3 (n2);  
    return5;  
}
```

## {Global} States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- ⇒  $2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- ⇒  $6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{6, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- ⇒  $13 \cdot 2^{2048}$  stack frames





# Example

```
int n;  
int fak(int n) {  
  if9 (n6 <= 817) return11 110;  
  return18 fak15 (n12 - 1413) *17 n16;  
}  
void main() {  
  n = 120;  
  int x = 43 fak5 (n2);  
  return;  
}
```

## {Global} States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- $\Rightarrow 2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{6, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{2048}$  stack frames



# Example

```
int n;  
int fak(int n) {  
    if9 (n6 <= 817) return11 110;  
    return18 fak15 (n12 - 1413) *17 n16;  
}  
void main() {  
    n = 120;  
    int x = 4 fak3 (n2);  
    return5;  
}
```

## {Global} States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- $\Rightarrow 2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{6, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{2048}$  stack frames



# Example

```
int n;  
int fak(int n) {  
  if9 (n6 <= 817) return11 110;  
  return18 fak15 (n12 - 1413) *17 n16;  
}  
void main() {  
  n = 120;  
  int x = 43 fak5 (n2);  
  return;  
}
```

## {Global} States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- $\Rightarrow 2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{6, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{2048}$  stack frames



# Example

```
int n;  
int fak(int n) {  
  if9 (n6 <= 817) return11 110;  
  return18 fak15 (n12 - 1413) *17 n16;  
}  
void main() {  
  n = 120;  
  int x = 43 fak5 (n2);  
  return5;  
}
```

## {Global} States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- $\Rightarrow 2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{6, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{2048}$  stack frames



# Example

```
int n;  
int fak(int n) {  
    if9 (n6 ≤8 17) return11 110;  
    return18 fak15 (n12 -14 113) *17 n16;  
}  
void main() {  
    n =1 20;  
    int x =4 fak3 (n2);  
    return5;  
}
```

n=0

0:x=-10

|    |    |   |
|----|----|---|
| 0  | 2  | 3 |
| -7 | 20 | 5 |

## {Global} States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- ⇒  $2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- ⇒  $6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{6, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- ⇒  $13 \cdot 2^{2048}$  stack frames



# Example

```
int n;  
int fak(int n) {  
  if9 (n6 <= 817) return11 110;  
  return18 fak15 (n12-14 113) *17 n16;  
}  
void main() {  
  n =1 20;  
  int x =4 fak3 (n2);  
  return5;  
}
```

n=0

0:x=-10

|    |    |   |
|----|----|---|
| 0  | 2  | 3 |
| -7 | 20 | 5 |

## {Global} States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- $\Rightarrow 2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{6, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{2048}$  stack frames



# Example

```
int n;  
int fak(int n) {  
    if9 (n6 <= 817) return11 110;  
    return18 fak15 (n12 - 1413) *17 n16;  
}  
void main() {  
    n = 12 * 20;  
    int x = 45 fak3 (n2);  
    return5;  
}
```

## {Global} States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- $\Rightarrow 2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{6, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{2048}$  stack frames

n=0

0:x=-10

|    |    |   |
|----|----|---|
| 0  | 2  | 3 |
| -7 | 20 | 5 |

n=2

3:x=-10

|   |   |   |
|---|---|---|
| 0 | 2 | 3 |
| 2 | 2 | 5 |



# Example

```
int n;  
int fak(int n) {  
    if9 (n6 <= 817) return11 110;  
    return18 fak15 (n12 - 1413) *17 n16;  
}  
void main() {  
    n = 12 0;  
    int x = 45 fak3 (n2);  
    return5;  
}
```

## {Global} States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- $\Rightarrow 2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{6, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{2048}$  stack frames

n=0

0:x=-10

|    |    |   |
|----|----|---|
| 0  | 2  | 3 |
| -7 | 20 | 5 |

n=2

3:x=-10

|   |   |   |
|---|---|---|
| 0 | 2 | 3 |
| 2 | 2 | 5 |





# Example

```
int n;  
int fak(int n) {  
    if9 (n6 <= 817) return11 110;  
    return18 fak15 (n12-14 113) *17 n16;  
}  
void main() {  
    n = 12 0;  
    int x = 45 fak3 (n2);  
    return5;  
}
```

## {Global} States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- $\Rightarrow 2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{6, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{2048}$  stack frames

n=0

|         |    |    |   |
|---------|----|----|---|
| 0:x=-10 | 0  | 2  | 3 |
|         | -7 | 20 | 5 |

n=2

|         |   |   |   |
|---------|---|---|---|
| 3:x=-10 | 0 | 2 | 3 |
|         | 2 | 2 | 5 |

n=2

|         |    |   |   |    |    |    |    |    |    |    |
|---------|----|---|---|----|----|----|----|----|----|----|
| 6:n=2   | 6  | 7 | 8 | 10 | 12 | 13 | 14 | 15 | 16 | 17 |
|         | 15 | 1 | 3 | 7  | 0  | 2  | 5  | -1 | 9  | 2  |
| 3:x=-10 | 0  | 2 | 3 |    |    |    |    |    |    |    |
|         | 2  | 2 | 5 |    |    |    |    |    |    |    |



# Example

```
int n;  
int fak(int n) {  
    if9 (n6 <= 817) return11 110;  
    return18 fak15 (n12 - 1413) *17 n16;  
}  
void main() {  
    n = 120;  
    int x = 4fak3(n2);  
    return5;  
}
```

## {Global} States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- $\Rightarrow 2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{6, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{2048}$  stack frames

n=0

|         |    |    |   |
|---------|----|----|---|
| 0:x=-10 | 0  | 2  | 3 |
|         | -7 | 20 | 5 |

n=2

|         |   |   |   |
|---------|---|---|---|
| 3:x=-10 | 0 | 2 | 3 |
|         | 2 | 2 | 5 |

n=2

|         |    |   |   |    |    |    |    |    |    |    |
|---------|----|---|---|----|----|----|----|----|----|----|
| 6:n=2   | 6  | 7 | 8 | 10 | 12 | 13 | 14 | 15 | 16 | 17 |
|         | 15 | 1 | 3 | 7  | 0  | 2  | 5  | -1 | 9  | 2  |
| 3:x=-10 | 0  | 2 | 3 |    |    |    |    |    |    |    |
|         | 2  | 2 | 5 |    |    |    |    |    |    |    |



# Example

```

int n;
int fak(int n) {
  if9 (n6 <= 817) return11 110;
  return18 fak15 (n12 - 1413) * 1716;
}
void main() {
  n = 12 0;
  int x = 43 fak5 (n2);
  return5;
}

```

## (Global) States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- $\Rightarrow 2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{6, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{2048}$  stack frames

n=0

|         |    |    |   |
|---------|----|----|---|
| 0:x=-10 | 0  | 2  | 3 |
|         | -7 | 20 | 5 |

n=2

|         |   |   |   |
|---------|---|---|---|
| 3:x=-10 | 0 | 2 | 3 |
|         | 2 | 2 | 5 |

n=2

|         |    |   |   |    |    |    |    |    |    |    |
|---------|----|---|---|----|----|----|----|----|----|----|
| 6:n=2   | 6  | 7 | 8 | 10 | 12 | 13 | 14 | 15 | 16 | 17 |
|         | 15 | 1 | 3 | 7  | 0  | 2  | 5  | -1 | 9  | 2  |
| 3:x=-10 | 0  | 2 | 3 |    |    |    |    |    |    |    |
|         | 2  | 2 | 5 |    |    |    |    |    |    |    |

n=2

|         |   |   |   |    |    |    |    |    |    |    |
|---------|---|---|---|----|----|----|----|----|----|----|
| 15:n=2  | 6 | 7 | 8 | 10 | 12 | 13 | 14 | 15 | 16 | 17 |
|         | 2 | 1 | 0 | 7  | 2  | 1  | 1  | -1 | 9  | 2  |
| 3:x=-10 | 0 | 2 | 3 |    |    |    |    |    |    |    |
|         | 2 | 2 | 5 |    |    |    |    |    |    |    |



# Example

```
int n;  
int fak(int n) {  
    if9 (n6 <= 81 17) return11 110;  
    return18 fak15 (n12 - 14 113) * 17 n16;  
}  
void main() {  
    n = 12 0;  
    int x = 43 fak3 (n2);  
    return5;  
}
```

## (Global) States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- $\Rightarrow 2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{6, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{2048}$  stack frames

n=0

|         |    |    |   |
|---------|----|----|---|
| 0:x=-10 | 0  | 2  | 3 |
|         | -7 | 20 | 5 |

n=2

|         |   |   |   |
|---------|---|---|---|
| 3:x=-10 | 0 | 2 | 3 |
|         | 2 | 2 | 5 |

n=2

|         |    |   |   |    |    |    |    |    |    |    |
|---------|----|---|---|----|----|----|----|----|----|----|
| 6:n=2   | 6  | 7 | 8 | 10 | 12 | 13 | 14 | 15 | 16 | 17 |
|         | 15 | 1 | 3 | 7  | 0  | 2  | 5  | -1 | 9  | 2  |
| 3:x=-10 |    |   |   |    |    |    |    | 0  | 2  | 3  |
|         |    |   |   |    |    |    |    | 2  | 2  | 5  |

n=2

|         |   |   |   |    |    |    |    |    |    |    |
|---------|---|---|---|----|----|----|----|----|----|----|
| 15:n=2  | 6 | 7 | 8 | 10 | 12 | 13 | 14 | 15 | 16 | 17 |
|         | 2 | 1 | 0 | 7  | 2  | 1  | 1  | -1 | 9  | 2  |
| 3:x=-10 |   |   |   |    |    |    |    | 0  | 2  | 3  |
|         |   |   |   |    |    |    |    | 2  | 2  | 5  |



# Example

```
int n;  
int fak(int n) {  
    if9 (n6 <= 817) return11 110;  
    return18 fak15 (n12 - 141 113) * 171 n16;  
}  
void main() {  
    n = 12 0;  
    int x = 41 fak3 (n2);  
    return5;  
}
```

## (Global) States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- $\Rightarrow 2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{6, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{2048}$  stack frames

n=0

|         |    |    |   |
|---------|----|----|---|
| 0:x=-10 | 0  | 2  | 3 |
|         | -7 | 20 | 5 |

n=2

|         |   |   |   |
|---------|---|---|---|
| 3:x=-10 | 0 | 2 | 3 |
|         | 2 | 2 | 5 |

n=2

|         |    |   |   |    |    |    |    |    |    |    |
|---------|----|---|---|----|----|----|----|----|----|----|
| 6:n=2   | 6  | 7 | 8 | 10 | 12 | 13 | 14 | 15 | 16 | 17 |
|         | 15 | 1 | 3 | 7  | 0  | 2  | 5  | -1 | 9  | 2  |
| 3:x=-10 | 0  | 2 | 3 |    |    |    |    |    |    |    |
|         | 2  | 2 | 5 |    |    |    |    |    |    |    |

n=2

|         |   |   |   |    |    |    |    |    |    |    |
|---------|---|---|---|----|----|----|----|----|----|----|
| 15:n=2  | 6 | 7 | 8 | 10 | 12 | 13 | 14 | 15 | 16 | 17 |
|         | 2 | 1 | 0 | 7  | 2  | 1  | 1  | -1 | 9  | 2  |
| 3:x=-10 | 0 | 2 | 3 |    |    |    |    |    |    |    |
|         | 2 | 2 | 5 |    |    |    |    |    |    |    |

n=2

|         |   |   |   |    |    |    |    |    |    |    |
|---------|---|---|---|----|----|----|----|----|----|----|
| 6:n=2   | 6 | 7 | 8 | 10 | 12 | 13 | 14 | 15 | 16 | 17 |
|         | 0 | 9 | 7 | -1 | 8  | 27 | 88 | -6 | 3  | 1  |
| 15:n=2  | 6 | 7 | 8 | 10 | 12 | 13 | 14 | 15 | 16 | 17 |
|         | 2 | 1 | 0 | 7  | 2  | 1  | 1  | -1 | 9  | 2  |
| 3:x=-10 | 0 | 2 | 3 |    |    |    |    |    |    |    |
|         | 2 | 2 | 5 |    |    |    |    |    |    |    |

# Example

```

int n;
int fak(int n) {
  if9 (n6 <= 817) return11 110;
  return18 fak15 (n12 - 1413) * 1716;
}
void main() {
  n = 12 0;
  int x = 43 fak5 (n2);
  return5;
}

```

## (Global) States:

- $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - No heap
- $\Rightarrow 2^{32}$  states

## Stack Frames for main:

- Local variables:  $\sigma : \{x\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{0, \dots, 5\}$
  - Registers:  $\rho : \{0, 2, 3\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 6 \cdot 2^{128}$  stack frames

## Stack Frames for fak:

- Local variables:  $\sigma : \{n\} \rightarrow \text{BIT}^{32}$
  - Program points:  $p \in \{6, \dots, 18\}$
  - Registers:  $\rho : \{6, 7, 8, 10, 12, \dots, 17\} \rightarrow \text{BIT}^{32}$
- $\Rightarrow 13 \cdot 2^{2048}$  stack frames

n=0

|         |    |    |   |
|---------|----|----|---|
| 0:x=-10 | 0  | 2  | 3 |
|         | -7 | 20 | 5 |

n=2

|         |   |   |   |
|---------|---|---|---|
| 3:x=-10 | 0 | 2 | 3 |
|         | 2 | 2 | 5 |

n=2

|         |    |   |   |    |    |    |    |    |    |    |
|---------|----|---|---|----|----|----|----|----|----|----|
| 6:n=2   | 6  | 7 | 8 | 10 | 12 | 13 | 14 | 15 | 16 | 17 |
|         | 15 | 1 | 3 | 7  | 0  | 2  | 5  | -1 | 9  | 2  |
| 3:x=-10 |    |   |   |    |    |    |    | 0  | 2  | 3  |
|         |    |   |   |    |    |    |    | 2  | 2  | 5  |

n=2

|         |   |   |   |    |    |    |    |    |    |    |
|---------|---|---|---|----|----|----|----|----|----|----|
| 15:n=2  | 6 | 7 | 8 | 10 | 12 | 13 | 14 | 15 | 16 | 17 |
|         | 2 | 1 | 0 | 7  | 2  | 1  | 1  | -1 | 9  | 2  |
| 3:x=-10 |   |   |   |    |    |    |    | 0  | 2  | 3  |
|         |   |   |   |    |    |    |    | 2  | 2  | 5  |

n=2

|         |   |   |   |    |    |    |    |    |    |    |
|---------|---|---|---|----|----|----|----|----|----|----|
| 6:n=2   | 6 | 7 | 8 | 10 | 12 | 13 | 14 | 15 | 16 | 17 |
|         | 0 | 9 | 7 | -1 | 8  | 27 | 88 | -6 | 3  | 1  |
| 15:n=2  | 6 | 7 | 8 | 10 | 12 | 13 | 14 | 15 | 16 | 17 |
|         | 2 | 1 | 0 | 7  | 2  | 1  | 1  | -1 | 9  | 2  |
| 3:x=-10 |   |   |   |    |    |    |    | 0  | 2  | 3  |
|         |   |   |   |    |    |    |    | 2  | 2  | 5  |

# Situation in other Programming Languages

## Java, C#

- Set of references is potentially infinite
- ⇒ Infinite data types
- ⇒ Halting problem is undecidable

## Java Bytecode, CLF

## Pascal, Ada, Modula-2



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## Basic Arithmetic



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## Based on: Mitchell



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# Conclusions

- The need of conservative program analysis approaches due to undecidability result is theoretically often wrong
- ⇒ From a practical point it is often necessary due to state space explosion
- Software model checking (LTL, CTL) on pushdown systems is possible
- ⇒ This enables the possibility to experimentally compare the quality of conservative approximations.





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