

Computing and Visualizing Closure Objects using Relation Algebra and RELVIEW

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Motivation

Closure systems and closure operations play an important role in both mathematics and computer science.

There are a number of concepts which are isomorphic to them. We refer to all such concepts as closure objects.

In this work we develop relation-algebraic algorithms

- to recognize several classes of closure objects,
- to compute the complete lattices they constitute and
- to transform any of these closure objects into another,

which can directly be translated into the language of the specific purpose computer algebra system `RELVIEW`.

We demonstrate that the system is well suited for computing and visualizing closure objects and their complete lattices.

Closure Objects

Closure objects base on a (finite) complete lattice (X, \leq) . The most important ones are:

- Closure systems: Subsets of X which contain the greatest element and are closed under (binary) greatest lower bounds.
- Closure operations: Functions on X , which are extensive, monotone and idempotent.
- Full implicational systems: Relations \rightarrow on X , which are transitive, a super relation of \geq and fulfill

$$x \rightarrow y, u \rightarrow v \Rightarrow x \sqcup u \rightarrow y \sqcup v$$

- Join congruences Equivalence relations \equiv on X , which fulfill

$$x \equiv y \Rightarrow x \sqcup z \equiv y \sqcup z$$

Further examples and specializations: Sperner villages, dependency relations and topologies (if (X, \leq) is a powerset lattice).

Relation Algebra

Notation

- R is a relation with domain X and range Y :

$$R : X \leftrightarrow Y$$

$X \leftrightarrow Y$ is the type of R .

- Instead of $(x, y) \in R$ we use Boolean matrix notation:

$$R_{x,y}$$

Signature of relation algebra

- Constants: $0, I, 1$.
- Operations: $R \cup S, R \cap S, RS, \bar{R}, R^T$.
- Tests: $R \subseteq S, R = S$.

Modelling Sets with Relations

Row-constant Relations

A relation $R : X \leftrightarrow Y$ is row-constant if $R = RL$.

Vectors

For row-constant relations the range is irrelevant. Therefore, the normal case is $v : X \leftrightarrow 1$, where $1 := \{\perp\}$ is a singleton set. Then we write v_x instead of $v_{x,\perp}$.

Vector-Representations of Sets

Given $v : X \leftrightarrow 1$, we define for subsets Y of X :

$$\begin{aligned} v \text{ represents } Y &\Leftrightarrow Y = \{x \in X : v_x\} \\ &\Leftrightarrow \forall x \in X : x \in Y \leftrightarrow v_x \end{aligned}$$

The Relation-Algebraic Tool RELVIEW

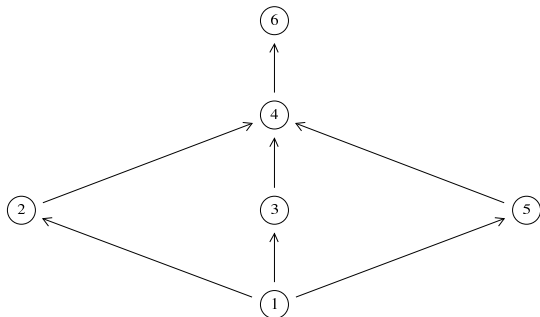
The screenshot displays the RELVIEW software interface, which is used for working with relations in a relation algebra. The interface is divided into several panels:

- Evaluate Term:** A panel for evaluating terms, featuring input fields for "Result:", "Term:", and "History:", along with an "Options" section containing a checkbox for "Check assertions?" and buttons for "Clear History", "Evaluate", and "Close".
- Kiel University Review:** A menu bar with "FILES", "OPTIONS", "INFO", and "QUIT". Below it are sections for "Editors:", "Directories:", "RELATION", "GRAPH", "XRV/PROG", "LABEL", and "Operations and user-defined functions:" with buttons for "DEFI", "EVAL", "ITER", "TESTS", and "OPS".
- Directory:** A panel showing a list of relations and programs. The "Relations" tab is active, displaying a table with columns "Name" and "Info".
- ClsysLat (NAME: ClsysLat DIM: 24 X 24):** A panel showing a 24x24 matrix representation of the ClsysLat relation. The matrix is sparse, with non-zero entries forming a pattern along the main diagonal and some off-diagonal elements.
- ClsysLat with 24 nodes:** A panel showing a directed graph representation of the ClsysLat relation. The graph has 24 nodes arranged in a diamond-like structure, with edges representing the relation between nodes.
- Filechooser:** A panel for selecting files, showing a list of files in the directory `/home/rub/ARBEITEN/RELATIONEN/2009-CASC/r/RELVIEW`. The files listed are `Beispiel.xrv`, `BeispielNonTopCl...`, `Bspflasis.xrv`, and `gegenbsp.xrv`. The file `Beispiel.xrv` is selected.

The Running Example

A Partial Order

Hasse diagram of a partial order visualized as RELVIEW graph



For a partial order $R : X \leftrightarrow X$ we often write $x \leq y$ for $R_{x,y}$.

Computing all Closure Systems

Specification Closure System (finite case)

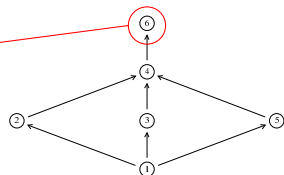
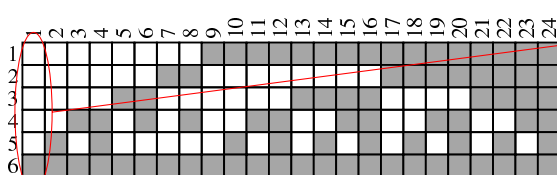
$S \subseteq X$ is closure system iff $\top \in S$ and $a, b \in S$ imply $a \sqcap b \in S$.

Computation of all Closure Systems of a partial order R :

$$\text{cls}(R) := \text{Minj}((\text{gel}(R, L)^T M \cap \overline{L(\pi M \cap \rho M \cap \overline{\text{Inf}(R) M})}^T)^T : X \leftrightarrow \mathfrak{G}$$

The 24 Closure Systems of the Example Order

Each system is shown as a column of a Boolean matrix.



Computing all Closure Systems

Specification Closure System (finite case)

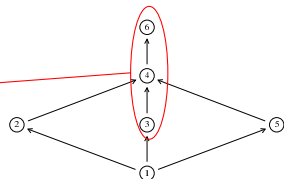
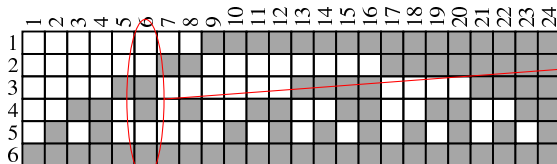
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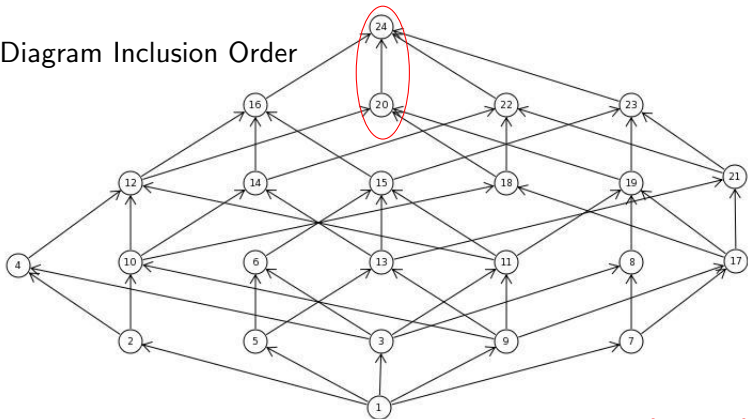
The 24 Closure Systems of the Example Order

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Inclusion Order of the 24 Closure Systems

Hasse Diagram Inclusion Order



Closure Systems

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
1																									
2																									
3																									
4																									
5																									
6																									

From Closure System to Closure Operation C

$$\begin{array}{c}
 x \leq y \Rightarrow C(x) \leq C(y) \\
 C \subseteq R
 \end{array}
 \quad \left| \quad
 \begin{array}{c}
 x \leq C(x) \\
 R \subseteq C R C^T
 \end{array}
 \quad \left| \quad
 \begin{array}{c}
 C(C(x)) = C(x) \\
 C C \subseteq C
 \end{array}$$

Four of the 24 closure operations and computation from a closure system s

	1	2	3	4	5	6
1						■
2						■
3						■
4						■
5						■
6						■

	1	2	3	4	5	6
1					■	
2					■	
3					■	
4					■	
5					■	
6					■	

	1	2	3	4	5	6
1				■		
2				■		
3				■		
4				■		
5				■		
6				■		

	1	2	3	4	5	6
1					■	
2					■	
3					■	
4					■	
5					■	
6					■	

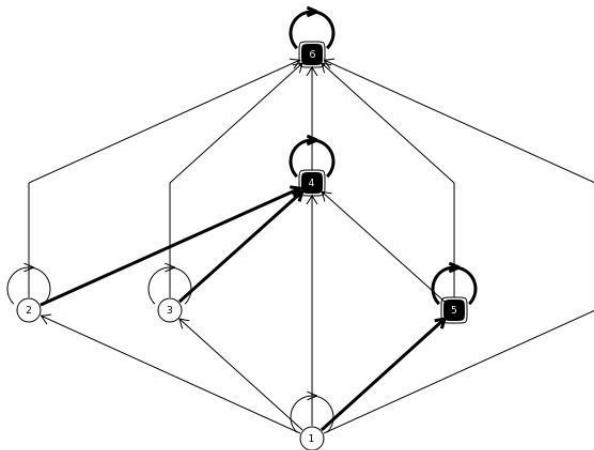
$$\text{ClsToClo}(s) := \text{glb}(R, sL \cap R^T)^T : X \leftrightarrow X$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
1																									
2																									
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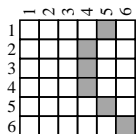
Closure Operations

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

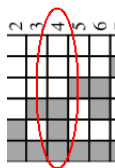
Closure Operation



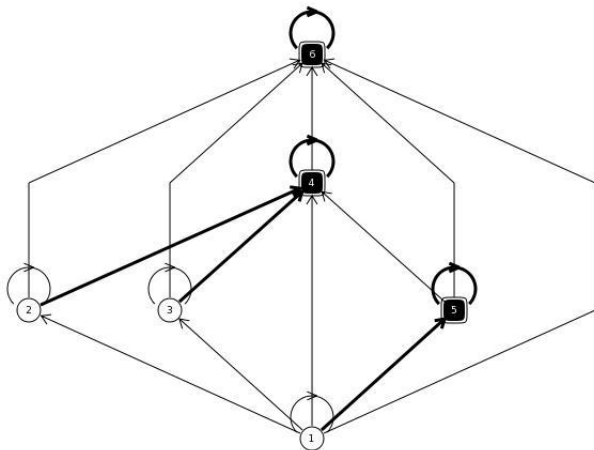
Closure Operations



Closure Operation



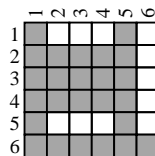
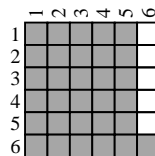
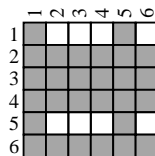
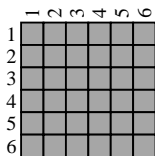
Corresponding Closure System



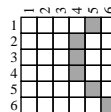
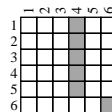
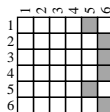
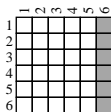
Full Implicational System \rightarrow (Finite Case)

- ① if $A \rightarrow B, B \rightarrow C$ then $A \rightarrow C$ $FF \subseteq F$
- ② if $A \supseteq B$ then $A \rightarrow B$ $R^T \subseteq F$
- ③ if $A \rightarrow B, C \rightarrow D$ then $A \cup C \rightarrow B \cup D$ $F \parallel F \subseteq \text{Sup}(R) F \text{Sup}(R)^T$

Computation of full implicational system from closure operation



$$\text{CloToFis}(C, R) = CR^T : X \leftrightarrow X$$



Join-Congruence J

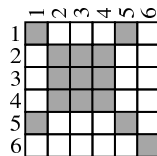
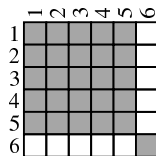
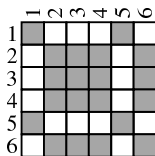
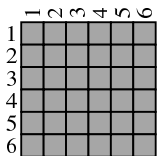
① J is equivalence relation

$$I \subseteq J \quad J = J^T \quad JJ \subseteq J$$

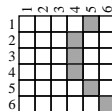
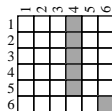
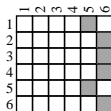
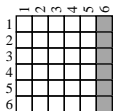
② if $J_{x,y}$ then $J_{x \sqcup z, y \sqcup z}$

$$J \parallel I \subseteq \text{Sup}(R) \quad J \text{Sup}(R)^T$$

Computation of join congruences from closure operations.



$$\text{CloToJc}(C) = CC^T : X \leftrightarrow X$$



Recognizing Closure Operations

A Function C is a Closure Operation iff

- **extensive** $\forall x \in X : x \leq C(x)$ $C \subseteq R$
- **monotonicity** $\forall x, y \in X : x \leq y \rightarrow C(x) \leq C(y)$ $R \subseteq CRCT^T$
- **idempotency** $\forall x \in X : C(C(x)) = C(x)$ $CC \subseteq C$

Development of first relational specification

$$\begin{aligned}
 & \forall x \in X : x \leq C(x) \\
 \Leftrightarrow & \forall x \in X : R_{x, C(x)} \\
 \Leftrightarrow & \forall x, y \in X : C(x) = y \rightarrow R_{x, y} \\
 \Leftrightarrow & \forall x, y \in X : C_{x, y} \rightarrow R_{x, y} \\
 \Leftrightarrow & C \subseteq R
 \end{aligned}$$

Recognizing Closure Operations

A Function C is a Closure Operation iff

- extensive $\forall x \in X : x \leq C(x)$ $C \subseteq R$
- **monotonicity** $\forall x, y \in X : x \leq y \rightarrow C(x) \leq C(y)$ $R \subseteq CRC^T$
- idempotency $\forall x \in X : C(C(x)) = C(x)$ $CC \subseteq C$

Development of second relational specification

$$\begin{aligned}
 & \forall x, y \in X : x \leq y \rightarrow C(x) \leq C(y) \\
 \Leftrightarrow & \forall x, y \in X : R_{x,y} \rightarrow R_{C(x),C(y)} \\
 \Leftrightarrow & \forall x, y \in X : R_{x,y} \rightarrow \exists a, b \in X : C(x) = a \wedge C(y) = b \wedge R_{a,b} \\
 \Leftrightarrow & \forall x, y \in X : R_{x,y} \rightarrow \exists a \in X : C_{x,a} \wedge \exists b \in X : R_{a,b} \wedge C_{b,y}^T \\
 \Leftrightarrow & \forall x, y \in X : R_{x,y} \rightarrow (CRC^T)_{x,y} \\
 \Leftrightarrow & R \subseteq CRC^T
 \end{aligned}$$

Recognizing Closure Operations

A Function C is a Closure Operation iff

- extensive $\forall x \in X : x \leq C(x)$ $C \subseteq R$
- monotonicity $\forall x, y \in X : x \leq y \rightarrow C(x) \leq C(y)$ $R \subseteq CRCT^T$
- idempotency $\forall x \in X : C(C(x)) = C(x)$ $CC \subseteq C$

Development of third relational specification

$$\begin{aligned}
 & \forall x \in X : C(C(x)) = C(x) \\
 \Leftrightarrow & \forall x, y, a \in X : C(x) = a \wedge C(a) = y \rightarrow C(x) = y \\
 \Leftrightarrow & \forall x, y, a \in X : C_{x,a} \wedge C_{a,y} \rightarrow C_{x,y} \\
 \Leftrightarrow & \forall x, y \in X : (\exists a \in X : C_{x,a} \wedge C_{a,y}) \rightarrow C_{x,y} \\
 \Leftrightarrow & \forall x, y \in X : (CC)_{x,y} \rightarrow C_{x,y} \\
 \Leftrightarrow & CC \subseteq C.
 \end{aligned}$$

Recognizing Closure Operations as RELVIEW programs

The Formulae: $C \subseteq R$ $R \subseteq CR C^T$ $CC \subseteq C$

Program in Declarative Style

```
isExt(C,R)  = incl(C,R).
isMon(C,R)  = incl(R,C*R*C^).
isIde(C)    = incl(C*C,C).
isClos(C,R) = isExt(C,R) & isMon(C,R) & isIde(C).
```

Program in Imperative Style

```
isClos(C,R)
DECL isExt, isMon, isIde
BEG  isExt = incl(C,R);
      isMon = incl(R,C*R*C^);
      isIde = incl(C*C,C)
      RETURN isExt & isMon & isIde
END.
```

Computing all Closure Systems

Computation of all Closure Systems of a partial order R

$S \subseteq X$ is closure system iff $\top \in S$ and $a, b \in S$ imply $a \sqcap b \in S$.

$$\text{cls}(R) := \text{Minj}((\text{gel}(R, L)^T M \cap \overline{\overline{L(\pi M \cap \rho M \cap \overline{\text{Inf}(R) M})}})^T)^T : X \leftrightarrow \mathfrak{G}$$

Condition 1: $\top \in S$

$$\begin{aligned} & \top \in S \\ \Leftrightarrow & \exists x \in X : M_{x,S} \wedge \text{gel}(R, L)_x \\ \Leftrightarrow & (M^T \text{gel}(R, L))_S \\ \Leftrightarrow & (\text{gel}(R, L)^T M)^T_S \end{aligned}$$

where

M is the membership relation

$\text{gel}(R, v)$ is greatest element of set represented by vector v

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Condition 2: $a, b \in S$ imply $a \sqcap b \in S$. Let $u = \langle u_1, u_2 \rangle$

$$\forall u \in X \times X : u_1 \in S \wedge u_2 \in S \rightarrow u_1 \sqcap u_2 \in S$$

$$\Leftrightarrow \forall u \in X \times X : u_1 \in S \wedge u_2 \in S \rightarrow \exists z \in X : u_1 \sqcap u_2 = z \wedge z \in S$$

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$$\Leftrightarrow \forall u \in X \times X : u_1 \in S \wedge u_2 \in S \rightarrow \exists z \in X : u_1 \sqcap u_2 = z \wedge z \in S$$

$$\Leftrightarrow \forall u \in X \times X : u_1 \in S \wedge u_2 \in S \rightarrow \exists z \in X : \text{Inf}(R)_{u,z} \wedge z \in S$$

Computing all Closure Systems

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$$\Leftrightarrow \forall u \in X \times X : (\pi M)_{u,S} \wedge (\rho M)_{u,S} \rightarrow \exists z \in X : \text{Inf}(R)_{u,z} \wedge M_{z,S}$$

Computing all Closure Systems

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$$\Leftrightarrow \forall u \in X \times X : u_1 \in S \wedge u_2 \in S \rightarrow \exists z \in X : u_1 \sqcap u_2 = z \wedge z \in S$$

$$\Leftrightarrow \forall u \in X \times X : u_1 \in S \wedge u_2 \in S \rightarrow \exists z \in X : \text{Inf}(R)_{u,z} \wedge z \in S$$

$$\Leftrightarrow \forall u \in X \times X : (\pi M)_{u,S} \wedge (\rho M)_{u,S} \rightarrow \exists z \in X : \text{Inf}(R)_{u,z} \wedge M_{z,S}$$

$$\Leftrightarrow \forall u \in X \times X : (\pi M)_{u,S} \wedge (\rho M)_{u,S} \rightarrow (\text{Inf}(R) M)_{u,S}$$

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$$\Leftrightarrow \forall u \in X \times X : (\pi M)_{u,S} \wedge (\rho M)_{u,S} \rightarrow \exists z \in X : \text{Inf}(R)_{u,z} \wedge M_{z,S}$$

$$\Leftrightarrow \forall u \in X \times X : (\pi M)_{u,S} \wedge (\rho M)_{u,S} \rightarrow (\overline{\text{Inf}(R) M})_{u,S}$$

$$\Leftrightarrow \neg \exists u \in X \times X : (\pi M)_{u,S} \wedge (\rho M)_{u,S} \wedge \overline{\text{Inf}(R) M}_{u,S}$$

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$$\text{cls}(R) := \text{Minj}((\text{gel}(R, L))^T M \cap \overline{L(\pi M \cap \rho M \cap \overline{\text{Inf}(R) M})}^T)^T : X \leftrightarrow \mathcal{G}$$

Condition 2: $a, b \in S$ imply $a \sqcap b \in S$. Let $u = \langle u_1, u_2 \rangle$

$$\forall u \in X \times X : u_1 \in S \wedge u_2 \in S \rightarrow u_1 \sqcap u_2 \in S$$

$$\Leftrightarrow \forall u \in X \times X : u_1 \in S \wedge u_2 \in S \rightarrow \exists z \in X : u_1 \sqcap u_2 = z \wedge z \in S$$

$$\Leftrightarrow \forall u \in X \times X : u_1 \in S \wedge u_2 \in S \rightarrow \exists z \in X : \text{Inf}(R)_{u,z} \wedge z \in S$$

$$\Leftrightarrow \forall u \in X \times X : (\pi M)_{u,S} \wedge (\rho M)_{u,S} \rightarrow \exists z \in X : \text{Inf}(R)_{u,z} \wedge M_{z,S}$$

$$\Leftrightarrow \forall u \in X \times X : (\pi M)_{u,S} \wedge (\rho M)_{u,S} \rightarrow (\text{Inf}(R) M)_{u,S}$$

$$\Leftrightarrow \neg \exists u \in X \times X : (\pi M)_{u,S} \wedge (\rho M)_{u,S} \wedge \overline{\text{Inf}(R) M}_{u,S}$$

$$\Leftrightarrow \neg \exists u \in X \times X : (\pi M \cap \rho M \cap \overline{\text{Inf}(R) M})^T_{S,u} \wedge L_u$$

Computing all Closure Systems

Computation of all Closure Systems of a partial order R

$S \subseteq X$ is closure system iff $\top \in S$ and $a, b \in S$ imply $a \sqcap b \in S$.

$$\text{cls}(R) := \text{Minj}((\text{gel}(R, L))^T M \cap \overline{L(\pi M \cap \rho M \cap \overline{\text{Inf}(R) M})}^T)^T : X \leftrightarrow \mathcal{G}$$

Condition 2: $a, b \in S$ imply $a \sqcap b \in S$. Let $u = \langle u_1, u_2 \rangle$

$$\forall u \in X \times X : u_1 \in S \wedge u_2 \in S \rightarrow u_1 \sqcap u_2 \in S$$

$$\Leftrightarrow \forall u \in X \times X : u_1 \in S \wedge u_2 \in S \rightarrow \exists z \in X : u_1 \sqcap u_2 = z \wedge z \in S$$

$$\Leftrightarrow \forall u \in X \times X : u_1 \in S \wedge u_2 \in S \rightarrow \exists z \in X : \text{Inf}(R)_{u,z} \wedge z \in S$$

$$\Leftrightarrow \forall u \in X \times X : (\pi M)_{u,S} \wedge (\rho M)_{u,S} \rightarrow \exists z \in X : \text{Inf}(R)_{u,z} \wedge M_{z,S}$$

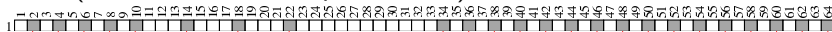
$$\Leftrightarrow \forall u \in X \times X : (\pi M)_{u,S} \wedge (\rho M)_{u,S} \rightarrow (\overline{\text{Inf}(R) M})_{u,S}$$

$$\Leftrightarrow \neg \exists u \in X \times X : (\pi M)_{u,S} \wedge (\rho M)_{u,S} \wedge \overline{\text{Inf}(R) M}_{u,S}$$

$$\Leftrightarrow \neg \exists u \in X \times X : (\pi M \cap \rho M \cap \overline{\text{Inf}(R) M})^T_{S,u} \wedge L_u$$

$$\Leftrightarrow \overline{(\pi M \cap \rho M \cap \overline{\text{Inf}(R) M})^T}_{L_S}$$

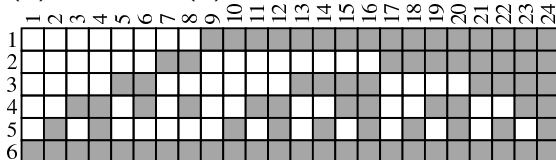
vector v (here shown in transposed form)



Membership relation M



Result of $M \text{inj}(v)^T$, since $\text{inj}(v) : \mathcal{G} \leftrightarrow 2^X$ is the identity function



Conclusion

The computer algebra system `RELVIEW` supports the study of closure objects in several ways:

- Visualization (Boolean matrices, graphs, layout algorithms, labeling, highlighting)
- Animation (via step-wise execution)
- Testing (random relations and graphs, specified properties and degree of filling, generation of sets of all candidates)

Most of the presented algorithms scale well and are applicable to large examples (due to `RELVIEW`'s ROBDD implementation).

Only exception: Computation of all possible closure systems as their number is exponential in general.