

Optimierende Compiler

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Jens Knoop



Technische Universität Wien
Institut für Computersprachen



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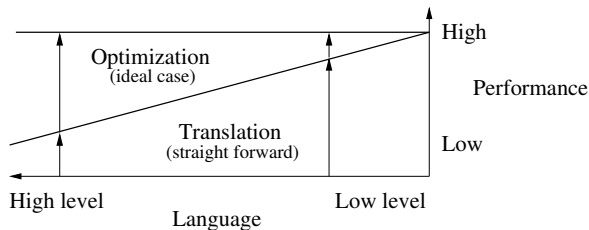
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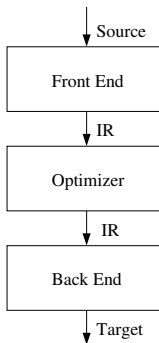
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Languages and Their Perceived Performance



- ▶ Common perception is that **high level languages/abstraction** gives **low level of performance**.
- ▶ Translation (straight forward) **preserves semantics** but **does not exploit specific opportunities of lower level language with respect to performance**.
- ▶ **Optimization improves performance** (misnomer: usually we do not achieve an “optimal” solution - but it is the ideal case)

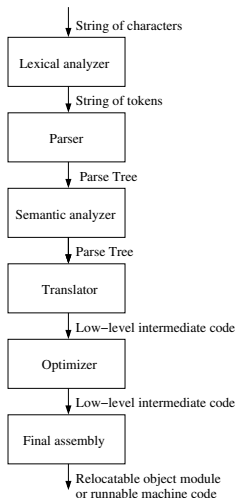
Generic Structure of an Optimizing Compiler



Goal of code optimization

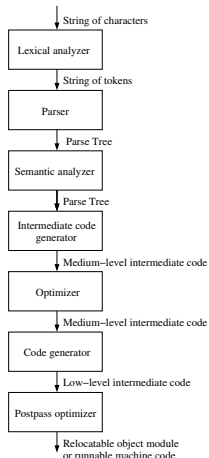
- ▶ Discover, **at compile-time**, information about the run-time behavior of the program and use that information to improve the code generated by the compiler.

Model of a Low Level Optimizer



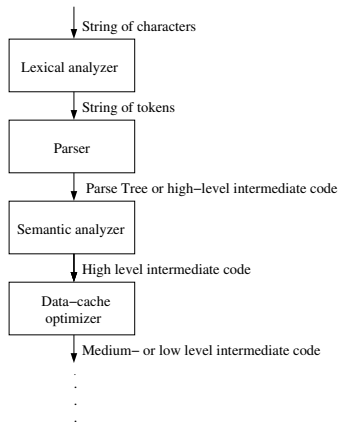
- All optimization is done on a **low level intermediate code**.

Model of a Mixed Level Optimizer



- ▶ Optimization is divided into two phases, one operating on a **medium level** and one on a **low level**.

Model of a High Level Cache Optimizer



Adding data-cache optimization to an optimizing compiler

- ▶ Data-cache optimizations are most effective when applied to a high-level intermediate form.

Examples

▶ High-Level optimizations

- ▶ IBM's PowerPC compiler: first translates to LL code (XIL) and then generates a HL representation (YIL) from it to do data-cache optimization.
- ▶ Source-To-Source Optimizer Tools: Sage++, LLNL-ROSE, JTransformer.

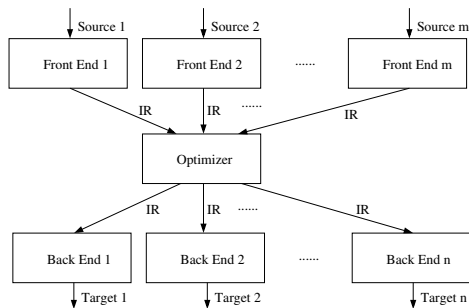
▶ Mixed model

- ▶ Sun Microsystem's compilers for SPARC
- ▶ Intel's compilers for the 386 architecture family
- ▶ Silicon Graphic's compilers for MIPS

▶ Low level model

- ▶ IBM's compilers for PowerPC
- ▶ Hewlett-Packard's compilers for PA-RISC.

Practice: m-2-n Compilers and Optimizers

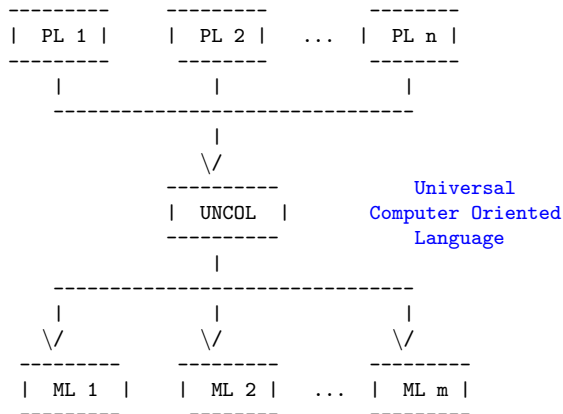


Idea: Decoupling of Compiler Front Ends from Back Ends

- ▶ **Without IR:** m source languages, n targets $\rightarrow m \times n$ compilers
- ▶ **With IR:** m Front Ends, n Back Ends
- ▶ **Problem:** Appropriate choice of the level of IR (possible solution: multiple levels of IR)

IR-Decoupling of Compiler Front/Back Ends

...is an application of the well-known UNCOL concept:



- ▶ Melvin E. Conway. [Proposal for an UNCOL](#). Communications of the ACM 1(3):5, 1958.

Intermediate Representation (IR)

- ▶ **High level**
 - ▶ quite close to source language
 - ▶ e.g. abstract syntax tree
 - ▶ code generation issues are quite clumsy at high-level
 - ▶ adequate for high-level optimizations (cache, loops)
- ▶ **Medium level**
 - ▶ represent source variables, temporaries, (and registers)
 - ▶ reduce control flow to conditional and unconditional branches
 - ▶ adequate to perform machine independent optimizations
- ▶ **Low level**
 - ▶ correspond to target-machine instructions
 - ▶ adequate to perform machine dependent optimizations

Different Kinds of Optimizations

...for different purposes, e.g.:

- ▶ Speeding up execution of compiled code
- ▶ Size of compiled code
 - ▶ when committed to read-only memory where size is an economic constraint
 - ▶ or code is transmitted over a limited-bandwidth communications channel
- ▶ Energy consumption
- ▶ Response to real-time events
- ▶ etc.

Considerations for Optimization

- ▶ **Safety**
 - ▶ **correctness**: generated code must have the same meaning as the input code
 - ▶ **meaning**: is the observable behavior of the program
- ▶ **Profitability**
 - ▶ improvement of code
 - ▶ trade offs between different kinds of optimizations
- ▶ **Problems**
 - ▶ reading past array bounds, pointer arithmetics, etc.

Scope of Optimization (1)

- ▶ Local
 - ▶ basic blocks
 - ▶ statements are executed sequentially
 - ▶ if any statement is executed the entire block is executed
 - ▶ limited to improvements that involve operations that all occur in the same block
- ▶ Intra-procedural (global)
 - ▶ entire procedure
 - ▶ procedure provides a natural boundary for both analysis and transformation
 - ▶ procedures are abstractions encapsulating and insulating run-time environments
 - ▶ opportunities for improvements that local optimizations do not have

Scope of Optimization (2)

- ▶ Inter-procedural (whole program)
 - ▶ entire program
 - ▶ exposes new opportunities but also new challenges
 - ▶ name-scoping
 - ▶ parameter binding
 - ▶ virtual methods
 - ▶ recursive methods (number of variables?)
 - ▶ scalability to program size

Optimization Taxonomy

Optimizations are categorized by the effect they have on the code.

- ▶ **Machine independent**
 - ▶ largely ignore the details of the target machine
 - ▶ in many cases profitability of a transformation depends on detailed machine-dependent issues, but those are ignored
- ▶ **Machine dependent**
 - ▶ explicitly consider details of the target machine
 - ▶ many of these transformations fall into the realm of code generation
 - ▶ some are within the scope of the optimizer (some cache optimizations, some expose instruction level parallelism)

Machine Independent Optimizations (1)

- ▶ **Dead code elimination**
 - ▶ eliminate useless or unreachable code
 - ▶ algebraic identities
- ▶ **Code motion**
 - ▶ move operation to place where it executes less frequently
 - ▶ loop invariant code motion, hoisting, constant propagation
- ▶ **Specialize**
 - ▶ to specific context in which an operation will execute
 - ▶ operator strength reduction, constant propagation, peephole optimization

Machine Independent Optimizations (2)

- ▶ **Eliminate redundancy**
 - ▶ replace redundant computation with a reference to previously computed value
 - ▶ e.g. common subexpression elimination, value numbering
- ▶ **Enable other transformations**
 - ▶ rearrange code to expose more opportunities for other transformations
 - ▶ e.g. inlining, cloning

Machine Dependent Optimizations

- ▶ Take advantage of special hardware features
 - ▶ Instruction Selection
- ▶ Manage or hide latency
 - ▶ Arrange final code in a way that hides the latency of some operations
 - ▶ Instruction Scheduling
- ▶ Manage bounded machine resources
 - ▶ Registers, functional units, cache memory, main memory

Case Study: C++STL Code Optimization

...on the impact of programming style and optimization on performance.

- ▶ Different programming styles for iterating on a container and performing operation on each element
- ▶ Use different levels of abstractions for iteration, container, and operation on elements
- ▶ Optimization levels O1-3 compared with GNU 4.0 compiler

Concrete example: We iterate on container 'mycontainer' and perform an operation on each element.

- ▶ Container is a vector
- ▶ Elements are of type `numeric_type` (double)
- ▶ Operation of adding 1 is applied to each element
- ▶ Evaluation Cases EC1-6

Acknowledgement: Joint work of Markus Schordan&Rene Heinzl.

Programming Styles - 1&2

EC1: Imperative Programming

```
for (unsigned int i = 0; i < mycontainer.size(); ++i)
{
    mycontainer[i] += 1.0;
}
```

EC2: Weakly Generic Programming

```
for (vector<numeric_type>::iterator
    it = mycontainer.begin();
    it != mycontainer.end();
    ++it)
{
    *it += 1.0;
}
```

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Programming Style - 3

EC3: Generic Programming

```
for_each(mycontainer.begin(),
         mycontainer.end(),
         plus_n<numeric_type>(1.0) );
```

Functor

```
template<class datatype>
struct plus_n
{
    plus_n(datatype member):member(member) {}
    void operator()(datatype& value) {
        value += member;
    }
private:
    datatype member;
};
```

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EC4: Functional Programming with STL

```
transform(mycontainer.begin(),  
         mycontainer.end(),  
         mycontainer.begin(),  
         bind2nd(std::plus<numeric_type>(), 1.0));
```

- ▶ plus: binary function object that returns the result of adding its first and second arguments
- ▶ bind2nd: Templated utility for binding values to function objects

Programming Styles - 5&6

EC5: Functional Programming with Boost::lambda

```
std::for_each(mycontainer.begin(),  
             mycontainer.end(),  
             boost::lambda::_1 += 1.0 );
```

EC6: Functional Programming with Boost::phoenix

```
std::for_each(mycontainer.begin(),  
             mycontainer.end(),  
             phoenix::arg1 += 1.0 );
```

- ▶ Use of unnamed function object.

Evaluation (EC1-6 without optimization)

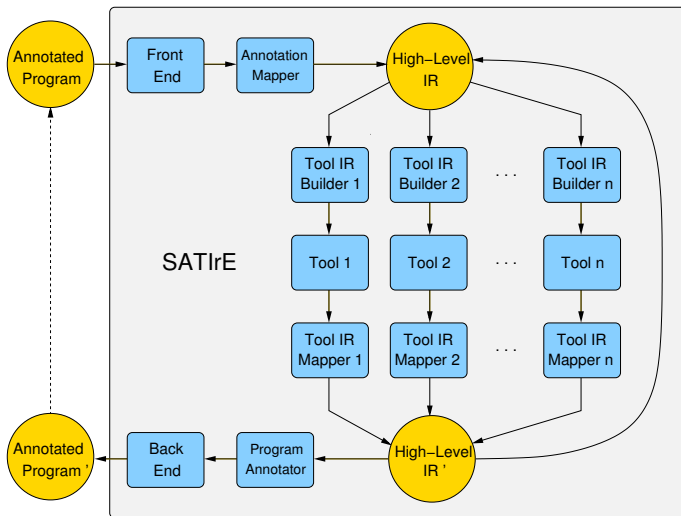
- ▶ Compiler: GNU g++ 4.0
- ▶ Evaluation Cases 1-6
- ▶ Time measured in milliseconds, container size: 1,000

Evaluation: Optimization Levels O1-3

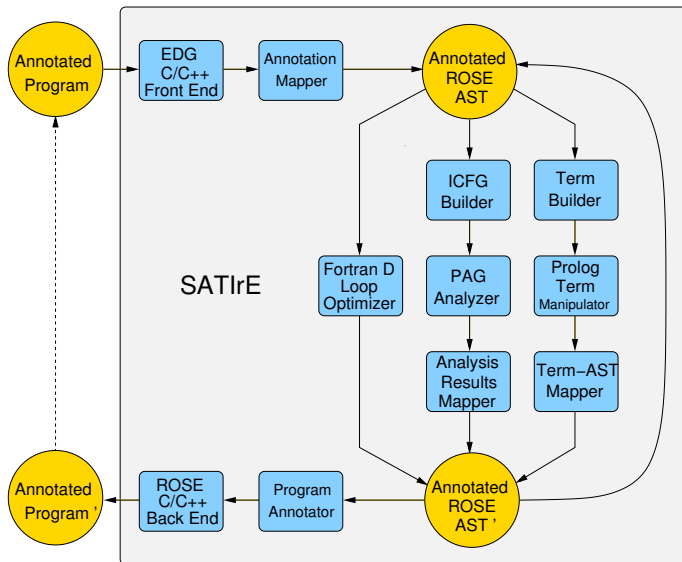
- ▶ Compiler: GNU g++ 4.0
- ▶ The actual run-time with different optimization levels -O1, -O2, -O3 for each programming style (EC1-6)
- ▶ An almost identical run-time is achieved at level -O3.

Static Analysis and Tool Integration Engine

SATIrE:



SATIrE: Concrete Architecture (Oct'07)



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SATIrE Components (1)

- ▶ **C/C++ Front End** (Edison Design Group)
- ▶ **Annotation Mapper** (maps source-code annotations to an accessible representation in the ROSE-AST)
- ▶ **Program Annotator** (annotates programs with analysis results; combined with the Annotation Mapper this allows to make analysis results persistent in source-code for subsequent analysis and optimization)
- ▶ **C/C++ Back End** (generates C++ code from ROSE-AST)

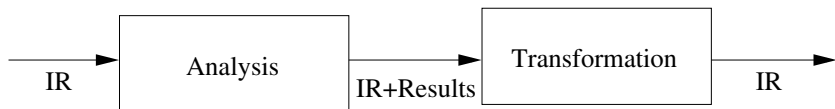
SATIrE Components (2)

- ▶ **Integration 1 (Loop Optimizer)**
 - ▶ **Loop Optimizer:** ported from the Fortran D compiler and integrated in LLNL-ROSE
- ▶ **Integration 2 (PAG)**
 - ▶ **ICFG Builder:** Interprocedural Control Flow Graph Generator, addresses full C++
 - ▶ **PAG Analyzer:** a program analyzer, generated with AbsInt's Program Analysis Generator (PAG) from a user-specified program analysis
 - ▶ **Analysis Results Mapper:** Maps Analysis Results from ICFG back to ROSE-AST, makes them available as AST-Attributes

SATIrE Components (3)

- ▶ Integration 3 (Termite)
 - ▶ **Term Builder:** generates an external textual term representation of the ROSE-AST (Term is in Prolog syntax)
 - ▶ **Term-AST Mapper:** parses the external textual program representation and translates it into a ROSE-AST

Optimization – Schematic View



- ▶ **Analysis**
 - ▶ determine properties of program
 - ▶ safe, pessimistic assumptions
- ▶ **Transformation**
 - ▶ based on analysis results

The Essence of Program Analysis

...**program analysis** offers techniques for predicting statically at compile-time safe and efficient **approximations** to the set of configurations or behaviors arising dynamically at run-time.

- ▶ **Safe:** faithful to the semantics
- ▶ **Efficient:** implementation with
 - ▶ good time performance
 - ▶ low space consumption

Typical Optimization Aspects

- ▶ **Avoid redundant computations**
 - ▶ reuse available results
 - ▶ move loop invariant computations outside loops
- ▶ **Avoid superfluous computations**
 - ▶ results known not to be needed
 - ▶ results known already at compile time

...to be demonstrated in some examples next.

Example: Lowering, IR, Address Computation

```
int a[m][n], b[m][n], c[m][n];
...
for(int i=0; i<m; ++i) {
    for(int j=0; j<n; ++j) {
        a[i][j]=b[i][j]+c[i][j];
    }
}
```

```
i=0;
while(i<m) {
    j=0;
    while(j<n) {
        temp=Base(a)+i*n+j;
        *(temp)=*(Base(b)+i*n+j)+*(Base(c)+i*n+j);
        j=j+1;
    }
    i=i+1;
}
```

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Analysis: Available Expressions Analysis

...determines for each program point, which expression **must** have already been computed, and not later modified, on all paths to the program point.

```
i=0;
while(i<m) {
    j=0;
    while(j<n) {
        temp = (Base(a)+i*n+j);
        *temp = *(Base(b)+i*n+j) + *(Base(c)+i*n+j);
        j=j+1;
    }
    i=i+1;
}
```

Optimization: Common Subexpression Elim.

- ▶ **Analysis:** Available Expressions Analysis
- ▶ **Transformation:** Eliminate recomputations of `i*n+j`
 - ▶ Introduce `t1=i*n+j`
 - ▶ Use `t1` instead of `i*n+j`

```
i=0;
while(i<m) {
    j=0;
    while(j<n) {
        temp = (Base(a)+ i*n+j );
        *temp = *(Base(b)+ i*n+j )
            + *(Base(c)+ i*n+j );
        j=j+1;
    }
    i=i+1;
}
```

```
i=0;
while(i<m) {
    j=0;
    while(j<n) {
        t1=i*n+j ;
        temp = (Base(a)+ t1 );
        *temp = *(Base(b)+ t1 )
            + *(Base(c)+ t1 );
        j=j+1;
    }
    i=i+1;
}
```

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Analysis: Loop Invariant Detection

...a **loop invariant** is an expression that is always computed to the same value in each iteration of the loop.

```
i=0;
while(i<m) {
    j=0;
    while(j<n) {
        t1=i*n+j;
        temp = (Base(a)+t1);
        *temp = *(Base(b)+t1) + *(Base(c)+t1);
        j=j+1;
    }
    i=i+1;
}
```


Optimization: Loop Invariant Code Motion

- ▶ **Analysis:** loop invariant detection
- ▶ **Transformation:** move loop invariant outside loop
 - ▶ introduce `t2=i*n` and replace `i*n` by `t2`
 - ▶ move `t2=i*n` outside loop

```
i=0;
while(i<m) {
    j=0;

    while(j<n) {
        t1=i*n+j;
        temp = (Base(a)+t1);
        *temp = *(Base(b)+t1)
            + *(Base(c)+t1);
        j=j+1;
    }
    i=i+1;
}
```

```
i=0;
while(i<m) {
    j=0;
    t2=i*n;
    while(j<n) {
        t1=t2+j;
        temp = (Base(a)+t1);
        *temp = *(Base(b)+t1)
            + *(Base(c)+t1);
        j=j+1;
    }
    i=i+1;
}
```

Analysis: Induction Variable Detection

```
i=0;
while(i<m) {
    j=0;
    t2=i*n;
    while(j<n) {
        t1=t2+j;
        temp = (Base(a)+t1);
        *temp = *(Base(b)+t1)
            + *(Base(c)+t1);
        j=j+1;
    }
    i=i+1;
}
```

Basic Induction Variables

- ▶ Variables i whose only definitions within a loop are of the form $i = i + c$ or $i = i - c$ and c is a loop invariant.

Derived Induction Variables

- ▶ Variables j defined only once in a loop whose value is a linear function of some basic induction variable.

Optimization: Strength Reduction (1)

...replaces a repeated series of **expensive** (“strong”) **operations** with a series of **inexpensive** (“weak”) **operations** that compute the same values.

Classical example:

- ▶ Replacing integer multiplications based on a loop index with equivalent additions.

Note: This particular case arises routinely from expansion of array and structure addresses in loops.

Optimization: Strength Reduction (2)

- ▶ **Analysis:** induction variable detection
- ▶ **Transformation:** move multiplication outside of loop
 - ▶ introduce `t3=i*n` before the loop, replace `i*n` by `t3`
 - ▶ add `t3=t3+i*c` at every update site of `i`

```
i=0;

while(i<m) {
    j=0;
    t2=i*n;
    while(j<n) {
        t1=t2+j;
        temp = (Base(a)+t1);
        *temp = *(Base(b)+t1)
            + *(Base(c)+t1);
        j=j+1;
    }
    i=i+1;
}
```

```
i=0;
t3=0;
while(i<m) {
    j=0;
    t2=t3;
    while(j<n) {
        t1=t2+j;
        temp = (Base(a)+t1);
        *temp = *(Base(b)+t1)
            + *(Base(c)+t1);
        j=j+1;
    }
    i=i+1;
    t3=t3+n;
}
```

Analysis: Copy Analysis

...determines for each program point, which copy statements $x = y$ that still are relevant (i.e. neither x nor y have been redefined) when control reaches that point.

```
i=0;
t3=0;
while(i<m) {
    j=0;
    t2=t3;
    while(j<n) {
        t1=t2+j;
        temp = (Base(a)+t1);
        *temp = *(Base(b)+t1)
            + *(Base(c)+t1);
        j=j+1;
    }
    i=i+1;
    t3=t3+n;
}
```

Optimization: Copy Propagation

- ▶ **Analysis:** Copy Analysis and def-use chains (ensure only one definition reaches the use of x)
- ▶ **Transformation:** Replace the use of x by y .

```
i=0;
t3=0;
while(i<m) {
    j=0;
    t2=t3;
    while(j<n) {
        t1=t2+j;
        temp = (Base(a)+t1);
        *temp = *(Base(b)+t1)
            + *(Base(c)+t1);
        j=j+1;
    }
    i=i+1;
    t3=t3+n;
}
```

```
i=0;
t3=0;
while(i<m) {
    j=0;
    t2=t3;
    while(j<n) {
        t1=t3+j;
        temp = (Base(a)+t1);
        *temp = *(Base(b)+t1)
            + *(Base(c)+t1);
        j=j+1;
    }
    i=i+1;
    t3=t3+n;
}
```

Live Variables, Dead Variables

- ▶ A variable is **live** at a program point if there is a path from this program point to a use of the variable that does not re-define the variable.
- ▶ If a variable is not live, it is **dead**.

A **live (dead) variable analysis**

- ▶ determines for each program point, which variable **may be live (is dead)** at the exit from that point.

Analysis: Dead Variable Analysis

```
i=0;
t3=0;
while(i<m) {
    j=0;
    t2=t3;
    while(j<n) {
        t1=t3+j;
        temp = (Base(a)+t1);
        *temp = *(Base(b)+t1)
            + *(Base(c)+t1);
        j=j+1;
    }
    i=i+1;
    t3=t3+n;
}
```

- ▶ Only **dead** variables are marked.

Optimization: Dead Code Elimination

- ▶ **Analysis:** dead variable analysis
- ▶ **Transformation:** remove all assignments to dead variables

```
i=0;
t3=0;
while(i<m) {
    j=0;
    t2=t3;
    while(j<n) {
        t1=t3+j;
        temp = (Base(a)+t1);
        *temp = *(Base(b)+t1)
            + *(Base(c)+t1);
        j=j+1;
    }
    i=i+1;
    t3=t3+n;
}
```

```
i=0;
t3=0;
while(i<m) {
    j=0;
    while(j<n) {
        t1=t3+j;
        temp = (Base(a)+t1);
        *temp = *(Base(b)+t1)
            + *(Base(c)+t1);
        j=j+1;
    }
    i=i+1;
    t3=t3+n;
}
```

Optimizations at a Glance

Analyses

Available expr. analysis
Loop invariant detection
Induction variable detection
Copy analysis
Live variables analysis

Transformations

Common subexpr. elim.
Invariant code motion
Strength reduction
Copy propagation
Dead code elimination

Further optimizations, i.e., analyses and transformations?

Pointer/Alias/Shape Analysis (1)

Problem

- ▶ Ambiguous memory references interfere with an optimizer's ability to improve code.
- ▶ One major source of ambiguity is the use of pointer-based values.

Goal of Pointer/Alias/Shape Analysis

- ▶ determine for each pointer the set of memory locations to which it may refer.

Pointer/Alias/Shape Analysis (2)

Without such analysis the compiler must assume that each pointer can refer to any addressable value, including

- ▶ any space allocated in the run-time heap
- ▶ any variable whose address is explicitly taken
- ▶ any variable passed as a call-by-reference parameter

Forms of Pointer Analysis

- ▶ points-to sets
- ▶ alias pairs
- ▶ shape analysis

Questions about Heap Contents (1)

Let **execution state** mean the set of cells in the heap, the connections between them (via pointer components of heap cells) and the values of pointer variables in the store.

- ▶ **NULL pointers**: Does a pointer variable or a pointer component of a heap cell contain NULL at the entry to a statement that dereferences the pointer or component?
 - ▶ **Yes (for every state)**. Issue an error message.
 - ▶ **No (for every state)**. Eliminate a check for **NULL**.
 - ▶ **Maybe**. Warn about the potential **NULL** dereference.
- ▶ **Memory leak**: Does a procedure or a program leave behind unreachable heap cells when it returns?
 - ▶ **Yes (in some state)**. Issue a warning.

Questions about Heap Contents (2)

- ▶ **Aliasing:** Do two pointer expressions reference the same heap cell?
 - ▶ **Yes (for every state).**
 - ▶ trigger a prefetch to improve cache performance
 - ▶ predict a cache hit to improve cache behavior prediction
 - ▶ increase the sets of uses and definitions for an improved liveness analysis
 - ▶ **No (for every state).** Disambiguate memory references and improve program dependence information.
- ▶ **Sharing:** Is a heap cell shared? (within the heap)
 - ▶ **Yes (for some state).** Warn about explicit deallocation, because the memory manager may run into an inconsistent state.
 - ▶ **No (for every state).** Explicitly deallocate the heap cell when the last pointer to ceases to exist.

Questions about Heap Contents (3)

- ▶ **Reachability:** Is a heap cell reachable from a specific variable or from any pointer variable?
 - ▶ **Yes (for every state).** Use this information for program verification.
 - ▶ **No (for every state).** Insert code at compile time that collects unreachable cells at run-time.
- ▶ **Disjointness:** Do two data structures pointed to by two distinct pointer variables ever have common elements?
 - ▶ **No (for every state).** Distribute disjoint data structures and their computations to different processors.
- ▶ **Cyclicity:** Is a heap cell part of a cycle?
 - ▶ **No (for every state).** Perform garbage collection of data structures by reference counting. Process all elements in an acyclic linked list in a doall-parallel fashion.

Optimizations f. Object-Oriented Languages (1)

Invoking a method in an object-oriented language requires looking up the address of the block of code which implements that method and passing control to it.

Opportunities for optimization




- ▶ Look-up may be performed at compile time
- ▶ Only one implementation of the method in the class and in its subclasses
- ▶ Language provides a declaration which forces the call to be non-virtual
- ▶ Compiler performs static analysis which can determine that a unique implementation is always called at a particular call-site.

Optimizations f. Object-Oriented Languages (2)




Related Optimizations

- ▶ Dispatch Table Compression
- ▶ Devirtualization
- ▶ Inlining
- ▶ **Escape Analysis** for allocating objects on the run-time stack (instead of the heap)

Further Reading for Chapter 1 (1)

-  Randy Allen, Ken Kennedy. *Optimizing Compilers for Modern Architectures*. Morgan Kaufman Publishers, 2002. (Chapter 1, Compiler Challenges for High-Performance Architectures)
-  Melvin E. Conway. *Proposal for an UNCOL*. Communications of the ACM 1(3):5, 1958.
-  Keith D. Cooper, Linda Torczon. *Engineering a Compiler*. Morgan Kaufman Publishers, 2004. (Chapter 1, Overview of Compilation; Chapter 8, Introduction to Code Optimization; Chapter 10, Scalar Optimizations)

Further Reading for Chapter 1 (2)

-  Donald E. Knuth. *An Empirical Study of Fortran Programs*. Software – Practice and Experience 1:105-13, 1971.
-  Stephen S. Muchnick. *Advanced Compiler Design Implementation*. Morgan Kaufman Publishers, 1997. (Chapter 1, Introduction to Advanced Topics)
-  Flemming Nielson, Hanne Riis Nielson, Chris Hankin. *Principles of Program Analysis*. 2nd edition, Springer-V., 2005. (Chapter 1, Introduction)

Chapter 2

Data Flow Analysis in a Nutshell

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Chapter 2.1

Program Analysis

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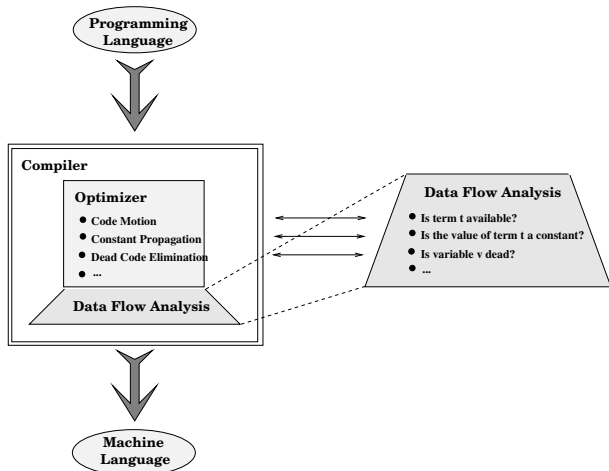
Typical Questions

...of program analysis, especially **data flow analysis**:

- ▶ What is the **value** of a variable at a program point?
~> Constant propagation and folding
- ▶ Is the value of an expression **available** at a program point?
~> (Partial) redundancy elimination
- ▶ Is a variable **dead** at a program point?
~> Elimination of (partially) dead code

Common (and also our) Application Scenario

...(program) analysis for (program) optimization:



Essential Issues

comprise...

fundamental ones

- ▶ What does **optimality** mean?
...in analysis and optimization?

as (apparently) minor ones:

- ▶ What is an **appropriate** and **suitable** program representation?

Outlook

In more detail we will distinguish:

- ▶ intraprocedural
- ▶ interprocedural
- ▶ parallel
- ▶ ...

data flow analysis (DFA).

Outlook (cont'd)

Ingredients of (intraprocedural) data flow analysis:

- ▶ (Local) abstract semantics
 1. A data flow analysis lattice $\hat{\mathcal{C}} = (\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \perp, \top)$
 2. A data flow analysis functional $\llbracket \] \! : E \rightarrow (\mathcal{C} \rightarrow \mathcal{C})$
 3. A Start information (start assertion) $c_s \in \mathcal{C}$
- ▶ Globalization strategies
 1. “Meet over all Paths” Approach (*MOP*)
 2. Maximum Fixed Point Approach (*MaxFP*)
- ▶ Generic Fixed Point Algorithm

Theory of Intraprocedural DFA

Main Results:

- ▶ Safety (Soundness) Theorem
- ▶ Coincidence (Completeness) Theorem

Plus:

- ▶ Effectivity (Termination) Theorem

Practice of Intraprocedural DFA

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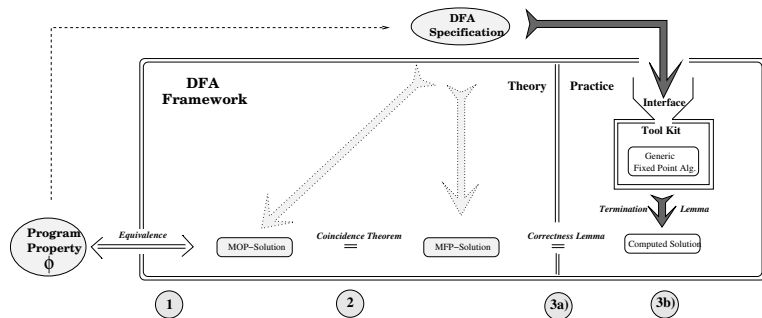
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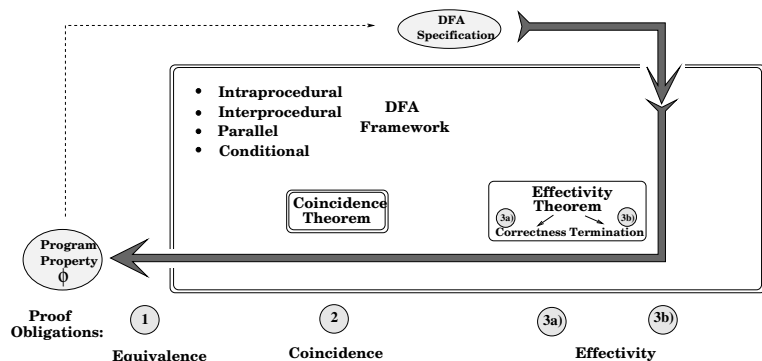
The Intraprocedural DFA Framework / DFA Toolkit View:



Practice of DFA

The constraint “intraprocedural” can be dropped.

The DFA Framework / DFA-Toolkit View holds generally:



Ultimate Goal

Optimal Program Optimization

...a white “Schimmel” (two twins) in computer science?

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There is no free Lunch!

In the diction of [optimizing compilation](#):

...w/out [analysis](#) no optimization!

Further Reading for Chapter 2.1

-  Alfred V. Aho, Monica S. Lam, Ravi Sethi, Jeffrey D. Ullman. *Compilers: Principles, Techniques, & Tools*. Addison-Wesley, 2nd edition, 2007. (Chapter 1.2, The Structure of a Compiler; Chapter 1.4, The Science of Building a Compiler; Chapter 1.4.2, The Science of Code Optimization; Chapter 9.1, The Principal Sources of Program Optimization)

Chapter 2.2

Forward Analyses

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Formalising the Development

- ▶ the programming language of interest
 - ▶ abstract syntax
 - ▶ labelled program fragments
- ▶ abstract flow graphs
 - ▶ control and data flow between labelled program fragments
- ▶ extract equations from the program
 - ▶ specify the information to be computed at entry and exit of labeled fragments
- ▶ compute the solution to the equations
 - ▶ work list algorithms
 - ▶ compute entry and exit information at entry and exit of labelled fragments

WHILE Language

Syntactic categories

$a \in \text{AExp}$ arithmetic expressions

$b \in \text{BExp}$ boolean expressions

$S \in \text{Stmt}$ statements

$x, y \in \text{Var}$ variables

$n \in \text{Num}$ numerals

$\ell \in \text{Lab}$ labels

$op_a \in \text{Op}_a$ arithmetic operators

$op_b \in \text{Op}_b$ boolean operators

$op_r \in \text{Op}_r$ relational operators

Abstract Syntax

$$\begin{aligned} a & ::= x \mid n \mid a_1 \text{ op}_a a_2 \\ b & ::= \text{true} \mid \text{false} \mid \text{not } b \mid b_1 \text{ op}_b b_2 \mid a_1 \text{ op}_r a_2 \\ S & ::= [x:=a]^\ell \mid [\text{skip}]^\ell \\ & \quad \mid \text{if } [b]^\ell \text{ then } S_1 \text{ else } S_2 \\ & \quad \mid \text{while}[b]^\ell \text{ do } S \text{ od} \\ & \quad \mid S_1; S_2 \end{aligned}$$

Assignments and **tests** are (uniquely) labelled to allow analyses to refer to these program fragments – the labels correspond to pointers into the syntax tree. We use abstract syntax and insert parenthesis to disambiguate syntax.

We will often refer to labelled fragments as **elementary blocks**.

Auxiliary Functions for Flow Graphs

- labels(S) set of nodes of flow graphs of S
- init(S) initial node of flow graph of S ; the unique node where execution of program starts
- final(S) final nodes of flow graph for S ; set of nodes where program execution may terminate
- flow(S) edges of flow graphs for S (used for forward analyses)
- flow^R(S) reverse edges of flow graphs for S (used for backward analyses)
- blocks(S) set of elementary blocks in a flow graph

Computing the Information (1)

S	$\text{labels}(S)$	$\text{init}(S)$	$\text{final}(S)$
$[x := a]^\ell$	$\{l\}$	l	$\{l\}$
$[\text{skip}]^\ell$	$\{l\}$	l	$\{l\}$
$S_1; S_2$	$\text{labels}(S_1) \cup \text{labels}(S_2)$	$\text{init}(S_1)$	$\text{final}(S_2)$
$\text{if } [b]^\ell \text{ then } (S_1) \text{ else } (S_2)$	$\{l\} \cup \text{labels}(S_1) \cup \text{labels}(S_2)$	l	$\text{final}(S_1) \cup \text{final}(S_2)$
$\text{while } [b]^\ell \text{ do } S \text{ od}$	$\{l\} \cup \text{labels}(S)$	l	$\{l\}$

Computing the Information (2)

S	$\text{flow}(S)$	$\text{blocks}(S)$
$[x := a]^\ell$	\emptyset	$\{[x := a]^\ell\}$
$[\text{skip}]^\ell$	\emptyset	$\{[\text{skip}]^\ell\}$
$S_1; S_2$	$\text{flow}(S_1) \cup \text{flow}(S_2) \cup \{(\ell, \text{init}(S_2)) \mid \ell \in \text{final}(S_1)\}$	$\text{blocks}(S_1) \cup \text{blocks}(S_2)$
$\text{if } [b]^\ell \text{ then } (S_1) \text{ else } (S_2)$	$\text{flow}(S_1) \cup \text{flow}(S_2) \cup \{(\ell, \text{init}(S_1)), (\ell, \text{init}(S_2))\}$	$\{[b]^\ell\} \cup \text{blocks}(S_1) \cup \text{blocks}(S_2)$
$\text{while } [b]^\ell \text{ do } S \text{ od}$	$\{(\ell, \text{init}(S))\} \cup \text{flow}(S) \cup \{(\ell', \ell) \mid \ell' \in \text{final}(S)\}$	$\{[b]^\ell\} \cup \text{blocks}(S)$

$$\text{flow}^R(S) = \{(\ell, \ell') \mid (\ell', \ell) \in \text{flow}(S)\}$$

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Program of Interest (1)

We shall use the following notation:

- ▶ S_* to represent the program being analyzed (the “top level” statement)
- ▶ Lab_* to represent the labels ($labels(S_*)$) appearing in S_*
- ▶ Var_* to represent the variables ($FV(S_*)$) appearing in S_*
- ▶ $Blocks_*$ to represent the elementary blocks ($blocks(S_*)$) occurring in S_*
- ▶ $AExp_*$ to represent the set of *non-trivial* arithmetic subexpressions in S_* ; an expression is trivial if it is a single variable or constant
- ▶ $AExp(a)$, $AExp(b)$ to refer to the set of non-trivial arithmetic subexpressions of a given arithmetic, respectively boolean, expression

Program of Interest (2)

Free Variables $FV(a)$

The free variables of an arithmetic expression, $a \in AExp$, are defined to be variables occurring in it.

Compositional definition of subset $FV(a)$ of Var

$$FV(n) = \emptyset$$

$$FV(x) = \{x\}$$

$$FV(a_1 + a_2) = FV(a_1) \cup FV(a_2)$$

$$FV(a_1 * a_2) = FV(a_1) \cup FV(a_2)$$

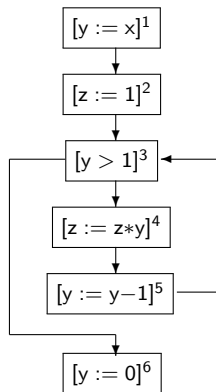
$$FV(a_1 - a_2) = FV(a_1) \cup FV(a_2)$$

Similarly for boolean expressions, $b \in BExp$, and statements, $S \in Stmt$, such that $Var_* = FV(S_*)$.

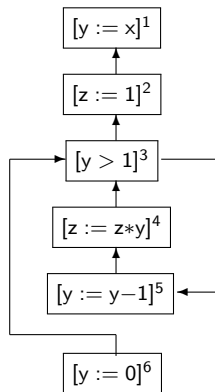
Flow Graphs – Example (1)

Example:

$[y := x]^1; [z := 1]^2; \text{while } [y > 1]^3 \text{ do } [z := z * y]^4; [y := y - 1]^5 \text{ od}; [y := 0]^6$



$\text{flow}(S_*) = \{(1, 2), (2, 3), (3, 4),$
 $(4, 5), (5, 3), (3, 6)\}$



$\text{flow}^R(S_*) = \{(6, 3), (3, 5), (5, 4),$
 $(4, 3), (3, 2), (2, 1)\}$

Flow Graphs – Example (2)

Example:

$[y := x]^1; [z := 1]^2; \text{while } [y > 1]^3 \text{ do } [z := z * y]^4; [y := y - 1]^5 \text{ od}; [y := 0]^6$

$$\text{labels}(S_*) = \{1, 2, 3, 4, 5, 6\}$$

$$\text{init}(S_*) = 1$$

$$\text{final}(S_*) = \{6\}$$

$$\text{flow}(S_*) = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 3), (3, 6)\}$$

$$\text{flow}^R(S_*) = \{(6, 3), (3, 5), (5, 4), (4, 3), (3, 2), (2, 1)\}$$

$$\text{blocks}(S_*) = \{[y := x]^1, [z := 1]^2, [y > 1]^3, \\ [z := z * y]^4, [y := y - 1]^5, [y := 0]^6\}$$

Simplifying Assumptions

The program of interest S_\star is often assumed to satisfy:

- ▶ S_\star has isolated entries if there are no edges leading into $\text{init}(S_\star)$:

$$\forall \ell : (\ell, \text{init}(S_\star)) \notin \text{flow}(S_\star)$$

- ▶ S_\star has isolated exits if there are no edges leading out of labels in $\text{final}(S_\star)$:

$$\forall \ell \in \text{final}(S_\star), \forall \ell' : (\ell, \ell') \notin \text{flow}(S_\star)$$

- ▶ S_\star is label consistent if

$$\forall B_1^{\ell_1}, B_2^{\ell_2} \in \text{blocks}(S_\star) : \ell_1 = \ell_2 \rightarrow B_1 = B_2$$

This holds if S_\star is uniquely labelled.

Reaching Definitions Analysis

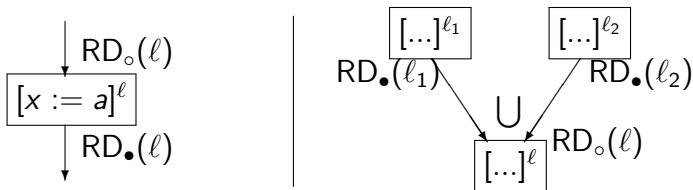
...determines for each program point, which assignments **may** have been made and not overwritten, when program execution reaches this point along some path.

Example:

$[y := x]^1; [z := 1]^2; \text{while } [y > 1]^3 \text{ do } [z := z * y]^4; [y := y - 1]^5 \text{ od}; [y := 0]^6$

- ▶ The assignments labelled 1,2,4,5 reach the entry at 4.
- ▶ Only the assignments labelled 1,4,5 reach the entry at 5.

Basic Idea



Analysis information: $RD_o(l), RD_\bullet(l) : \text{Lab}_* \rightarrow \mathcal{P}\text{Var}_* \times \text{Lab}_*^?$

- ▶ $RD_o(l)$: the definitions that reach **entry** of block l .
- ▶ $RD_\bullet(l)$: the definitions that reach **exit** of block l .

Analysis properties:

- ▶ Direction: forward
- ▶ May analysis with combination operator \cup

Analysis of Elementary Blocks

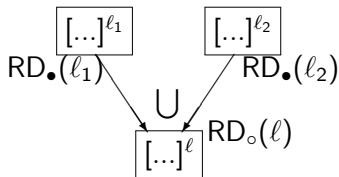
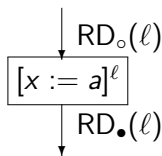
$$\begin{aligned}\text{kill}_{\text{RD}}([x := a]^\ell) &= \{(x, ?)\} \cup \{(x, \ell') \mid B^{\ell'} \text{ is assignment to } x\} \\ \text{kill}_{\text{RD}}([\text{skip}]^\ell) &= \emptyset \\ \text{kill}_{\text{RD}}([b]^\ell) &= \emptyset \\ \text{gen}_{\text{RD}}([x := a]^\ell) &= \{(x, \ell)\} \\ \text{gen}_{\text{RD}}([\text{skip}]^\ell) &= \emptyset \\ \text{gen}_{\text{RD}}([b]^\ell) &= \emptyset\end{aligned}$$

Example:

$[x := y]^1; [x := x + 3]^2;$

- ▶ $\text{kill}_{\text{RD}}([x := y]^1) = \{(x, ?)\} \cup \{(x, 1), (x, 2)\}$
- ▶ $\text{gen}_{\text{RD}}([x := y]^1) = \{(x, 1)\}$

Analysis of the Program



$$\begin{aligned}
 RD_o(l) &= \begin{cases} \{(x, ?) \mid x \in FV(S_*)\} & : \text{ if } l = \text{init}(S_*) \\ \bigcup \{RD_•(l') \mid (l', l) \in \text{flow}(S_*)\} & : \text{ otherwise} \end{cases} \\
 RD_•(l) &= (RD_o(l) \setminus \text{kill}_{RD}(B^l)) \cup \text{gen}_{RD}(B^l) \quad \text{where } B^l \in \text{blocks}(S_*)
 \end{aligned}$$

Example

Example:

$[y := x]^1; [z := 1]^2; \text{while } [y > 1]^3 \text{ do } [z := z * y]^4; [y := y - 1]^5 \text{ od}; [y := 0]^6$

Equations: Let

$$S_1 = \{(y, ?), (y, 1), (y, 5), (y, 6)\}, S_2 = \{(z, ?), (z, 2), (z, 4)\}$$

$$RD_o(1) = \{(x, ?), (y, ?), (z, ?)\} \quad RD_\bullet(1) = RD_o(1) \setminus S_1 \cup \{(y, 1)\}$$

$$RD_o(2) = RD_\bullet(1) \quad RD_\bullet(2) = RD_o(2) \setminus S_2 \cup \{(z, 2)\}$$

$$RD_o(3) = RD_\bullet(2) \cup RD_\bullet(5) \quad RD_\bullet(3) = RD_o(3)$$

$$RD_o(4) = RD_\bullet(3) \quad RD_\bullet(4) = RD_o(4) \setminus S_2 \cup \{(z, 4)\}$$

$$RD_o(5) = RD_\bullet(4) \quad RD_\bullet(5) = RD_o(5) \setminus S_1 \cup \{(y, 5)\}$$

$$RD_o(6) = RD_\bullet(3) \quad RD_\bullet(6) = RD_o(6) \setminus S_1 \cup \{(y, 6)\}$$

ℓ	$RD_o(\ell)$	$RD_\bullet(\ell)$
1	$\{(x, ?), (y, ?), (z, ?)\}$	$\{(x, ?), (y, 1), (z, ?)\}$
2	$\{(x, ?), (y, 1), (z, ?)\}$	$\{(x, ?), (z, 2), (y, 1)\}$
3	$\{(x, ?), (z, 4), (z, 2), (y, 5), (y, 1)\}$	$\{(x, ?), (z, 4), (z, 2), (y, 5), (y, 1)\}$
4	$\{(x, ?), (z, 4), (z, 2), (y, 5), (y, 1)\}$	$\{(z, 4), (x, ?), (y, 5), (y, 1)\}$
5	$\{(z, 4), (x, ?), (y, 5), (y, 1)\}$	$\{(z, 4), (x, ?), (y, 5)\}$
6	$\{(x, ?), (z, 4), (z, 2), (y, 5), (y, 1)\}$	$\{(z, 4), (x, ?), (z, 2), (y, 6)\}$

Solving RD Equations

Input

- ▶ a set of reaching definitions equations

Output

- ▶ the **least solution** to the equations: RD_{\circ}

Data structures

- ▶ The current analysis result for block entries: RD_{\circ}
- ▶ The worklist W : a list of pairs (ℓ, ℓ') indicating that the current analysis result has changed at the entry to the block ℓ and hence the information must be recomputed for ℓ' .

Solving RD Equations – Algorithm

```
W:=nil;
foreach ( $\ell, \ell'$ )  $\in$  flow( $S_*$ ) do W := cons( $(\ell, \ell')$ ,W); od;
foreach  $\ell \in$  labels( $S_*$ ) do
  if  $\ell \in$  init( $S_*$ ) then
    RD $_o$ ( $\ell$ ) :=  $\{(x, ?) \mid x \in$  FV( $S_*$ ) $\}$ 
  else
    RD $_o$ ( $\ell$ ) :=  $\emptyset$ 
  fi
od
while W  $\neq$  nil do
  ( $\ell, \ell'$ ) := head(W);
  W := tail(W);
  if (RD $_o$ ( $\ell$ ) \ kill $_{RD}$ ( $B^{\ell}$ ))  $\cup$  gen $_{RD}$ ( $B^{\ell}$ )  $\not\subseteq$  RD $_o$ ( $\ell'$ ) then
    RD $_o$ ( $\ell'$ ) := RD $_o$ ( $\ell'$ )  $\cup$  (RD $_o$ ( $\ell$ ) \ kill $_{RD}$ ( $B^{\ell}$ ))  $\cup$  gen $_{RD}$ ( $B^{\ell}$ );
    foreach  $\ell''$  with ( $\ell', \ell''$ ) in flow( $S_*$ ) do
      W := cons( $(\ell', \ell'')$ ,W);
    od
  fi
od
```

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Use-Definition and Definition-Use Chains

► Use-Definition chains or *ud* chains

each use of a variable is linked to all assignments that *reach* it


$[x := 0]^1; [x := 5]^2; [y := x]^3; [z := x]^4$



► Definition-Use chains or *du* chains

each assignment of a variable is linked to all uses of it

$[x := 0]^1; [x := 5]^2; [y := x]^3; [z := x]^4$



UD/DU Chains – Defined via RDs

$$\text{UD, DU} : \text{Var}_* \times \text{Lab}_* \rightarrow \mathcal{P}(\text{Lab}_*)$$

are defined by

$$\text{UD}(x, \ell) = \begin{cases} \{\ell' \mid (x, \ell') \in \text{RD}_o(\ell)\} & : \text{ if } x \in \text{used}(B^\ell) \\ \emptyset & : \text{ otherwise} \end{cases}$$

where $\text{used}([x := a]^\ell) = \text{FV}(a)$, $\text{used}([b]^\ell) = \text{FV}(b)$,
 $\text{used}([\text{skip}]^\ell) = \emptyset$

and

$$\text{DU}(x, \ell) = \{\ell' \mid \ell \in \text{UD}(x, \ell')\}$$

Available Expressions Analysis

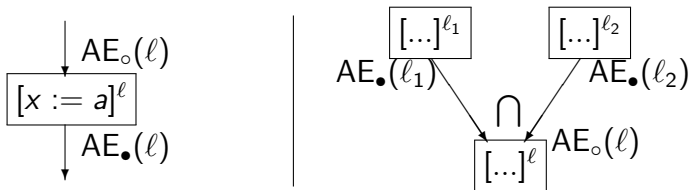
...determines for each program point, which expressions **must** have already been computed, and not later modified, on all paths to the program point.

Example:

$[x := a+b]^1; [y := a*x]^2; \text{while } [y > a+b]^3 \text{ do } [a := a + 1]^4; [x := a + b]^5 \text{ od}$

- ▶ No expression is available at the start of the program.
- ▶ An expression is considered available if no path kills it.
- ▶ The expression $a+b$ is available every time execution reaches the test in the loop at 3.

Basic Idea



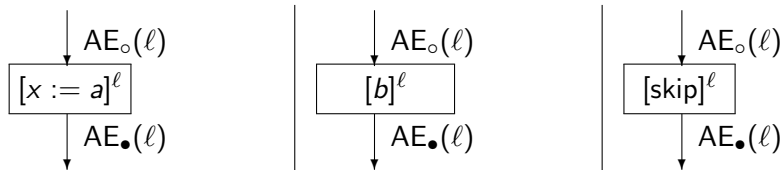
Analysis information: $AE_o(l), AE_\bullet(l) : \text{Lab}_* \rightarrow \mathcal{PAExp}_*$

- ▶ $AE_o(l)$: the expressions that have been comp. at **entry** of block l .
- ▶ $AE_\bullet(l)$: the expressions that have been comp. at **exit** of block l .

Analysis properties:

- ▶ Direction: forward
- ▶ Must analysis with combination operator \cap

Analysis of Elementary Blocks



$$\text{kill}_{AE}([x := a]^\ell) = \{a' \in AExp_\star \mid x \in FV(a')\}$$

$$\text{kill}_{AE}([\text{skip}]^\ell) = \emptyset$$

$$\text{kill}_{AE}([b]^\ell) = \emptyset$$

$$\text{gen}_{AE}([x := a]^\ell) = \{a' \in AExp(a) \mid x \notin FV(a')\}$$

$$\text{gen}_{AE}([\text{skip}]^\ell) = \emptyset$$

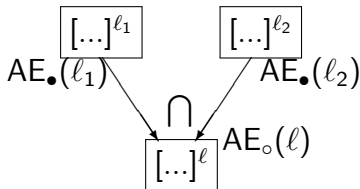
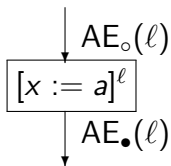
$$\text{gen}_{AE}([b]^\ell) = AExp(b)$$

Example: $[x := a+b]^1; [y := a*x]^2;$

► $\text{kill}_{AE}([x := a+b]^1) = \{a*x\}$

► $\text{gen}_{AE}([x := a+b]^1) = \{a+b\}$

Analysis of the Program



$$\begin{aligned}
 AE_o(\ell) &= \begin{cases} \emptyset & : \text{ if } \ell = \text{init}(S_\star) \\ \bigcap \{AE_\bullet(\ell') \mid (\ell', \ell) \in \text{flow}(S_\star)\} & : \text{ otherwise} \end{cases} \\
 AE_\bullet(\ell) &= (AE_o(\ell) \setminus \text{kill}_{AE}(B^\ell)) \cup \text{gen}_{AE}(B^\ell) \quad \text{where } B^\ell \in \text{blocks}(S_\star)
 \end{aligned}$$

Example

Example:

$[x := a+b]^1; [y := a*x]^2; \text{while } [y > a+b]^3 \text{ do } [a := a + 1]^4; [x := a + b]^5 \text{ od}$

Equations:

$$\begin{array}{ll} \text{AE}_o(1) = \emptyset & \text{AE}_\bullet(1) = \text{AE}_o(1) \setminus \{a * x\} \cup \{a + b\} \\ \text{AE}_o(2) = \text{AE}_\bullet(1) & \text{AE}_\bullet(2) = \text{AE}_o(2) \setminus \emptyset \cup \{a * x\} \\ \text{AE}_o(3) = \text{AE}_\bullet(2) \cap \text{AE}_\bullet(5) & \text{AE}_\bullet(3) = \text{AE}_o(3) \setminus \emptyset \cup \{a + b\} \\ \text{AE}_o(4) = \text{AE}_\bullet(3) & \text{AE}_\bullet(4) = \text{AE}_o(4) \setminus \{a + b, a * x, a + 1\} \cup \emptyset \\ \text{AE}_o(5) = \text{AE}_\bullet(4) & \text{AE}_\bullet(5) = \text{AE}_o(5) \setminus \{a * x\} \cup \{a + b\} \end{array}$$

ℓ	$\text{AE}_o(\ell)$	$\text{AE}_\bullet(\ell)$
1	\emptyset	$\{a+b\}$
2	$\{a+b\}$	$\{a+b, a*x\}$
3	$\{a+b\}$	$\{a+b\}$
4	$\{a+b\}$	\emptyset
5	\emptyset	$\{a+b\}$

Remark: predefined AE Analysis in PAG/WWW includes boolean expressions

Solving AE Equations

Input

- ▶ a set of available expressions equations

Output

- ▶ the **largest solution** to the equations: AE_{\circ}

Data structures

- ▶ The current analysis result for block entries: AE_{\circ}
- ▶ The worklist W : a list of pairs (ℓ, ℓ') indicating that the current analysis result has changed at the entry to the block ℓ and hence the information must be recomputed for ℓ' .

Solving AE Equations – Algorithm

```
W:=nil;
foreach  $(\ell, \ell') \in \text{flow}(S_*)$  do W := cons( $(\ell, \ell')$ ,W); od;
foreach  $\ell \in \text{labels}(S_*)$  do
  if  $\ell \in \text{init}(S_*)$  then
     $\text{AE}_o(\ell) := \emptyset$ 
  else
     $\text{AE}_o(\ell) := \text{AExp}_*$ 
  fi
od
while  $W \neq \text{nil}$  do
   $(\ell, \ell') := \text{head}(W)$ ;
   $W := \text{tail}(W)$ ;
  if  $(\text{AE}_o(\ell) \setminus \text{kill}_{\text{AE}}(B^\ell)) \cup \text{gen}_{\text{AE}}(B^\ell) \not\subseteq \text{AE}_o(\ell')$  then
     $\text{AE}_o(\ell') := \text{AE}_o(\ell') \cap (\text{AE}_o(\ell) \setminus \text{kill}_{\text{AE}}(B^\ell)) \cup \text{gen}_{\text{AE}}(B^\ell)$ ;
    foreach  $\ell''$  with  $(\ell', \ell'')$  in  $\text{flow}(S_*)$  do
       $W := \text{cons}((\ell', \ell''), W)$ ;
    od
  fi
od
```

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Common Subexpression Elimination (CSE)

...aims at finding computations that are always performed at least twice on a given execution path and to eliminate the second and later occurrences; it uses [Available Expressions Analysis](#) to determine the redundant computations.

Example:

```
[x := a+b]1; [y := a*x]2; while [y > a+b]3 do [a := a + 1]4; [x := a + b]5 od
```

- ▶ Expression $a+b$ is computed at 1 and 5 and recomputation can be eliminated at 3.

The Optimization – CSE

Let S_\star^N be the normalized form of S_\star such that there is at most one operator on the right hand side of an assignment.

For each $[...a...]^\ell$ in S_\star^N with $a \in \text{AE}_o(\ell)$ do

- ▶ determine the set $\{[y_1 := a]^{\ell_1}, \dots, [y_k := a]^{\ell_k}\}$ of elementary blocks in S_\star^N “defining” a that **reaches** $[...a...]^\ell$
- ▶ create a fresh variable u and
 - ▶ replace each occurrence of $[y_i := a]^{\ell_i}$ with $[u := a]^{\ell_i}; [y_i := u]^{\ell_i}$ for $1 \leq i \leq k$
 - ▶ replace $[...a...]^\ell$ with $[...u...]^\ell$

$[x := a]^{\ell'}$ **reaches** $[...a...]^\ell$ if there is a path in $\text{flow}(S_\star^N)$ from ℓ' to ℓ that does not contain *any* assignments with expression a on the right hand side and no variable of a is modified.

Computing the “reaches” Information

$[x := a]^{\ell'}$ **reaches** $[...a...]^{\ell}$ if there is a path in $\text{flow}(S_{\star}^N)$ from ℓ' to ℓ that does not contain **any** assignments with expression a on the right hand side and no variable of a is modified.

The set of elementary blocks that **reaches** $[...a...]^{\ell}$ can be computed as $\text{reaches}_{\circ}(a, \ell)$ where

$$\begin{aligned} \text{reaches}_{\circ}(a, \ell) &= \begin{cases} \emptyset & : \text{ if } \ell = \text{init}(S_{\star}) \\ \bigcup \text{reaches}_{\bullet}(a, \ell') & : \text{ otherwise} \end{cases} \\ \text{reaches}_{\bullet}(a, \ell) &= \begin{cases} \{B^{\ell}\} & : \text{ if } B^{\ell} \text{ has the form } [x := a]^{\ell} \text{ and } x \notin \text{FV}(a) \\ \emptyset & : \text{ if } B^{\ell} \text{ has the form } [x := \dots]^{\ell} \text{ and } x \in \text{FV}(a) \\ \text{reaches}_{\circ}(a, \ell) & : \text{ otherwise} \end{cases} \end{aligned}$$

Example – CSE

Example:

$[x := a+b]^1; [y := a*x]^2; \text{while } [y > a+b]^3 \text{ do } [a := a + 1]^4; [x := a + b]^5 \text{ od}$

ℓ	$AE_o(\ell)$
1	\emptyset
2	$\{a+b\}$
3	$\{a+b\}$
4	$\{a+b\}$
5	\emptyset

$$\text{reaches}(a+b,3) = \{[x := a + b]^1, [x := a + b]^5\}$$

Result of CSE optimization wrt $\text{reaches}(a+b,3)$:

$[u := a+b]^1; [x := u]^1; [y := a*x]^2; \text{while } [y > u]^3 \text{ do } [a := a + 1]^4; [u := a + b]^5; [x := u]^5 \text{ od}$

Copy Analysis

...aims at determining for each program point ℓ' , which copy statements $[x := y]^\ell$ that still are relevant (i.e. neither x nor y have been redefined) when control reaches point ℓ' .

Example:

$[a := b]^1; \text{if } [x > b]^2 \text{ then } ([y := a]^3) \text{ else } ([b := b + 1]^4; [y := a]^5); [\text{skip}]^6$

ℓ	$C_o(\ell)$	$C_\bullet(\ell)$
1	\emptyset	$\{(a,b)\}$
2	$\{(a,b)\}$	$\{(a,b)\}$
3	$\{(a,b)\}$	$\{(y,a),(a,b)\}$
4	$\{(a,b)\}$	\emptyset
5	\emptyset	$\{(y,a)\}$
6	$\{(y,a)\}$	$\{(y,a)\}$

Copy Propagation (CP) (1)

...aims at finding copy statements $[x := y]^{\ell_j}$ and eliminating them if possible.

If x is used in $B^{\ell'}$ then x can be replaced by y in $B^{\ell'}$ provided that

- ▶ $[x := y]^{\ell_j}$ is the only kind of definition of x that reaches $B^{\ell'}$ – this information can be obtained from the def-use chain.
- ▶ on every path from ℓ_j to ℓ' (including paths going through ℓ' several times but only once through ℓ_j) there are no redefinitions of y ; this can be detected by Copy Analysis.

Copy Propagation (CP) (2)

Example 1

$[u := a+b]^{1'}$; $[x := u]^1$; $[y := a*x]^2$; while $[y > u]^3$ do $[a := a + 1]^4$; $[u := a + b]^{5'}$; $[x := u]^5$ od

becomes after CP

$[u := a+b]^{1'}$; $[y := a*u]^2$; while $[y > u]^3$ do $[a := a + 1]^4$; $[u := a + b]^{5'}$; $[x := u]^5$ od

The Optimization – CP

For each copy statement $[x := y]^{\ell_j}$ in S_* do

- ▶ determine the set $\{[\dots x \dots]^{\ell_1}, \dots, [\dots x \dots]^{\ell_i}\}, 1 \leq i \leq k$, of elementary blocks in S_* that uses $[x := y]^{\ell_j}$ – this can be computed from $DU(x, \ell_j)$
- ▶ for each $[\dots x \dots]^{\ell_i}$ in this set determine whether $\{(x', y') \in C_o(\ell_i) \mid x' = x\} = \{(x, y)\}$; if so then $[x := y]$ is the only kind of definition of x that reaches ℓ_i from all ℓ_j .
- ▶ if this holds for all i ($1 \leq i \leq k$) then
 - ▶ remove $[x := y]^{\ell_j}$
 - ▶ replace $[\dots x \dots]^{\ell_i}$ with $[\dots y \dots]^{\ell_i}$ for $1 \leq i \leq k$.

Examples – CP

Example 2

$[a := 2]^1; \text{if } [y > u]^2 \text{ then } ([a := a + 1]^3; [x := a]^4;) \text{ else } ([a := a * 2]^5; [x := a]^6;) [y := y * x]^7$

becomes after CP

$[a := 2]^1; \text{if } [y > u]^2 \text{ then } ([a := a + 1]^3; \quad ;) \text{ else } ([a := a * 2]^5; \quad ;) [y := y * a]^7;$

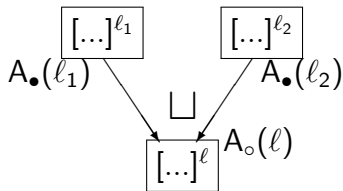
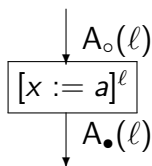
Example 3

$[a := 10]^1; [b := a]^2; \text{while } [a > 1]^3 \text{ do } [a := a - 1]^4; [b := a]^5; \text{od } [y := y * b]^6;$

becomes after CP

$[a := 10]^1; \quad ; \text{while } [a > 1]^3 \text{ do } [a := a - 1]^4; \quad ; \text{od } [y := y * a]^6;$

Summary: Forward Analyses



$$\begin{aligned}
 A_o(l) &= \begin{cases} \iota_A & : \text{ if } l = \text{init}(S_*) \\ \bigsqcup_A \{A_\bullet(l') \mid (\ell', l) \in \text{flow}(S_*)\} & : \text{ otherwise} \end{cases} \\
 A_\bullet(l) &= (A_o(l) \setminus \text{kill}_A(B^\ell)) \cup \text{gen}_A(B^\ell) \quad \text{where } B^\ell \in \text{blocks}(S_*)
 \end{aligned}$$

where

Analysis	RD	AE
ι_A	$\{(x, ?) \mid x \in FV(S_*)\}$	\emptyset
\bigsqcup_A	\cup	\cap

Further Reading for Chapter 2.2

-  Flemming Nielson, Hanne Riis Nielson, Chris Hankin.
Principles of Program Analysis. 2nd edition, Springer-V.,
2005. (Chapter 2, Data Flow Analysis)

Chapter 2.3

Backward Analyses

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Live Variable Analysis

Definition Live Variables

A variable is **live at the exit from a label** if there is a path from the label to a use of the variable that does not re-define the variable.

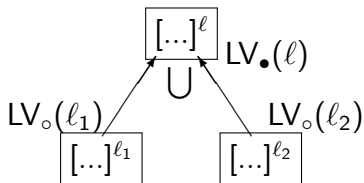
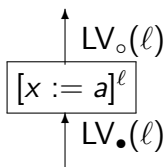
The Aim of the Live Variables Analysis is to determine for each program point, which variables may be live at the exit from the point.

Example

$[y := 0]^0; [u := a+b]^1; [y := a*u]^2; \text{while } [y > u]^3 \text{ do } [a := a + 1]^4; [u := a + b]^5; [x := u]^6 \text{ od}$

- ▶ y is dead (i.e., not live) at the exit from label 0
- ▶ x is dead (i.e., not live) at the exit from label 6

Basic Idea



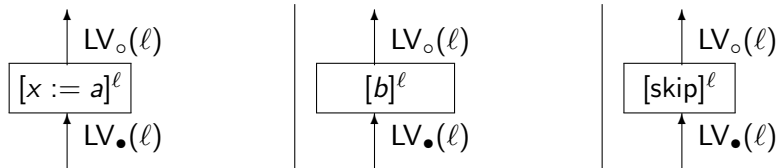
Analysis information: $LV_o(\ell), LV_\bullet(\ell) : \text{Lab}_* \rightarrow \mathcal{P}\text{Var}_*$

- ▶ $LV_o(\ell)$: the variables that are live at **entry** of block ℓ .
- ▶ $LV_\bullet(\ell)$: the variables that are live at **exit** of block ℓ .

Analysis properties:

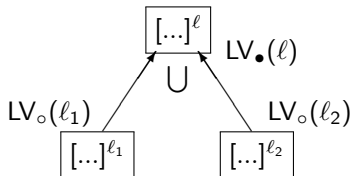
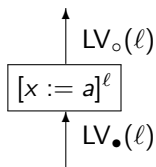
- ▶ Direction: backward
- ▶ May analysis with combination operator \cup

Analysis of Elementary Blocks



$$\begin{aligned}\text{kill}_{LV}([x := a]^\ell) &= \{x\} \\ \text{kill}_{LV}([\text{skip}]^\ell) &= \emptyset \\ \text{kill}_{LV}([b]^\ell) &= \emptyset \\ \text{gen}_{LV}([x := a]^\ell) &= FV(a) \\ \text{gen}_{LV}([\text{skip}]^\ell) &= \emptyset \\ \text{gen}_{LV}([b]^\ell) &= FV(b)\end{aligned}$$

Analysis of the Program



$$\begin{aligned}
 LV_{\circ}(l) &= (LV_{\bullet}(l) \setminus \text{kill}_{LV}(B^l)) \cup \text{gen}_{LV}(B^l) && \text{where } B^l \in \text{blocks}(S_{\star}) \\
 LV_{\bullet}(l) &= \begin{cases} \emptyset & : \text{ if } l = \text{final}(S_{\star}) \\ \bigcup \{LV_{\circ}(l') \mid (l', l) \in \text{flow}^R(S_{\star})\} & : \text{ otherwise} \end{cases}
 \end{aligned}$$

Example

Program	$LV_{\bullet}(\ell)$	$LV_{\circ}(\ell)$	ℓ	$kill_{LV}(\ell)$	$gen_{LV}(\ell)$
$[y := 0]^0;$	$\{a, b\}$	$\{a, b\}$	0	$\{y\}$	\emptyset
$[u := a+b]^1;$	$\{u, a, b\}$	$\{a, b\}$	1	$\{u\}$	$\{a, b\}$
$[y := a*u]^2;$	$\{u, a, b, y\}$	$\{u, a, b\}$	2	$\{y\}$	$\{a, u\}$
$while [y > u]^3 do$	$\{a, b, y\}$	$\{u, a, b, y\}$	3	\emptyset	$\{y, u\}$
$[a := a + 1]^4;$	$\{a, b, y\}$	$\{a, b, y\}$	4	$\{a\}$	$\{a\}$
$[u := a + b]^5;$	$\{u, a, b, y\}$	$\{a, b, y\}$	5	$\{u\}$	$\{a, b\}$
$[x := u]^6 od$	$\{u, a, b, y\}$	$\{u, a, b, y\}$	6	$\{x\}$	$\{u\}$
$[skip]^7$	\emptyset	\emptyset	7	\emptyset	\emptyset

Dead Code Elimination (DCE)

An assignment $[x := a]^\ell$ is **dead** if the value of x is not used before it is redefined. Dead assignments can be eliminated.

- ▶ **Analysis:** Live Variables Analysis
- ▶ **Transformation:** For each $[x := a]^\ell$ in S_\star with $x \notin \text{LV}_\bullet(\ell)$ (i.e. dead) eliminate $[x := a]^\ell$ from the program.

Example:

Before DCE:

$[y := 0]^0; [u := a+b]^1; [y := a*u]^2; \text{while } [y > u]^3 \text{ do } [a := a + 1]^4; [u := a + b]^5; [x := u]^6 \text{ od}$

After DCE:

$[u := a+b]^1; [y := a*u]^2; \text{while } [y > u]^3 \text{ do } [a := a + 1]^4; [u := a + b]^5; \text{od}$

Combining Optimizations

...usually strengthens the overall impact.

Example:

$[x := a+b]^1; [y := a*x]^2; \text{while } [y > a+b]^3 \text{ do } [a := a + 1]^4; [x := a + b]^5 \text{ od}$

1. Common Subexpression Elimination gives

$[u := a+b]^{1'}; [x := u]^1; [y := a*x]^2; \text{while } [y > u]^3 \text{ do } [a := a + 1]^4; [u := a + b]^{5'}; [x := u]^5 \text{ od}$

2. Copy Propagation gives

$[u := a+b]^{1'}; [y := a*u]^2; \text{while } [y > u]^3 \text{ do } [a := a + 1]^4; [u := a + b]^{5'}; [x := u]^5 \text{ od}$

3. Dead Code Elimination gives

$[u := a+b]^{1'}; [y := a*u]^2; \text{while } [y > u]^3 \text{ do } [a := a + 1]^4; [u := a + b]^{5'}; \text{od}$

What are the results for other optimization sequences?

Faint Variables

...generalize the notion of **dead variables**.

Consider the following program consisting of three statements:

```
[x := 1]1; [x := 2]2; [y := x]3;
```

Clearly **x is dead at the exit from 1** and **y is dead at the exit of 3**. But **x is live at the exit of 2** although it is only used to calculate a new value for y that turns out to be dead.

We shall say that a variable is a **faint variable** if it is dead or if it is only used to calculate new values for faint variables; otherwise it is **strongly live**.

Very Busy Expressions Analysis

Definition Very Busy Expressions

An expression is **very busy** at the exit from a label if, no matter what path is taken from the label, the expression is always used before any of the variables occurring in it are redefined.

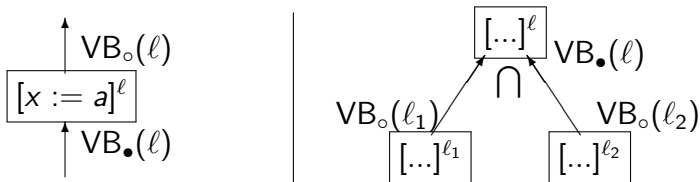
The Aim of the Very Busy Expression Analysis is to determine for each program point, which expressions **must** be very busy at the exit from the point.

Example

if $[a > b]^1$ then $([x := b-a]^2; [y := a-b]^3)$ else $([y := b-a]^4; [x := a-b]^5)$

- ▶ $b-a$ and $a-b$ are very busy at the exit from label 1

Basic Idea



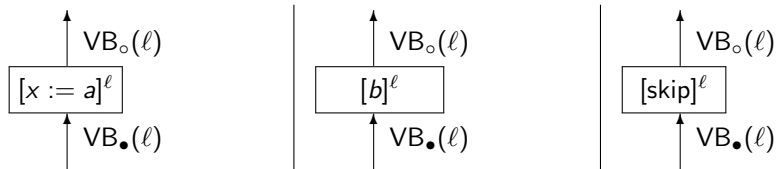
Analysis information: $VB_o(l), VB_b(l) : \text{Lab}_* \rightarrow \mathcal{PAExp}_*$

- ▶ $VB_o(l)$: the expressions that are very busy at **entry** of block l .
- ▶ $VB_b(l)$: the expressions that are very busy at **exit** of block l .

Analysis properties:

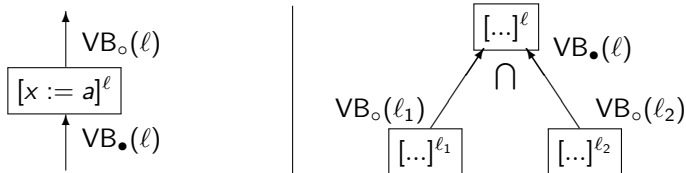
- ▶ Direction: backward
- ▶ Must analysis with combination operator \cap

Analysis of Elementary Blocks



$$\begin{aligned}\text{kill}_{VB}([x := a]^{\ell}) &= \{a' \in \text{AExp}_{\star} \mid x \in FV(a')\} \\ \text{kill}_{VB}([\text{skip}]^{\ell}) &= \emptyset \\ \text{kill}_{VB}([b]^{\ell}) &= \emptyset \\ \text{gen}_{VB}([x := a]^{\ell}) &= \text{AExp}(a) \\ \text{gen}_{VB}([\text{skip}]^{\ell}) &= \emptyset \\ \text{gen}_{VB}([b]^{\ell}) &= \text{AExp}(b)\end{aligned}$$

Analysis of the Program



$$\begin{aligned}
 VB_o(\ell) &= (VB_{\bullet}(\ell) \setminus \text{kill}_{VB}(B^\ell)) \cup \text{gen}_{VB}(B^\ell) && \text{where } B^\ell \in \text{blocks}(S_\star) \\
 VB_{\bullet}(\ell) &= \begin{cases} \emptyset & : \text{ if } \ell = \text{final}(S_\star) \\ \bigcap \{VB_o(\ell') \mid (\ell', \ell) \in \text{flow}^R(S_\star)\} & : \text{ otherwise} \end{cases}
 \end{aligned}$$

Example

if $[a > b]^1$ then ($[x := b-a]^2; [y := a-b]^3$) else ($[y := b-a]^4; [x := a-b]^5$)

ℓ	$VB_{\bullet}(\ell)$	$VB_{\circ}(\ell)$	ℓ	$kill_{VB}(\ell)$	$gen_{VB}(\ell)$
1	$\{a-b, b-a\}$	$\{a-b, b-a\}$	1	\emptyset	\emptyset
2	$\{a-b\}$	$\{a-b, b-a\}$	2	\emptyset	$\{b-a\}$
3	\emptyset	$\{a-b\}$	3	\emptyset	$\{a-b\}$
4	$\{a-b\}$	$\{a-b, b-a\}$	4	\emptyset	$\{b-a\}$
5	\emptyset	$\{a-b\}$	5	\emptyset	$\{a-b\}$

Code Hoisting (CH)

...finds expressions that are always evaluated following some point in the program regardless of the execution path – and moves them to the earliest point (in execution order) beyond which they would always be executed.

Example:

Before CH:

```
if [a > b]1 then ([x := b-a]2; [y := a-b]3) else ([y := b-a]4; [x := a-b]5)
```

After CH:

```
[t1 := a-b]0; [t2 := b-a]0';  
if [a > b]1 then ([x := t2]2; [y := t1]3) else ([y := t2]4; [x := t1]5)
```

Further Reading for Chapter 2.3

-  Flemming Nielson, Hanne Riis Nielson, Chris Hankin.
Principles of Program Analysis. 2nd edition, Springer-V.,
2005. (Chapter 2, Data Flow Analysis)

Chapter 3

Taxonomy of DFA-Analyses

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Taxonomy of Classical DFA-Analyses

Analysis	may	must
Forward	Reaching Definitions	Available Expressions
Backward	Live Variables	Very Busy Expressions
Analysis	may	must
Combination Op.	\cup	\cap
Solution of equ.	smallest	largest
Analysis	Extremal labels set	Abstract flow graph
Forward	$\{\text{init}(S_*)\}$	$\text{flow}(S_*)$
Backward	$\text{final}(S_*)$	$\text{flow}^R(S_*)$

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Bit Vectors and Bit Vector Analyses

The **classical analyses** operate over elements of $\mathcal{P}(D)$ where D is a finite set.

The elements can be represented as **bit vectors**. Each element of D can be assigned a unique bit position i ($1 \leq i \leq n$). A subset S of D is then represented by a **vector of n bits**:

- ▶ if the i 'th element of D is in S then the i 'th bit is 1.
- ▶ if the i 'th element of D is not in S then the i 'th bit is 0.

Then we have **efficient implementations** of

- ▶ **set union** as logical 'or'
- ▶ **set intersection** as logical 'and'

More Bit Vector Framework Examples

- ▶ **Dual available expressions** determines for each program point which expressions may not be available when execution reaches that point (**forward may analysis**).
- ▶ **Copy analysis** determines whether there on every execution path from a copy statement $x := y$ to a use of x there are no assignments to y (**forward must analysis**).
- ▶ **Dominators** determines for each program point which program points are guaranteed to have been executed before the current one is reached (**forward must analysis**).
- ▶ **Upwards exposed uses** determines for a program point, what uses of a variable are reached by a particular definition (assignment) (**backward may analysis**).




Some Non-Bit Vector Framework Examples (1)

- ▶ **Constant propagation** determines for each program point whether or not a variable has a constant value whenever execution reaches that point (**forward must analysis**).
- ▶ **Detection of signs analysis** determines for each program point the possible signs that the values of the variables may have whenever execution reaches that point (**forward must analysis**).
- ▶ **Faint variables** determines for each program point which variables are faint: a variable is faint if it is dead or it is only used to compute new values of faint variables (**backward must analysis**).


Some Non-Bit Vector Framework Examples (2)

- ▶ **May be uninitialized** determines for each program point which variables have dubious values: a variable has a dubious value if either it is not initialized or its value depends on variables with dubious values (**forward may analysis**).

Further Reading for Chapter 3 (1)

-  Alfred V. Aho, Monica S. Lam, Ravi Sethi, Jeffrey D. Ullman. *Compilers: Principles, Techniques, & Tools*. Addison-Wesley, 2nd edition, 2007. (Chapter 9.2, Introduction to Data-Flow Analysis; Chapter 9.3, Foundations of Data-Flow Analysis)
-  Keith D. Cooper, Linda Torczon. *Engineering a Compiler*. Morgan Kaufman Publishers, 2004. (Chapter 10.2, A Taxonomy for Transformations — Machine-Independent Transformations, Machine-Dependent Transformations)
-  Stephen S. Muchnick. *Advanced Compiler Design Implementation*. Morgan Kaufman Publishers, 1997. (Chapter 8.3, Taxonomy of Data-Flow Problems and Solution Methods)

Further Reading for Chapter 3 (2)

-  Flemming Nielson, Hanne Riis Nielson, Chris Hankin.
Principles of Program Analysis. 2nd edition, Springer-V.,
2005. (Chapter 1, Introduction; Chapter 2, Data Flow
Analysis; Chapter 6, Algorithms)

Part II

Intraprocedural Data Flow Analysis

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Chapter 4

Flow Graphs

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Programs as Flow Graphs

For program analysis, especially **data flow analysis**, it is usual to

- ▶ represent programs in terms of (non-deterministic) **flow graphs**

Flow Graphs

A (non-deterministic) **flow graph** is a tuple $G = (N, E, s, e)$ with

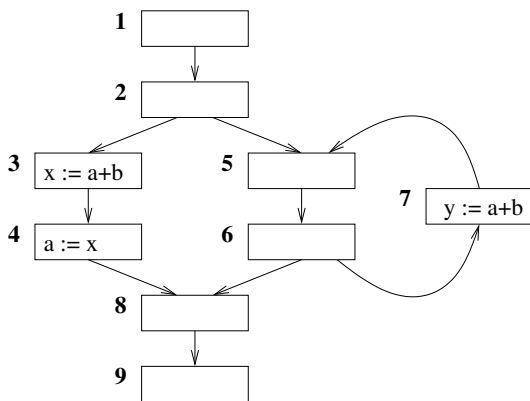
- ▶ node set N
- ▶ edge set $E \subseteq N \times N$
- ▶ distinguished **start node** s w/out any predecessors
- ▶ distinguished **end node** e w/out successors

Nodes represent **program points**, **edges** represent the **branching structure**. Program statements (assignments, tests) can be represented by

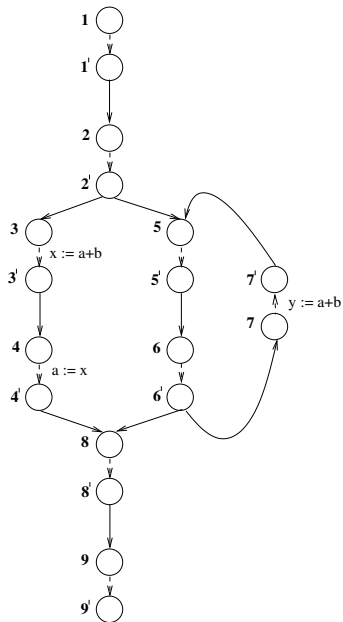
- ▶ nodes: **node-labelled** flow graph
- ▶ edges: **edge-labelled** flow graph

where nodes and edges are labelled by **single instructions** or **basic blocks**, respectively.

A Node-Labelled Flow Graph



An Edge-Labelled Flow Graph



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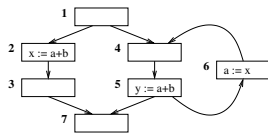
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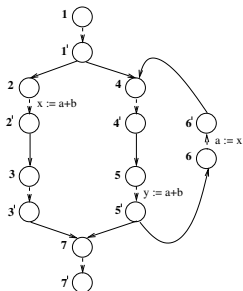
Single Instruction Flow Graphs

Node-labelled vs. edge-labelled:

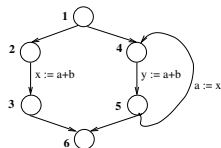
a)



b)



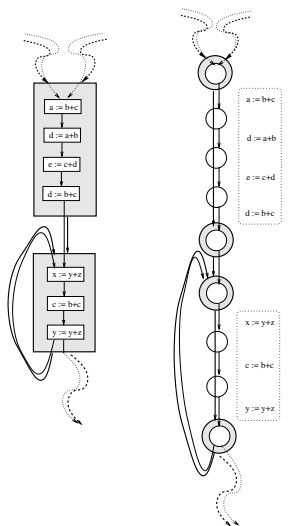
i) Schematisch



ii) "Optimiert"

Basic Block Flow Graphs

Node-labelled vs. edge-labelled:



Node-labelled (BB-) Graph

Edge-labelled (BB-) Graph

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Summing up

We distinguish:

- ▶ **Node-labelled flow graphs**
 - ▶ Single instruction graphs (SI graphs)
 - ▶ Basic block graphs (BB graphs)
- ▶ **Edge-labelled flow graphs**
 - ▶ Single instruction graphs (SI graphs)
 - ▶ Basic block graphs (BB graphs)

Later on we will preferably deal w/ edge-labelled SI graphs.

Notations

Let $G = (N, E, s, e)$ be a flow graph, let m, n be two nodes of N . Then:

- ▶ $\mathbf{P}_G[m, n]$ denotes the set of all paths from m to n (including m and n)
- ▶ $\mathbf{P}_G[m, n[$ denotes the set of all paths from m to a predecessor of n
- ▶ $\mathbf{P}_G]m, n]$ denotes the set of all paths from a successor of m to n
- ▶ $\mathbf{P}_G]m, n[$ denotes the set of all paths from a successor of m to a predecessor of n

Note: If G is uniquely determined by the context, then we drop the index and simply write \mathbf{P} instead of \mathbf{P}_G .

Further Reading for Chapter 4

-  Keith D. Cooper, Linda Torczon. *Engineering a Compiler*. Morgan Kaufman Publishers, 2004. (Appendix B.3.1, Graphical Intermediate Representations)
-  Jens Knoop, Dirk Koschützki, Bernhard Steffen. *Basic-block Graphs: Living Dinosaurs?* In Proceedings of the 7th International Conference on Compiler Construction (CC'98), Springer-V., LNCS 1383, 65-79, 1998.
-  Stephen S. Muchnick. *Advanced Compiler Design Implementation*. Morgan Kaufman Publishers, 1997. (Chapter 7, Control-Flow Analysis)

Chapter 5

The Intraprocedural DFA Framework

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DFA Specification, DFA Problem

Definition (5.1, DFA Specification)

A DFA specification is given by

- ▶ a (local) abstract semantics consisting of
 1. a DFA lattice $\hat{\mathcal{C}} = (\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \perp, \top)$
 2. a DFA functional $\llbracket \cdot \rrbracket : E \rightarrow (\mathcal{C} \rightarrow \mathcal{C})$
- ▶ a start information/assertion: $c_s \in \mathcal{C}$

Definition (5.2, DFA Problem)

A DFA specification defines a DFA problem.

Practically Relevant

...are DFA problems that are

- ▶ monotonic
- ▶ distributive/additive

and satisfy the

- ▶ ascending/descending chain condition

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Properties

...of DFA functionals, DFA problems:

Definition (5.3)

A DFA functional $\llbracket \cdot \rrbracket : E \rightarrow (\mathcal{C} \rightarrow \mathcal{C})$ is **monotonic/distributive/additive** iff for all $e \in E$ the local semantic function $\llbracket e \rrbracket$ is monotonic/distributive/additive.

Definition (5.4)

A DFA problem is **monotonic/distributive/additive** iff the DFA functional $\llbracket \cdot \rrbracket$ of the underlying DFA specification $(\hat{\mathcal{C}}, \llbracket \cdot \rrbracket, c_s)$ is monotonic/distributive/additive.

These properties induce a **taxonomy** of DFA problems.

Monotonicity, Distributivity, Additivity (1)

Definition (5.5, Properties of DFA Functionals)

Let $\hat{\mathcal{C}} = (\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \perp, \top)$ be a complete (DFA) lattice and $f : \mathcal{C} \rightarrow \mathcal{C}$ a function on \mathcal{C} . Then f is

1. **monotonic** iff $\forall c, c' \in \mathcal{C}. c \sqsubseteq c' \Rightarrow f(c) \sqsubseteq f(c')$
(Preservation of the order of elements)
2. **distributive** iff $\forall C' \subseteq \mathcal{C}. f(\sqcap C') = \sqcap \{f(c) \mid c \in C'\}$
(Preservation of greatest lower bounds)
3. **additive** gdw $\forall C' \subseteq \mathcal{C}. f(\sqcup C') = \sqcup \{f(c) \mid c \in C'\}$
(Preservation of least upper bounds)

Monotonicity vs. Distributivity and Additivity

We have:

Lemma (5.6)

Let $\hat{\mathcal{C}} = (\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \perp, \top)$ be a complete (DFA) lattice and $f : \mathcal{C} \rightarrow \mathcal{C}$ a function on \mathcal{C} . Then:

$$\begin{aligned} f \text{ is monotonic} &\iff \forall C' \subseteq \mathcal{C}. f(\bigsqcap C') \sqsubseteq \bigsqcap \{f(c) \mid c \in C'\} \\ &\iff \forall C' \subseteq \mathcal{C}. f(\bigsqcup C') \supseteq \bigsqcup \{f(c) \mid c \in C'\} \end{aligned}$$

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Chain Condition

Definition (5.7 Chain Condition)

A (DFA) lattice $\hat{\mathcal{C}} = (\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \perp, \top)$ satisfies the

1. **descending chain condition**, if every descending chain gets stationary, i.e. for each chain $c_1 \supseteq c_2 \supseteq \dots \supseteq c_n \supseteq \dots$ there is an index $m \geq 1$ such that $c_m = c_{m+j}$ holds for all $j \in \mathbb{N}$
2. **ascending chain condition**, if every ascending chain gets stationary, i.e. for each chain $c_1 \sqsubseteq c_2 \sqsubseteq \dots \sqsubseteq c_n \sqsubseteq \dots$ there is an index $m \geq 1$ such that $c_m = c_{m+j}$ holds for all $j \in \mathbb{N}$

Next Step

...globalizing a local abstract semantics from statements to flow graphs.

For that we introduce two (globalization) strategies:

- ▶ Meet over all Paths Approach (*MOP*)
 \rightsquigarrow yields the specifying solution of a DFA problem
- ▶ Maximum Fixed Point (*MaxFP*) Approach
 \rightsquigarrow yields a computable solution of a DFA problem

Chapter 5.1

The *MOP* Approach

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The *MOP* Approach

Essential for the *MOP* approach:

Definition (5.1.1, Path extension of $\llbracket \cdot \rrbracket$)

The extension of a local abstract semantics $\llbracket \cdot \rrbracket$ onto paths $p = \langle e_1, e_2 \dots, e_q \rangle$ is defined by

$$\llbracket p \rrbracket =_{df} \begin{cases} Id_{\mathcal{C}} & \text{falls } q < 1 \\ \llbracket \langle e_2, \dots, e_q \rangle \rrbracket \circ \llbracket e_1 \rrbracket & \text{otherwise} \end{cases}$$

where $Id_{\mathcal{C}}$ denotes the identity on \mathcal{C} .

The *MOP* Solution

Definition (5.1.2, *MOP* Solution)

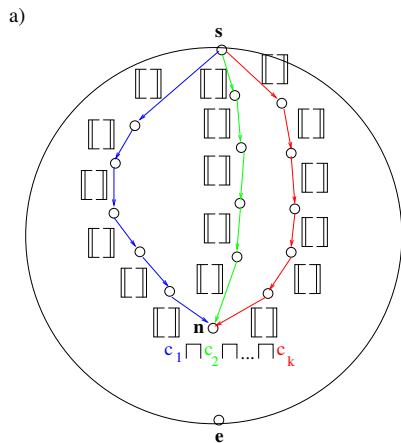
Let $(\hat{C}, \llbracket \cdot \rrbracket, c_s)$ be the specification of a DFA problem. Then, for all nodes $n \in N$ the *MOP solution* is defined by:

$$MOP_{(\hat{C}, \llbracket \cdot \rrbracket, c_s)}(n) = \bigsqcap \{ \llbracket p \rrbracket(c_s) \mid p \in \mathbf{P}[s, n] \}$$

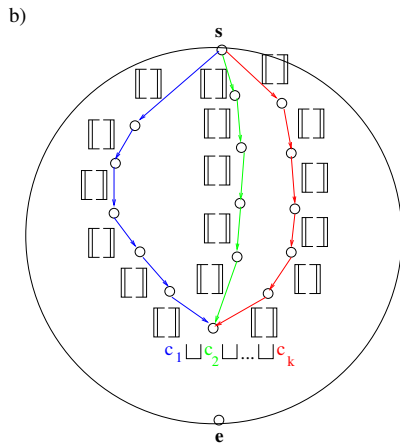
Essential:

- ▶ The *MOP solution* is considered the *specifying* solution of the DFA problem given by $(\hat{C}, \llbracket \cdot \rrbracket, c_s)$.

Illustration



a) Traditional DFA view:
'Meeting' infos: *MOP*
(Universally quantified)



b) Traditional AI view:
'Joining' infos: *JOP*
(Existentially quantified)

Unfortunately

...the following negative result holds:

Theorem (5.1.1, Undecidability)

There is no algorithm A satisfying:

- ▶ *The input of A are*
 - ▶ *algorithms for the computation of the meet, the equality test, and the application of functions on the lattice elements of a monotonic DFA framework*
 - ▶ *a DFA problem p specified by an instance of a DFA specification $(C, [\] , c_s)$*
- ▶ *The output of A is the MOP solution of p .*

(John B. Kam and Jeffrey D. Ullman. [Monotone Data Flow Analysis Frameworks](#). Acta Informatica 7, 305-317, 1977)

Towards the *MaxFP* Approach

Because of the preceding negative result we introduce a [second globalization approach](#) of a local abstract semantics, the *MaxFP* approach.

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Chapter 5.2

The *MaxFP* Approach

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The *MaxFP* Approach

Essential for the *MaxFP* approach:

Definition (5.2.1, *MaxFP* Equation System)

The *MaxFP* equation system is given by:

$$\text{inf}(n) = \begin{cases} c_s & \text{if } n = \mathbf{s} \\ \bigcap \{ \llbracket (m, n) \rrbracket (\text{inf}(m)) \mid m \in \text{pred}(n) \} & \text{otherwise} \end{cases}$$

The *MaxFP* Solution

Definition (5.2.2, *MaxFP* Solution)

Let $(\hat{C}, \llbracket \ \rrbracket, c_s)$ be the specification of a DFA problem. Then, for all nodes $n \in N$ the *MaxFP solution* is defined by:

$$\text{MaxFP}_{(\hat{C}, \llbracket \ \rrbracket, c_s)}(n) =_{df} \text{inf}_{c_s}^*(n)$$

where $\text{inf}_{c_s}^*$ denotes the greatest solution of the *MaxFP* Equation System 5.2.1 wrth $(\hat{C}, \llbracket \ \rrbracket, c_s)$.

Essential:

- ▶ The *MaxFP solution* is effectively computable under the conditions of the Termination Theorem 5.2.4). It is thus considered the *computable solution* of a DFA problem.

The Generic Fixed Point Algorithm 5.2.3 (1)

Input: (1) A flow graph $G = (N, E, s, e)$, (2) a DFA problem given by a (local) abstract semantics consisting of a DFA lattice \mathcal{C} , a DFA functional $[[\]]: E \rightarrow (\mathcal{C} \rightarrow \mathcal{C})$, and (3) a start information $c_s \in \mathcal{C}$.

Output: The *MaxFP* solution, if the preconditions of the Termination Theorem 5.2.4 hold. Depending on the properties of the DFA functional we have:

- (i) $[[\]]$ is **distributive**: The variable *inf* stores for each node the strongest post-condition wrt the start information c_s .
- (ii) $[[\]]$ is **monotonic**: The variable *inf* stores for each node a safe (i.e. lower) approximation of the strongest post-condition wrt the start information c_s .

Remark: The variable *workset* controls the iterative process. Its elements are nodes of G , whose annotation has recently been updated.

The Generic Fixed Point Algorithm 5.2.3 (2)

(Prologue: Initializing *inf* and *workset*)

FORALL $n \in N \setminus \{s\}$ DO $inf[n] := \top$ OD;

$inf[s] := c_s$;

$workset := N$;

(Main loop: The iterative fixed point computation)

WHILE $workset \neq \emptyset$ DO

 CHOOSE $m \in workset$;

$workset := workset \setminus \{m\}$;

 (Update the annotations of all successors of node m)

 FORALL $n \in succ(m)$ DO

$meet := \llbracket (m, n) \rrbracket (inf[m]) \sqcap inf[n]$;

 IF $inf[n] \sqsupseteq meet$

 THEN

$inf[n] := meet$;

$workset := workset \cup \{n\}$

 FI

 OD ESOOHC OD.

Theorem (5.2.4, Termination)

The *Generic Fixed Point Algorithm 5.2.3* terminates with the $\text{MaxFP}_{(\hat{C}, \llbracket \cdot \rrbracket, c_s)}$ solution, if

- the DFA functional $\llbracket \cdot \rrbracket$ is monotonic
- the DFA lattice \hat{C} satisfies the descending chain condition.

Note

The [Generic Fixed Point Algorithm 5.2.3](#) is formulated for

- ▶ universally quantified (“distributive”) forward problems

The other [three variants of DFA problems](#) of

- ▶ existentially quantified (“additive”) problems
- ▶ backward problems

can be captured and solved with the [Generic Fixed Point Algorithm 5.2.3](#) by “turning round”

- ▶ the lattice (by replacing of \sqsubseteq by \supseteq)
- ▶ the flow graph (by [reversing](#) all edges)

Chapter 5.3

Coincidence and Safety Theorem

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Main Results: Soundness and Completeness

The relationship of *MOP* and *MaxFP* solution

- ▶ Soundness
- ▶ Completeness

In Detail:

- ▶ **Soundness:**

Does always hold $MaxFP_{(\hat{c}, \llbracket \cdot \rrbracket, c_s)} \sqsubseteq MOP_{(\hat{c}, \llbracket \cdot \rrbracket, c_s)}$?

- ▶ **Completeness:**

Does always hold $MaxFP_{(\hat{c}, \llbracket \cdot \rrbracket, c_s)} \sqsupseteq MOP_{(\hat{c}, \llbracket \cdot \rrbracket, c_s)}$?

Theorem (5.3.1, Safety)

The MaxFP solution is a safe (conservative), i.e. lower approximation of the MOP solution, i.e.,

$$\forall c_s \in \mathcal{C} \quad \forall n \in \mathbb{N}. \text{MaxFP}_{(\hat{c}, \llbracket \cdot \rrbracket, c_s)}(n) \sqsubseteq \text{MOP}_{(\hat{c}, \llbracket \cdot \rrbracket, c_s)}(n)$$

if the DFA functional $\llbracket \cdot \rrbracket$ is monotonic.

Completeness (and simultaneously Soundness)

Theorem (5.3.2, Coincidence)

The MaxFP solution coincides with the MOP solution, i.e.,

$$\forall c_s \in \mathcal{C} \quad \forall n \in \mathbb{N}. \text{MaxFP}_{(\hat{C}, \llbracket \cdot \rrbracket, c_s)}(n) = \text{MOP}_{(\hat{C}, \llbracket \cdot \rrbracket, c_s)}(n)$$

if the DFA functional $\llbracket \cdot \rrbracket$ is distributive.

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Note

In the context of DFA

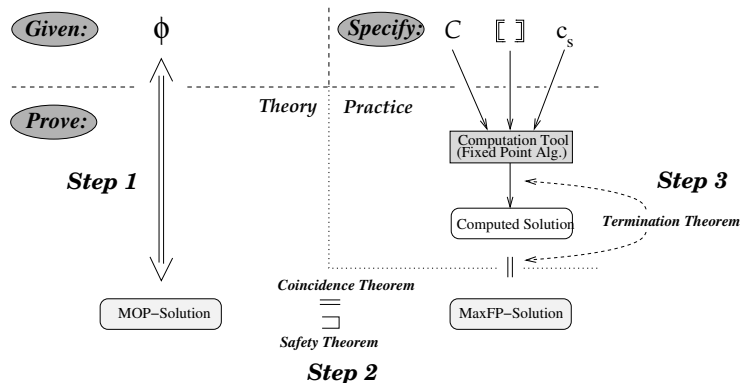
- ▶ Safety
- ▶ Coincidence

are traditionally used instead of

- ▶ Soundness
- ▶ Completeness

Intraprocedural DFA at a Glance (1)

The schematic view:



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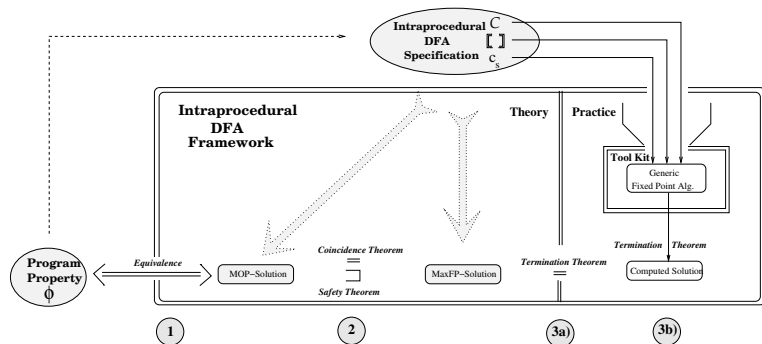
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Intraprocedural DFA at a Glance (2)

Focused on the framework/toolkit view:



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Chapter 5.4

Examples: Available Expressions, Simple Constants

Two Prototypical DFA Problems

- ▶ Available Expressions

↪ a canonical example of a **distributive** DFA problem

- ▶ Simple Constants

↪ a canonical example of a **monotonic** DFA problem

Chapter 5.4.1

Available Expressions

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Available Expressions

...a typical distributive DFA problem.

- ▶ Local abstract semantics for available expressions:

1. DFA lattice:

$$(\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \perp, \top) =_{df} (\mathbb{B}, \wedge, \vee, \leq, \mathbf{false}, \mathbf{true})$$

2. DFA functional: $\llbracket \cdot \rrbracket_{av} : E \rightarrow (\mathbb{B} \rightarrow \mathbb{B})$ defined by

$$\forall e \in E. \llbracket e \rrbracket_{av} =_{df} \begin{cases} Cst_{\mathbf{true}} & \text{if } Comp_e \wedge Transp_e \\ Id_{\mathbb{B}} & \text{if } \neg Comp_e \wedge Transp_e \\ Cst_{\mathbf{false}} & \text{otherwise} \end{cases}$$

Notations

- ▶ $\hat{\mathbf{B}}=_{df} (\mathbf{B}, \wedge, \vee, \leq, \mathbf{false}, \mathbf{true})$: The lattice of Boolean values w/ $\mathbf{false} \leq \mathbf{true}$ and the logical \wedge and \vee as meet operation and join operation \sqcap and \sqcup , respectively.
- ▶ $Cst_{\mathbf{true}}$ and $Cst_{\mathbf{false}}$: The constant functions “true” and “false” on $\hat{\mathbf{B}}$, respectively.
- ▶ $Id_{\mathbf{B}}$: The identity function on $\hat{\mathbf{B}}$.

...and for a fixed candidate expression t :

- ▶ $Comp_e$: t is **computed** by the instruction attached to edge e (i.e., t is a subexpression of the right-hand side expression)
- ▶ $Transp_e$: no operand of t is assigned a new value by the instruction attached to edge e (i.e. no operand of t occurs on the left-hand side: e is **transparent** for t)

Main Results

Lemma (5.4.1.1)

$\llbracket \cdot \rrbracket_{av}$ is distributive.

Corollary (5.4.1.2)

The MOP solution and the MaxFP solution coincide for available expressions.

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Chapter 5.4.2

Simple Constants

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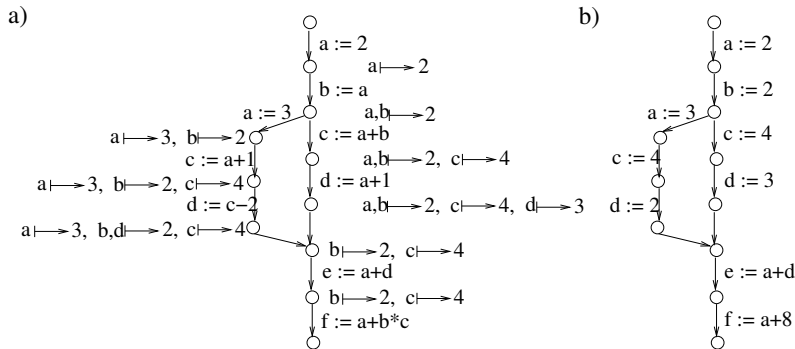
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Simple Constants

...a typical monotonic (but non distributive) DFA problem.



Abstract Semantics for Simple Constants

► Local abstract semantics for **simple constants**:

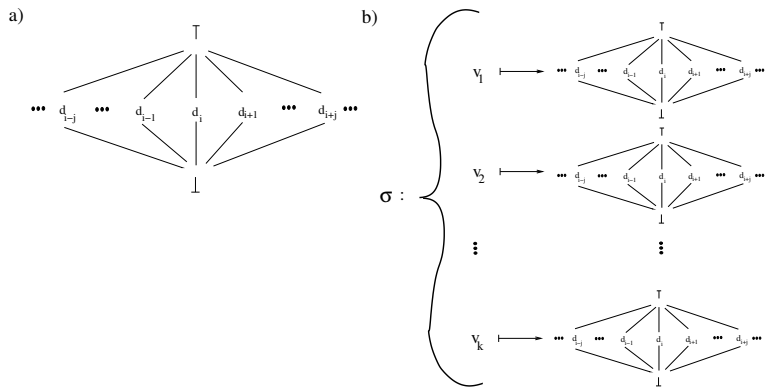
1. **DFA lattice**: $(\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \perp, \top) =_{df} (\Sigma, \sqcap, \sqcup, \sqsubseteq, \sigma_{\perp}, \sigma_{\top})$

2. **DFA functional**: $\llbracket \cdot \rrbracket_{sc} : E \rightarrow (\Sigma \rightarrow \Sigma)$ defined by

$$\forall e \in E. \llbracket e \rrbracket_{sc} =_{df} \theta_e$$

DFA Lattice for Simple Constants

The “canonical” lattice for constant propagation and folding:



The Semantics of Terms

The **semantics** of terms $t \in \mathbf{T}$ is given by the inductively defined **evaluation function**

$$\mathcal{E} : \mathbf{T} \rightarrow (\Sigma \rightarrow \mathbf{D})$$

$$\forall t \in \mathbf{T} \forall \sigma \in \Sigma. \mathcal{E}(t)(\sigma) \stackrel{\text{df}}{=} \begin{cases} \sigma(x) & \text{if } t = x \in \mathbf{V} \\ l_0(c) & \text{if } t = c \in \mathbf{C} \\ l_0(\text{op})(\mathcal{E}(t_1)(\sigma), \dots, \mathcal{E}(t_r)(\sigma)) & \text{if } t = \text{op}(t_1, \dots, t_r) \end{cases}$$

Some Yet to be defined Notions

...to complete the definition of the semantics of terms:

- ▶ Term syntax
- ▶ Interpretation
- ▶ State

The Syntax of Terms (1)

Let

- ▶ \mathbf{V} be a set of variables
- ▶ \mathbf{Op} be a set of n -ary operators, $n \geq 0$, and $\mathbf{C} \subseteq \mathbf{Op}$ be the set of 0-ary operators, the so-called **constants** in \mathbf{Op} .

The Syntax of Terms (2)

We legen fest:

1. Each variable $v \in \mathbf{V}$ and each constant $c \in \mathbf{C}$ is a **term**.
2. If $op \in \mathbf{Op}$ is an n -ary operator, $n \geq 1$, and t_1, \dots, t_n are terms, then $op(t_1, \dots, t_n)$ is a **term**, too.
3. There are no other **terms** in addition to those that can be constructed by the above two rules.

The set of all terms is denoted by \mathbf{T} .

Interpretation

Let \mathbf{D}' be a suitable data domain (e.g. the set of integers), let \perp and \top be two distinguished elements w/ $\perp, \top \notin \mathbf{D}'$, and let $\mathbf{D} =_{df} \mathbf{D}' \cup \{\perp, \top\}$.

An **interpretation** on \mathbf{T} and \mathbf{D} is a tuple $I \equiv (\mathbf{D}, I_0)$, where

- ▶ I_0 is a function, which associates w/ each 0-ary operator $c \in \mathbf{Op}$ a datum $I_0(c) \in \mathbf{D}'$ and w/ each n -ary operator $op \in \mathbf{Op}$, $n \geq 1$, a total function $I_0(op) : \mathbf{D}^n \rightarrow \mathbf{D}$, which is assumed to be **strict** (i.e. $I_0(op)(d_1, \dots, d_n) = \perp$, if there is a $j \in \{1, \dots, n\}$ w/ $d_j = \perp$)

Set of States

$$\Sigma =_{df} \{ \sigma \mid \sigma : \mathbf{V} \rightarrow \mathbf{D} \}$$

...denotes the set of **states**, i.e. the set of mappings σ from the set of variables \mathbf{V} to a suitable data domain \mathbf{D} (that is not specified in more detail here).

In particular

- ▶ σ_{\perp} : ...denotes the **totally undefined** state of Σ that is defined as follows: $\forall v \in \mathbf{V}. \sigma_{\perp}(v) = \perp$

The State Transformation Function

The **state transformation function**

$$\theta_\iota : \Sigma \rightarrow \Sigma, \quad \iota \equiv x := t$$

is defined by:

$$\forall \sigma \in \Sigma \quad \forall y \in \mathbf{V}. \theta_\iota(\sigma)(y) =_{df} \begin{cases} \mathcal{E}(t)(\sigma) & \text{falls } y = x \\ \sigma(y) & \text{sonst} \end{cases}$$

Main Results

Lemma (5.4.2.1)





$\llbracket \cdot \rrbracket_{sc}$ is monotonic.

Note: Distributivity does not hold! (Exercise)





Corollary (5.4.2.2)

The MOP solution and the MaxFP solution do in general not coincide. The MaxFP solution, however, is always a safe approximation of the MOP solution for simple constants.

Further Reading for Chapter 5 (1)

-  Alfred V. Aho, Monica S. Lam, Ravi Sethi, Jeffrey D. Ullman. *Compilers: Principles, Techniques, & Tools*. Addison-Wesley, 2nd edition, 2007. (Chapter 1, Introduction; Chapter 9.2, Introduction to Data-Flow Analysis; Chapter 9.3, Foundations of Data-Flow Analysis)
-  F. E. Allen, J. A. Cocke. *A Program Data Flow Analysis Procedure*. Communications of the ACM 19(3):137-147, 1976.
-  Randy Allen, Ken Kennedy. *Optimizing Compilers for Modern Architectures*. Morgan Kaufman Publishers, 2002. (Chapter 4.4, Data Flow Analysis)
-  Keith D. Cooper, Linda Torczon. *Engineering a Compiler*. Morgan Kaufman Publishers, 2004. (Chapter 1, Overview of Compilation; Chapter 8, Introduction to Code Optimization; Chapter 9, Data Flow Analysis)




Further Reading for Chapter 5 (2)

-  C. Fecht, Helmut Seidl. *An Even Faster Solver for General Systems of Equations*. In Proceedings SAS'96, LNCS 1145, 189-204, 1996.
-  C. Fecht, Helmut Seidl. *Propagating Differences: An Efficient New Fixpoint Algorithm for Distributive Constraint Systems*. In Proceedings ESOP'98, LNCS 1381, 90-104, 1998.
-  C. Fecht, Helmut Seidl. *A Faster Solver for General Systems of Equations*. *Science of Computer Programming* 35(2):137-161, 1999.
-  Matthew S. Hecht. *Flow Analysis of Computer Programs*. Elsevier, North-Holland, 1977.





Further Reading for Chapter 5 (3)

-  Susan Horwitz, A. Demers, T. Teitelbaum. *An Efficient General Iterative Algorithm for Dataflow Analysis*. Acta Informatica 24:679-694, 1987.
-  John B. Kam, Jeffrey D. Ullman. *Global Data Flow Analysis and Iterative Algorithms*. Journal of the ACM 23:158-171, 1976.
-  John B. Kam, Jeffrey D. Ullman. *Monotone Data Flow Analysis Frameworks*. Acta Informatica 7:305-317, 1977.
-  Gary A. Kildall. *A Unified Approach to Global Program Optimization*. In Conference Record of the 1st Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages (POPL'73), 194-206, 1973.




Further Reading for Chapter 5 (4)

-  Marion Klein, Jens Knoop, Dirk Koschützki, Bernhard Steffen. *DFA&OPT-METAFrame: A Toolkit for Program Analysis and Optimization*. In Proceedings TACAS'96, Springer-V., LNCS 1055, 422-426, 1996.
-  Jens Knoop. *From DFA-Frameworks to DFA-Generators: A Unifying Multiparadigm Approach*. In Proceedings of the 5th International Conference on Tools and Algorithms for the Construction and Analysis of Systems (TACAS'99), Springer-V., LNCS 1579, 360-374, 1999.
-  Janusz Laski, William Stanley. *Software Verification and Analysis*. Springer-V., 2009. (Chapter 7, What can one tell about a Program without its Execution: Static Analysis)



Further Reading for Chapter 5 (5)

-  Thomas J. Marlowe, Barbara G. Ryder. *Properties of Data Flow Frameworks*. Acta Informatica 28(2):121-163, 1990.
-  Florian Martin. *PAG - An Efficient Program Analyzer Generator*. Journal of Software Tools for Technology Transfer 2(1):46-67, 1998.
-  Robert Morgan. *Building an Optimizing Compiler*. Digital Press, 1998.
-  Stephen S. Muchnick. *Advanced Compiler Design Implementation*. Morgan Kaufman Publishers, 1997. (Chapter 1, Introduction to Advanced Topics; Chapter 4, Intermediate Representations; Chapter 7, Control-Flow Analysis; Chapter 8, Data Flow Analysis; Chapter 11, Introduction to Optimization; Chapter 12, Early Optimizations)

Further Reading for Chapter 5 (6)

-  Flemming Nielson. *Semantics-directed Program Analysis: A Tool-maker's Perspective*. In Proceedings SAS'96, Springer-V., LNCS 1145, 2-21, 1996.
-  Hanne Riis Nielson, Flemming Nielson. *Semantics with Applications: A Formal Introduction*. Wiley, 1992. (Chapter 5, Static Program Analysis)
-  Hanne Riis Nielson, Flemming Nielson. *Semantics with Applications: An Appetizer*. Springer-V., 2007. (Chapter 7, Program Analysis; Chapter 8, More on Program Analysis; Appendix B, Implementation of Program Analysis)

Further Reading for Chapter 5 (7)

-  Flemming Nielson, Hanne Riis Nielson, Chris Hankin. *Principles of Program Analysis*. 2nd edition, Springer-V., 2005. (Chapter 1, Introduction; Chapter 2, Data Flow Analysis; Chapter 6, Algorithms)
-  Barry K. Rosen. *High-level Data Flow Analysis*. Communications of the ACM 20(10):141-156, 1977.

Chapter 6

Partial Redundancy Elimination

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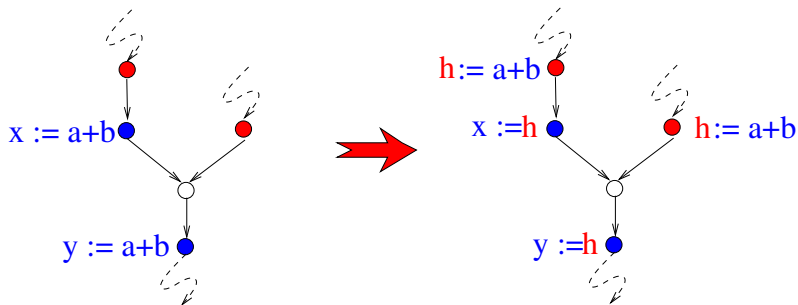
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Partial Redundancy Elimination (PRE)

What's it all about?

...avoiding multiple (re-) computations of the same value!



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Motivation

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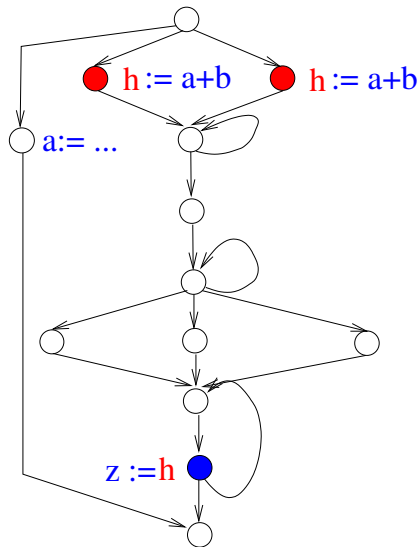
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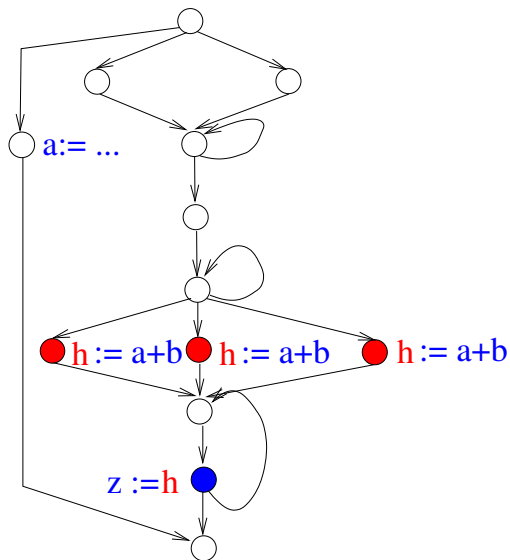
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A Computationally Optimal Program

...w/out any redundancy at all!



Often there is more than one!



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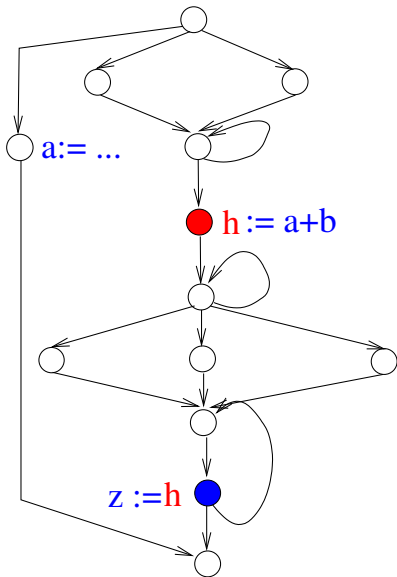
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Which one shall PRE deliver?



The (Optimization) Goals make the Difference!

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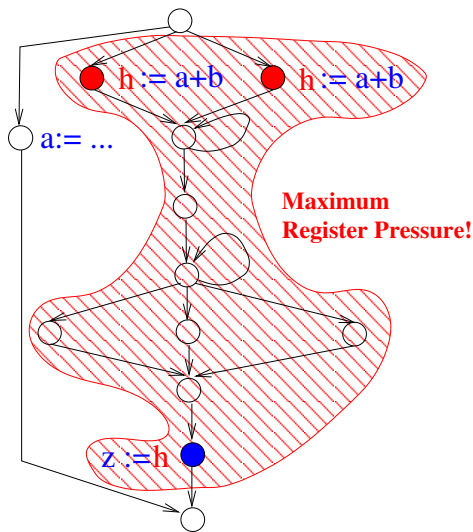
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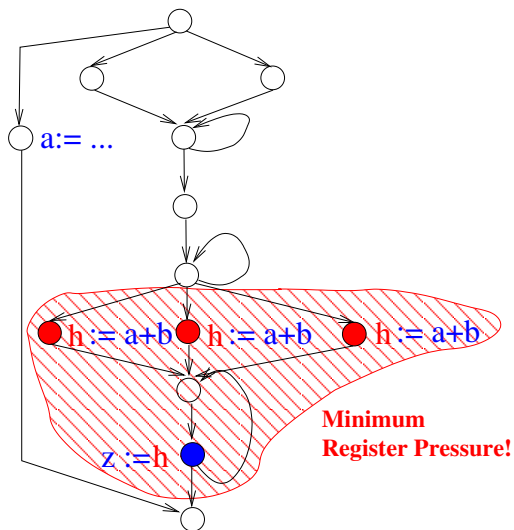
The first Transformation

...no redundancies but **maximum** register pressure!



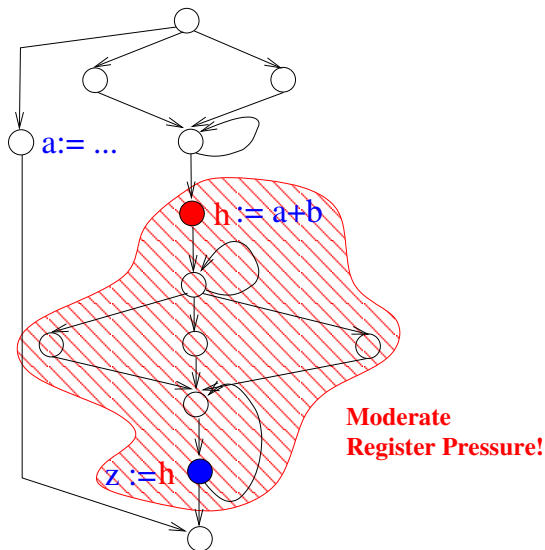
The second Transformation

...no redundancies, too, but **minimum** register pressure!



The third Transformation

...no redundancies, moderate register pressure, no code replication!



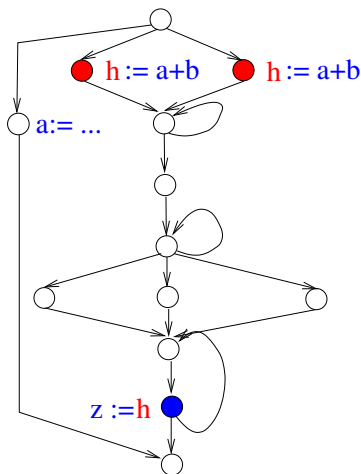
The (Optimization) Goals make the Difference!

In our running example:

- ▶ **Performance:** Avoiding unnecessary (re-) computations
 \rightsquigarrow Computational quality, **computational optimality**
- ▶ **Register pressure:** Avoiding unnecessary code motion
 \rightsquigarrow Lifetime quality, **lifetime optimality**
- ▶ **Space:** Avoiding unnecessary code replication
 \rightsquigarrow Code size quality, **code size optimality**

The Result of Busy Code Motion

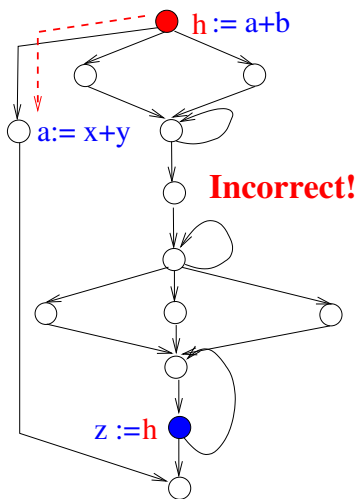
...placing computations as early as possible!



...yields **computationally optimal** programs.

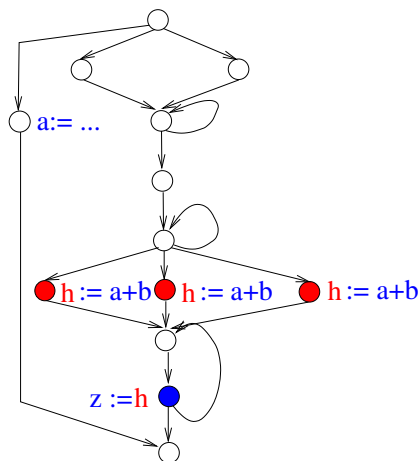
Note: As Early as Possible

...means earliest indeed but not earlier as earliest.



The Result of Lazy Code Motion

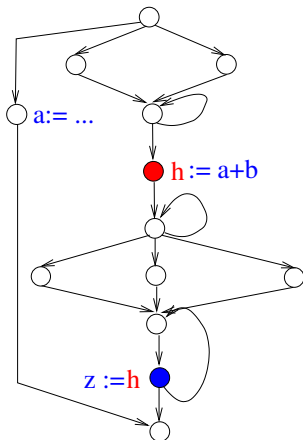
...placing computations as late as possible!



...yields **computationally and lifetime optimal** programs.

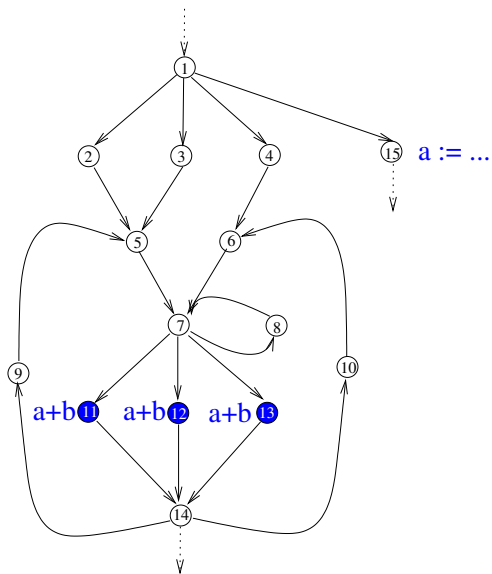
The Result of Sparse Code Motion

...placing computations as late as possible but as early as necessary!



...yields comp. and lifetime best **code-size optimal** programs.

A More Complex Example (1)



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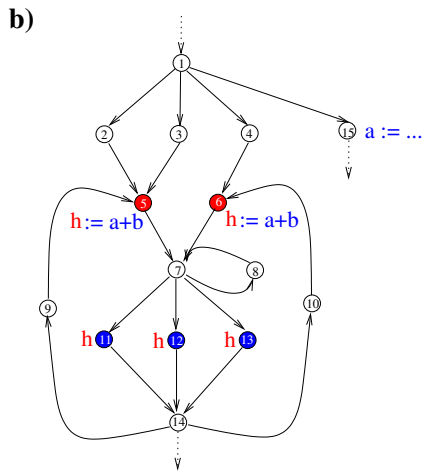
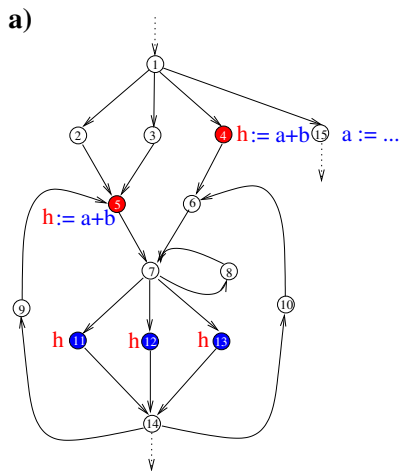
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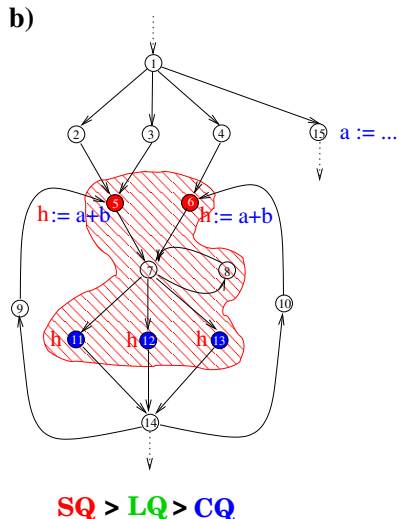
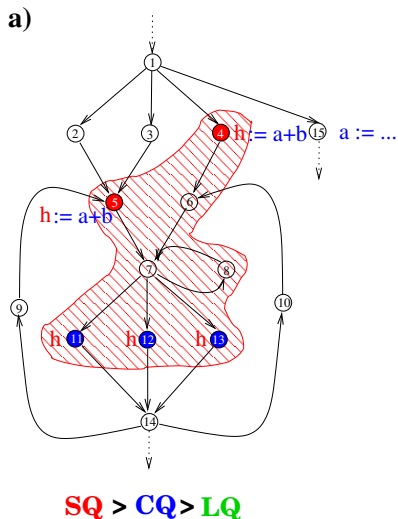
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A More Complex Example (2)



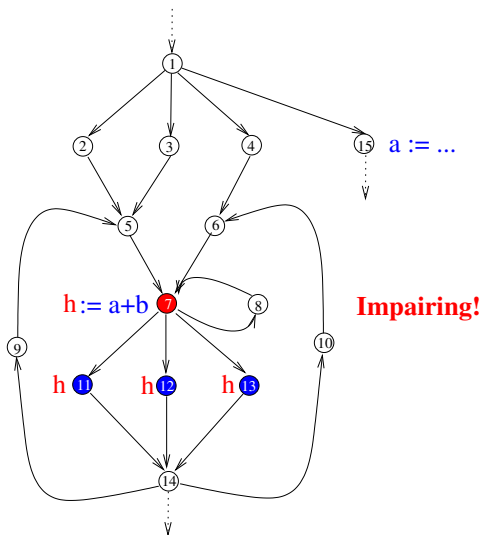
Two Code-size Optimal Programs

A More Complex Example (3)



A More Complex Example (4)

Note: The below transformation is not desired!



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Summing up

The previous examples demonstrate that in general we can not achieve

- ▶ computational, lifetime, and space optimality at the same time.

Think, however, about the following problem ([homework](#)):

- ▶ Let P be a program containing partially redundant computations.

Can you imagine an algorithm that always succeeds to transform P into a program P' such that P and P' have the same semantics and that P' is free of any partially redundant computation?

Chapter 6.2

The PRE Algorithm of Morel&Renvoise

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The Groundbreaking Algorithm for PRE

PRE is intrinsically tied to Etienne Morel und Claude Renvoise. The PRE algorithm they presented in 1979 can be considered the *prime father* of all [code motion \(CM\) algorithms](#) and was until the early 1990s the “state of the art” [PRE](#) algorithm.

Technically, the PRE algorithm of Morel and Renvoise is composed of:

- ▶ 3 uni-directional bitvector analyses (AV, ANT, PAV)
- ▶ 1 bi-directional bitvector analysis (PP)

The PRE Algorithm of Morel&Renvoise (1)

- ▶ Availability:

$$\mathbf{AVIN}(n) = \begin{cases} \mathbf{false} & \text{if } n = \mathbf{s} \\ \prod_{m \in \text{pred}(n)} \mathbf{AVOUT}(m) & \text{otherwise} \end{cases}$$

$$\mathbf{AVOUT}(n) = \text{TRANSP}(n) * (\text{COMP}(n) + \mathbf{AVIN}(n))$$

The PRE Algorithm of Morel&Renvoise (2)

- ▶ Very Busyess (Anticipability):

$$\mathbf{ANTIN}(n) = \text{COMP}(n) + \text{TRANSP}(n) * \mathbf{ANTOUT}(n)$$

$$\mathbf{ANTOUT}(n) = \begin{cases} \mathbf{false} & \text{if } n = \mathbf{e} \\ \prod_{m \in \text{succ}(n)} \mathbf{ANTIN}(m) & \text{otherwise} \end{cases}$$

The PRE Algorithm of Morel&Renvoise (3)

► Partial Availability:

$$\mathbf{PAVIN}(n) = \begin{cases} \mathbf{false} & \text{if } n = \mathbf{s} \\ \sum_{m \in \text{pred}(n)} \mathbf{PAVOUT}(m) & \text{otherwise} \end{cases}$$

$$\mathbf{PAVOUT}(n) = \text{TRANSP}(n) * (\text{COMP}(n) + \mathbf{PAVIN}(n))$$

The PRE Algorithm of Morel&Renvoise (4)

- Placement Possible:

$$\mathbf{PPIN}(n) = \begin{cases} \mathbf{false} & \text{if } n = \mathbf{s} \\ \mathbf{CONST}(n)* \\ \left(\prod_{m \in \text{pred}(n)} (\mathbf{PPOUT}(m) + \mathbf{AVOUT}(m)) \right)* \\ (\mathbf{COMP}(n) + \mathbf{TRANSP}(n) * \mathbf{PPOUT}(n)) \\ \text{otherwise} \end{cases}$$

$$\mathbf{PPOUT}(n) = \begin{cases} \mathbf{false} & \text{if } n = \mathbf{e} \\ \prod_{m \in \text{succ}(n)} \mathbf{PPIN}(m) & \text{otherwise} \end{cases}$$

where

$$\mathbf{CONST}(n) =_{df} \mathbf{ANTIN}(n) * (\mathbf{PAVIN}(n) + \neg \mathbf{COMP}(n) * \mathbf{TRANSP}(n))$$

The PRE Algorithm of Morel&Renvoise (5)

- ▶ Initializing temporaries where:

$$\mathbf{INSIN}(n) =_{df} \mathbf{false}$$

$$\mathbf{INSOUT}(n) =_{df} \mathbf{PPOUT}(n) * \neg \mathbf{AVOUT}(n) * (\neg \mathbf{PPIN}(n) + \neg \mathbf{TRANSP}(n))$$

- ▶ Replacing original computations where:

$$\mathbf{REPLACE}(n) =_{df} \mathbf{COMP}(n) * \mathbf{PPIN}(n)$$

Summing up (1)

Achievements and merits of Morel&Renvoise's PRE algorithm:

- ▶ First systematic algorithm for PRE
- ▶ State-of-the-art PRE algorithm for about 15 years

Summing up (2)

Short-comings of Morel&Renvoise's PRE algorithm:

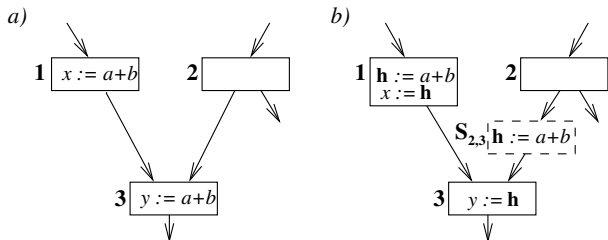
- ▶ **Conceptually**
 - ▶ Fails computational optimality
 \rightsquigarrow only, however, because of not splitting critical edges
 - ▶ Fails lifetime optimality
 \rightsquigarrow Register pressure is heuristically dealt with
 - ▶ Fails code-size optimality
 \rightsquigarrow Not considered at all (in the early days of PRE)
- ▶ **Technically**
 - ▶ Bi-directional
 \rightsquigarrow conceptually and computationally thus more complex

...the transformation result lies (unpredictably) between those of the **BCM** transformation and the **LCM** transformation.

Critical Edges

An edge is called **critical**, if it connects a branching node with a join node.

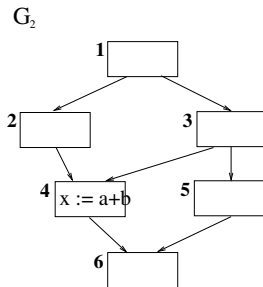
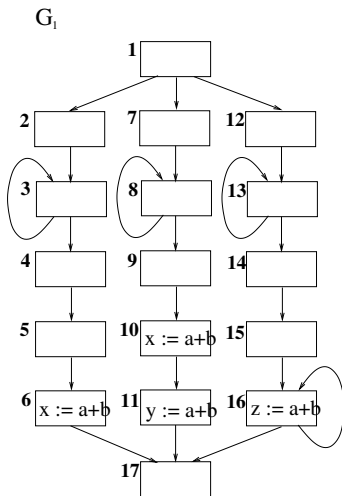
Illustration:






...by introducing the **synthetic** node $S_{2,3}$, the critical edge from node **2** to node **3** is split which allows to eliminate the partially redundant computation of $a + b$ at node **3**.

Instructive




...optimizing the following two programs using the PRE algorithm of Morel&Renvoise:






Further Reading for Chapter 6 (1)

-  Alfred V. Aho, Monica S. Lam, Ravi Sethi, Jeffrey D. Ullman. *Compilers: Principles, Techniques, & Tools*. Addison-Wesley, 2nd edition, 2007. (Chapter 9.5, Partial-Redundancy Elimination)
-  Keith D. Cooper, Linda Torczon. *Engineering a Compiler*. Morgan Kaufman Publishers, 2004. (Chapter 8.6, Global Redundancy Elimination)
-  D. M. Dhamdhere. *Practical Adaptation of the Global Optimization Algorithm of Morel and Renvoise*. ACM Transactions on Programming Languages and Systems 13(2):291-294, 1991, Technical Correspondence.

Further Reading for Chapter 6 (2)

-  D. M. Dhamdhere. *E-path_pre: Partial Redundancy Elimination Made Easy*. ACM SIGPLAN Notices 37(8):53-65, 2002.
-  K.-H. Drechsler, M. P. Stadel. *A Solution to a Problem with Morel and Renvoise's "Global Optimization by Suppression of Partial Redundancies"*. ACM Transactions on Programming Languages and Systems 10(4):635-640, 1988, Technical Correspondence.
-  Andrei P. Ershov. *On Programming of Arithmetic Operations*. Communications of the ACM 1(8):3-6, 1958. (Three figures from this article are in CACM 1(9):16).


Further Reading for Chapter 6 (3)

-  R. Nigel Horspool, H. C. Ho. *Partial Redundancy Elimination Driven by a Cost-benefit Analysis*. In Proceedings of the 8th Israeli Conference on Computer Systems and Software Engineering (CSSE'97), 111-118, 1997.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Lazy Code Motion*. In Proceedings of the ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI'92), ACM SIGPLAN Notices 27(7):224-234, 1992.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Optimal Code Motion: Theory and Practice*. ACM Transactions on Programming Languages and Systems 16(4):1117-1155, 1994.

Further Reading for Chapter 6 (4)

-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Retrospective: Lazy Code Motion*. In “20 Years of the ACM SIGPLAN Conference on Programming Language Design and Implementation (1979 - 1999): A Selection”, ACM SIGPLAN Notices 39(4):460-461&462-472, 2004.
-  Etienne Morel, Claude Renvoise. *Global Optimization by Suppression of Partial Redundancies*. Communications of the ACM 22(2):96-103, 1979.
-  Stephen S. Muchnick. *Advanced Compiler Design Implementation*. Morgan Kaufman Publishers, 1997. (Chapter 13, Redundancy Elimination)

Further Reading for Chapter 6 (5)

-  Oliver R uthing, Jens Knoop, Bernhard Steffen. *Sparse Code Motion*. In Conference Record of the 27th Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages (POPL 2000), 170-183, 2000.

Chapter 7

Busy Code Motion

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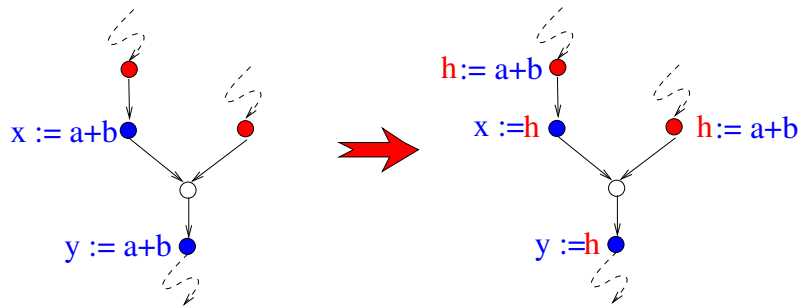
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The Very Idea

...of Code Motion (CM) – often synonymously used with Partial Redundancy Elimination (PRE) – recalled:

...avoiding multiple (re-) computations of the same value!



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Preliminaries

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Notations

Given a flow graph $G = (N, E, \mathbf{s}, \mathbf{e})$ let

- ▶ $pred(n) =_{df} \{m \mid (m, n) \in E\}$ denote the set of all **predecessors**
- ▶ $succ(n) =_{df} \{m \mid (n, m) \in E\}$ denote the set of all **successors**
- ▶ $source(e), dest(e)$ denote the **start node** and **end node** of an edge
- ▶ a sequence of edges (e_1, \dots, e_k) with $dest(e_i) = source(e_{i+1})$ for all $1 \leq i < k$ denote a **finite path**.

Note: Instead of edge sequences we also consider node sequences as paths, where reasonable.

Notations (Cont'd)

More specifically:

- ▶ $p = \langle e_1, \dots, e_k \rangle$ denotes a **path from m to n** , if $source(e_1) = m$ and $dest(e_k) = n$
- ▶ $\mathbf{P}[m, n]$ denotes the set of **all paths from m to n**
- ▶ λ_p denotes the **length** of p , i.e., the number of edges of p
- ▶ ε denotes the path of length 0
- ▶ $N_J \subseteq N$ denotes the set of **join** nodes, i.e., the set of nodes w/ more than one predecessor
- ▶ $N_B \subseteq N$ denotes the set of **branch** nodes, i.e. the set of nodes w/ more than one successor

Convention

W/out losing generality we assume:

- ▶ Each node of a flow graph lies on a path from **s** to **e**

Intuition: There are no unreachable parts within a flow graph.

...this is a typical and usual assumption for analysis and optimization!

An additional CM specific Convention

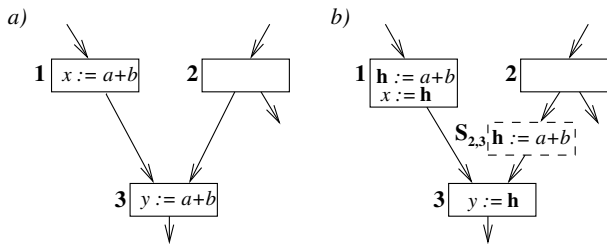
W/out losing generality we focus in the following on flow graphs given

- ▶ as node labelled SI graphs
- ▶ where **all** edges leading to a join node are split by inserting a so-called synthetic node (i.e., not just critical edges are split)

Reminder: Critical Edges

An edge is called **critical**, if it connects a branching node with a join node.

Illustration: ...by introducing the **synthetic** node $S_{2,3}$, the critical edge from node **2** to node **3** is split.



In the following we assume that also edges like the one from node **1** to node **3** are split by introducing a new node $S_{1,3}$.

Background and Motivation

...underlying this convention:

- ▶ The CM process becomes simpler.
 - ~> **computationally optimal** results can be achieved by initializing temporaries always at node entries.

Note

Computationally optimal results can also be achieved, if only critical edges are split.

This, however, requires that a PRE algorithm is able to perform initializations both at node entries (N-initializations) and at node exits (X-Initializations).

This is not a problem at all. Agreeing, however, on the above assumption simplifies the presentation of the CM algorithm even more.

Work Plan

In the following we will define:

- ▶ The set of CM transformations
- ▶ The set of **admissible** CM transformations
- ▶ The set of **computationally optimal** CM transformations
- ▶ The **BCM transformation** as a specific computationally optimal CM transformation
- ▶ The **LCM transformation** as the one and only computationally and lifetime optimal CM transformation (Chapter 8)

The Generic Pattern of a CM Transformations

The generic 3-step (transformation) pattern for a term t :

- ▶ Introduce a fresh temporary h for t in G
- ▶ Insert at some nodes of G the assignment statement $h := t$
- ▶ Replace some of the original occurrences of t in G by h

Remark: t is often called a **candidate expression**.

Observation

Two predicates (defined on nodes)

- ▶ $Insert_{CM}$
- ▶ $Repl_{CM}$

suffice to specify a CM (resp. PRE) transformation completely

(the first step of declaring the temporary h is the same for each CM transformation and thus does not need to be considered explicitly).

The Set of CM Transformations

...let \mathcal{CM}_t denote the set of all CM transformations (for the candidate expression t).

In the following we will consider a fixed candidate expression t and thus drop the index t .

Observation

Obviously, some transformations in \mathcal{CM} do not preserve the semantics and are thus not acceptable.

This leads us to the notion of **admissible** CM transformations.

Admissible CM Transformations

Let $CM \in \mathcal{CM}$.

CM is called **admissible**, if CM is **safe** and **correct**.

Intuitively:

- ▶ **Safe:** There is no path, on which by inserting an initialization a new value is computed.
- ▶ **Correct:** Wherever the temporary is used, it stores the “right” value, i.e., it stores the same value that a recomputation of t at the use site yields.

Formalising this

...requires two (local) predicates:

- ▶ $Comp_t(n)$: the candidate expression t is **computed** at n .
- ▶ $Transp_t(n)$: n is **transparent** for t , i.e., n does not modify any operand of t .

Note: In the following we will drop the index t .

Moreover, it is useful to introduce a third (local) predicate:

- ▶ $Comp_{CM}(n) =_{df} Insert_{CM}(n) \vee Comp(n) \wedge \neg Repl_{CM}(n)$:
The candidate expression t is computed after the application of CM .

Extending Predicates from Nodes to Paths

Let p be a path and let p_i denote the i th node of p .

Then we define:

- ▶ $Predicate^{\forall}(p) \iff \forall 1 \leq i \leq \lambda_p. Predicate(p_i)$
- ▶ $Predicate^{\exists}(p) \iff \exists 1 \leq i \leq \lambda_p. Predicate(p_i)$

Safety and Correctness

Definition (7.1.1, Safety and Correctness)

Let $n \in N$. Then:

1. $\text{Safe}(n) \iff_{df} \forall \langle n_1, \dots, n_k \rangle \in \mathbf{P}[s, e] \forall i. (n_i = n) \Rightarrow$
 - i) $\exists j < i. \text{Comp}(n_j) \wedge \text{Transp}^{\forall}(\langle n_j, \dots, n_{i-1} \rangle) \vee$
 - ii) $\exists j \geq i. \text{Comp}(n_j) \wedge \text{Transp}^{\forall}(\langle n_i, \dots, n_{j-1} \rangle)$
2. Let $CM \in \mathcal{CM}$. Then:
 $\text{Correct}_{CM}(n) \iff_{df} \forall \langle n_1, \dots, n_k \rangle \in \mathbf{P}[s, n]$
 $\exists i. \text{Insert}_{CM}(n_i) \wedge \text{Transp}^{\forall}(\langle n_i, \dots, n_{k-1} \rangle)$

Up-Safety and Down-Safety

Constraining the definition of **safety** to condition (i) resp. (ii) leads to the notions of

- ▶ **up-safety** (availability)
- ▶ **down-safety** (anticipability, very busyness)

Intuition

A computation of t at program point n is

- ▶ **up-safe**, if t is computed on all paths p from \mathbf{s} to n and the last computation of t on p is not followed by a modification of (an operand of) t .
- ▶ **down-safe**, if t is computed on all paths p from n to \mathbf{e} and the first computation of t on p is not preceded by a modification of (an operand of) t .

Up-Safety and Down-Safety

Definition (7.1.2, Up-Safety and Down-Safety)

1. $\forall n \in \mathbb{N}. U\text{-Safe}(n) \iff_{df}$
 $\forall p \in \mathbf{P}[s, n] \exists i < \lambda_p. \text{Comp}(p_i) \wedge \text{Transp}^\forall(p[i, \lambda_p[)$
2. $\forall n \in \mathbb{N}. D\text{-Safe}(n) \iff_{df}$
 $\forall p \in \mathbf{P}[n, e] \exists i \leq \lambda_p. \text{Comp}(p_i) \wedge \text{Transp}^\forall(p[1, i[)$

Admissible CM-Transformations

This allows us to define:

Definition (7.1.3, Admissible CM-Transformation)

A CM-transformation $CM \in \mathcal{CM}$ is **admissible** iff for every node $n \in N$ holds:

1. $Insert_{CM}(n) \Rightarrow Safe(n)$
2. $Repl_{CM}(n) \Rightarrow Correct_{CM}(n)$

The set of all admissible CM-transformations is denoted by \mathcal{CM}_{Adm} .

First Results

Lemma (7.1.4, Safety)

$$\forall n \in N. \text{ Safe}(n) \iff D\text{-Safe}(n) \vee U\text{-Safe}(n)$$

Lemma (7.1.5, Correctness)

$$\forall CM \in \mathcal{CM}_{Adm} \forall n \in N. \text{ Correct}_{CM}(n) \Rightarrow \text{ Safe}(n)$$

Computationally Better

Definition (7.1.6, Computationally Better)

A CM-transformation $CM \in \mathcal{CM}_{Adm}$ is **computationally better** as a CM-transformation $CM' \in \mathcal{CM}_{Adm}$ iff

$$\forall p \in \mathbf{P}[s, e]. \quad |\{i \mid Comp_{CM}(p_i)\}| \leq |\{i \mid Comp_{CM'}(p_i)\}|$$

Note: The relation “computationally better” is a quasi-order, i.e., a reflexive and transitive relation.

Computational Optimality

Definition (7.1.7, Comp. Optimal CM-Transf.)

An admissible CM-transformation $CM \in \mathcal{CM}_{Adm}$ is **computationally optimal** iff CM is computationally better than any other admissible CM-transformation.

We denote the set of all computationally optimal CM-transformations by \mathcal{CM}_{CmpOpt} .

Properties of Relations – A Reminder

Let M be a set and R be a relation on M , i.e., $R \subseteq M \times M$.

Then R is called

- ▶ **reflexive** iff $\forall m \in M. m R m$
- ▶ **transitive** iff $\forall m, n, p \in M. m R n \wedge n R p \Rightarrow m R p$
- ▶ **anti-symmetric** iff
 $\forall m, n \in M. m R n \wedge n R m \Rightarrow m = n$
- ▶ **quasi order** iff R is reflexive and transitive
- ▶ **partial order** iff R is reflexive, transitive and anti-symmetric

Chapter 7.2

The *BCM*-Transformation

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Conceptually

...CM can be considered a two-stage process consisting of:

1. **Hoisting expressions**

...hoisting expressions to “**earlier**” safe computation points

2. **Eliminating totally redundant expressions**

...elimination computations that became totally redundant by hoisting expressions

The Earliestness Principle

...induces an extreme placing (i.e., hoisting) strategy:

Placing computations **as early as possible**...

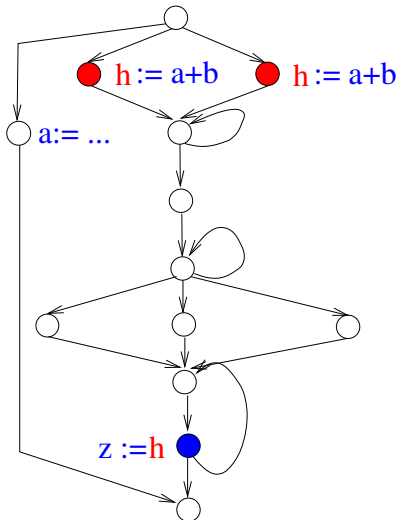
- ▶ **Theorem (Computational Optimality)**
...hoisting computations to their **earliest** safe computation points yields **computationally optimal** programs.

~> ...known as the **Busy Code Motion**

Earliestness Principle

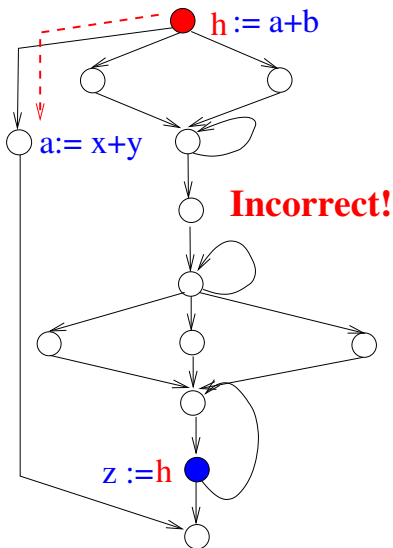
Placing computations **as early as possible**...

yields **computationally optimal** programs.



Note

...earliest means indeed as early as possible, but not earlier!



Busy Code Motion

Intuitively:

Place computations **as early as possible** in a program w/out violating safety and correctness!

Note: Following this principle computations are moved as far as possible in the opposite direction of the control flow

~> ...motivates the choice of the term **busy**.

Earliestness

Definition (7.2.1, Earliestness)

$\forall n \in N. Earliest(n) =_{df}$

$$Safe(n) \wedge \begin{cases} \mathbf{true} & \text{if } n = \mathbf{s} \\ \bigvee_{m \in pred(n)} \neg Transp(m) \vee \neg Safe(m) & \text{otherwise} \end{cases}$$

The *BCM* Transformation

The *BCM* Transformation is defined by:

- ▶ $Insert_{BCM}(n) =_{df} Earliest(n)$
- ▶ $Repl_{BCM}(n) =_{df} Comp(n)$

The *BCM* Theorem

Theorem (7.2.2, *BCM* Theorem)

The BCM-Transformation is computationally optimal, i.e.,
 $BCM \in \mathcal{CM}_{CmpOpt}$.

The proof of the *BCM* Theorem 7.2.2 relies on the [Earliest-ness Lemma 7.2.3](#) and the *BCM* Lemma 7.2.4.

The Earliestness Lemma

Lemma (7.2.3, Earliestness Lemma)

Let $n \in N$. Then we have:

1. $Safe(n) \Rightarrow \forall p \in \mathbf{P}[s, n] \exists i \leq \lambda_p.$
 $Earliest(p_i) \wedge Transp^{\forall}(p[i, \lambda_p[))$
2. $Earliest(n) \iff$
 $D-Safe(n) \wedge \bigwedge_{m \in pred(n)} (\neg Transp(m) \vee \neg Safe(m))$
3. $Earliest(n) \iff$
 $Safe(n) \wedge \forall CM \in$
 $\mathcal{CM}_{Adm}. Correct_{CM}(n) \Rightarrow Insert_{CM}(n)$

The BCM Lemma

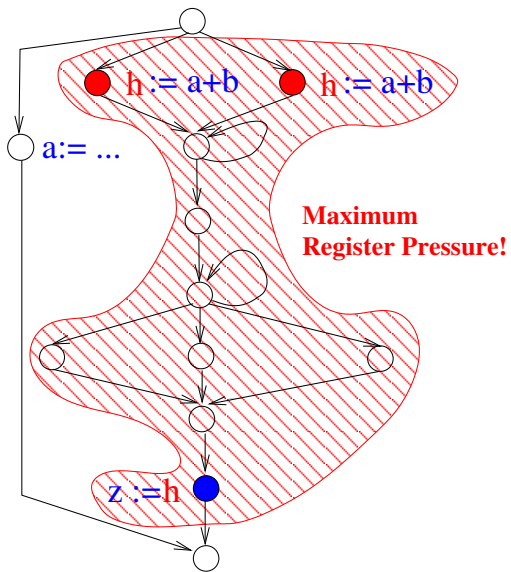
Lemma (7.2.4, BCM Lemma)

Let $p \in \mathbf{P}[s, e]$. Then we have:



1. $\forall i \leq \lambda_p. \text{Insert}_{BCM}(p_i) \iff \exists j \geq i. p[i, j] \in \text{FU-LtRg}(BCM)$
2. $\forall CM \in \mathcal{CM}_{Adm} \forall i, j \leq \lambda_p. p[i, j] \in \text{LtRg}(BCM) \Rightarrow \text{Comp}_{CM}^{\exists}(p[i, j])$
3. $\forall CM \in \mathcal{CM}_{CmpOpt} \forall i \leq \lambda_p. \text{Comp}_{CM}(p_i) \Rightarrow \exists j \leq i \leq l. p[j, l] \in \text{FU-LtRg}(BCM)$

The Result of the *BCM* Transformation

...computationally optimal, but **maximum register pressure.**



Further Reading for Chapter 7

-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Lazy Code Motion*. In Proceedings of the ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI'92), ACM SIGPLAN Notices 27(7):224-234, 1992.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Optimal Code Motion: Theory and Practice*. ACM Transactions on Programming Languages and Systems 16(4):1117-1155, 1994.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Retrospective: Lazy Code Motion*. In “20 Years of the ACM SIGPLAN Conference on Programming Language Design and Implementation (1979 - 1999): A Selection”, ACM SIGPLAN Notices 39(4):460-461&462-472, 2004.

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Lazy Code Motion

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The Latestness Principle

...induces an extreme dual placing strategy:

Placing computations **as late as possible**...

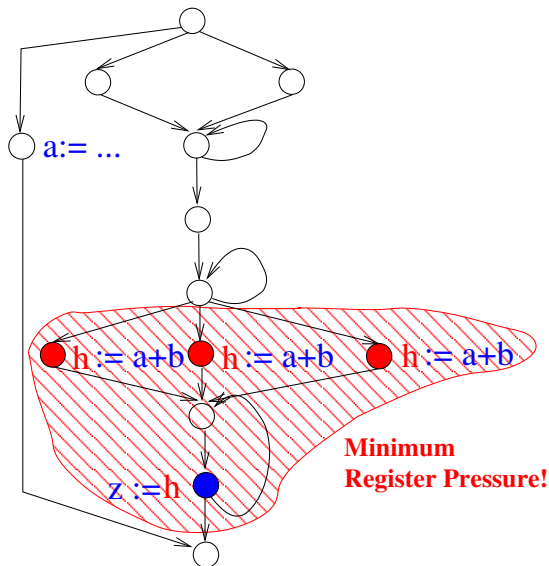
- ▶ **Theorem (Lifetime Optimality)**

...hoisting computations as little as possible, but as far as necessary (to achieve computational optimality), yields **computationally optimal programs w/ minimum register pressure**.

↪ ...known as the **Lazy Code Motion**

The Result of the *LCM* Transformation

...computationally optimal w/ minimum register pressure!



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Lazy Code Motion

Intuitively:

Place computations **as late as possible** in a program w/out violating safety, correctness and computational optimality!

Note: Following this principle computations are moved as little as possible in the opposite direction of the control flow

~> ...motivates the choice of the term **lazy**.

Work Plan

Next we will define:

- ▶ The set of **lifetime optimal** CM transformations
- ▶ The *LCM* transformation as the unique determined sole lifetime optimal CM transformation

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Central to Capture Register Pressure Formally

...is the notion of a (first-use) lifetime range.

Definition (8.1.1, Lifetime Ranges)

Let $CM \in \mathcal{CM}$.

- ▶ Lifetime range

$$LtRg(CM) =_{df} \{p \mid Insert_{CM}(p_1) \wedge Repl_{CM}(p_{\lambda_p}) \wedge \neg Insert_{CM}^{\exists}(p]1, \lambda_p)\}$$

- ▶ First-use lifetime range

$$FU-LtRg(CM) =_{df} \{p \in LtRg(CM) \mid \forall q \in LtRg(CM). (q \sqsubseteq p) \Rightarrow (q = p)\}$$

First Result

Lemma (8.1.2, First-Use Lifetime-Range Lemma)

Let $CM \in \mathcal{CM}$, $p \in \mathbf{P}[s, e]$, and let i_1, i_2, j_1, j_2 indexes such that $p[i_1, j_1] \in \text{FU-LtRg}(CM)$ and $p[i_2, j_2] \in \text{FU-LtRg}(CM)$. Then we have:

- ▶ either $p[i_1, j_1]$ and $p[i_2, j_2]$ coincide, i.e., $i_1 = i_2$ and $j_1 = j_2$, or
- ▶ $p[i_1, j_1]$ and $p[i_2, j_2]$ are disjoint, i.e., $j_1 < i_2$ or $j_2 < i_1$.

Lifetime Better

Definition (8.1.3, Lifetime Better)

A CM-transformation $CM \in \mathcal{CM}$ is **lifetime better** than a CM-transformation $CM' \in \mathcal{CM}$ iff

$$\forall p \in LtRg(CM) \exists q \in LtRg(CM'). p \sqsubseteq q$$

Note: The relation “lifetime better” is a partial order, i.e., a reflexive, transitive, and antisymmetric relation.

Lifetime Optimality

Definition (8.1.4, Lifetime Optimal CM-Transf.)

A computationally optimal CM-transformation $CM \in \mathcal{CM}_{CompOpt}$ is **lifetime optimal** iff CM is lifetime better than every other computationally optimal CM-transformation.

We denote the set of all lifetime optimal CM-transformations by \mathcal{CM}_{LtOpt} .

We have

Lemma (8.1.5)

$\forall CM \in \mathcal{CM}_{CmpOpt} \forall p \in LtRg(CM) \exists q \in LtRg(BCM). p \sqsubseteq q$

Intuitively:

- ▶ No computationally optimal CM-transformation places computations earlier as the *BCM* transformation
- ▶ The *BCM* transformation is that computationally optimal CM-transformation w/ maximum register pressure

Uniqueness of Lifetime Optimal PRE

Obviously we have:

$$\mathcal{CM}_{LtOpt} \subseteq \mathcal{CM}_{CmpOpt} \subseteq \mathcal{CM}_{Adm} \subset \mathcal{CM}$$

Actually, we have even more:

Theorem (8.1.6, Uniqueness of Lifetime Optimal CM-Transformations)

$$|\mathcal{CM}_{LtOpt}| \leq 1$$

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The *ALCM*-Transformation

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Delayability

Definition (8.2.1, Delayability)

$$\forall n \in \mathbf{N}. \text{Delayed}(n) \iff_{df} \forall p \in \mathbf{P}[s, n] \exists i \leq \lambda_p. \text{Earliest}(p_i) \wedge \neg \text{Comp}^{\exists}(p[i, \lambda_p[])$$

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The Delayability Lemma

Lemma (8.2.2, Delayability Lemma)

1. $\forall n \in N. \text{Delayed}(n) \Rightarrow \text{D-Safe}(n)$
2. $\forall p \in \mathbf{P}[s, e] \forall i \leq \lambda_p. \text{Delayed}(p_i) \Rightarrow \exists j \leq i \leq l. p[j, l] \in \text{FU-LtRg}(BCM)$
3. $\forall CM \in \mathcal{CM}_{\text{CompOpt}} \forall n \in N. \text{Comp}_{CM}(n) \Rightarrow \text{Delayed}(n)$

Definition (8.2.3, Latestness)

$$\forall n \in N. \text{Latest}(n) =_{df} \text{Delayed}(n) \wedge (\text{Comp}(n) \vee \bigvee_{m \in \text{succ}(n)} \neg \text{Delayed}(m))$$

The Latestness Lemma

Lemma (8.2.4, Latestness Lemma)

1. $\forall p \in LtRg(BCM) \exists i \leq \lambda_p. Latest(p_i)$
2. $\forall p \in LtRg(BCM) \forall i \leq \lambda_p. Latest(p_i) \Rightarrow$
 $\neg Delayed^{\exists}(p]i, \lambda_p]$

The *ALCM* Transformation

The “Almost Lazy Code Motion” Transformation is defined by:

- ▶ $Insert_{ALCM}(n) =_{df} Latest(n)$
- ▶ $Repl_{ALCM}(n) =_{df} Comp(n)$

Almost Lifetime Optimal

Definition (8.2.5, Almost Lifetime Optimal CM-Transformation)

A computationally optimal CM-transformation $CM \in \mathcal{CM}_{CmpOpt}$ is **almost lifetime optimal** iff

$$\forall p \in LtRg(CM). \lambda_p \geq 2 \Rightarrow \\ \forall CM' \in \mathcal{CM}_{CmpOpt} \exists q \in LtRg(CM'). p \sqsubseteq q$$

We denote the set of all almost lifetime optimal CM-transformations by \mathcal{CM}_{ALtOpt} .

The *ALCM* Theorem

Theorem (8.2.6, *ALCM* Theorem)

The ALCM transformation is almost lifetime optimal, i.e.,
 $ALCM \in \mathcal{CM}_{ALtOpt}$.

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Chapter 8.3

Lazy Code Motion

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Isolated Computations

Definition (8.3.1, *CM*-Isolation)

$$\forall CM \in \mathcal{CM} \forall n \in \mathbb{N}. \text{Isolated}_{CM}(n) \iff_{df}$$
$$\forall p \in \mathbf{P}[n, e] \forall 1 < i \leq \lambda_p. \text{Repl}_{CM}(p_i) \Rightarrow \text{Insert}_{CM}^{\exists}(p[1, i])$$

The Isolation Lemma

Lemma (8.3.2, Isolation Lemma)

1. $\forall CM \in \mathcal{CM} \forall n \in N. \text{Isolated}_{CM}(n) \iff$
 $\forall p \in \text{LtRg}(CM). \langle n \rangle \sqsubseteq p \Rightarrow \lambda_p = 1$
2. $\forall CM \in \mathcal{CM}_{\text{CompOpt}} \forall n \in N. \text{Latest}(n) \Rightarrow$
 $(\text{Isolated}_{CM}(n) \iff \text{Isolated}_{BCM}(n))$

The *LCM* Transformation

The *LCM* Transformation is defined by:

- ▶ $Insert_{LCM}(n) =_{df} Latest(n) \wedge \neg Isolated_{BCM}(n)$
- ▶ $Repl_{LCM}(n) =_{df} Comp(n) \wedge \neg(Latest(n) \wedge Isolated_{BCM}(n))$

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The *LCM* Theorem

Theorem (8.3.3, *LCM* Theorem)

The LCM transformation is lifetime optimal, i.e.,
 $LCM \in \mathcal{CM}_{LtOpt}$.

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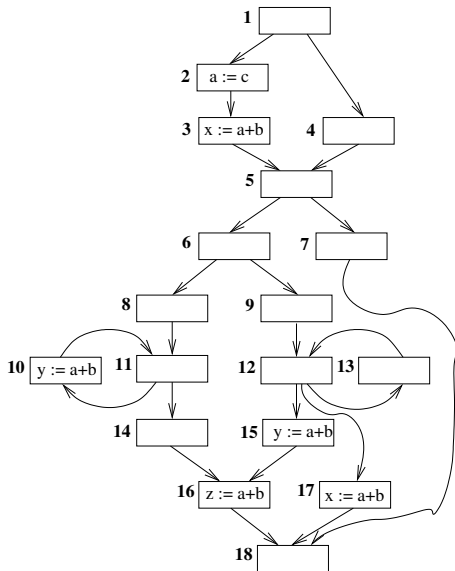
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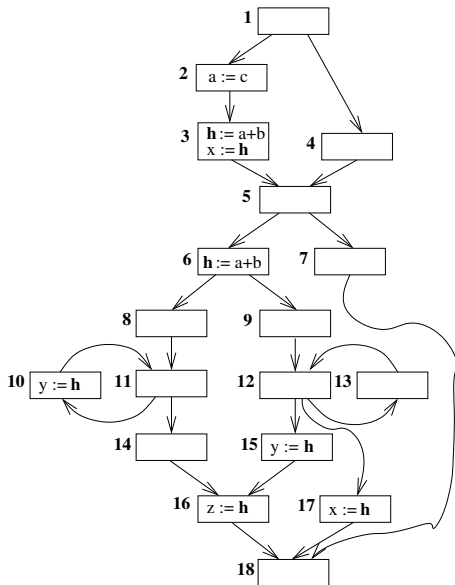
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The Original Program



The Result of the *BCM* Transformation



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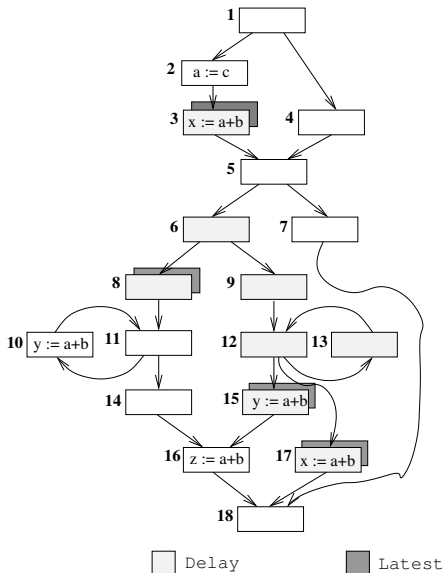
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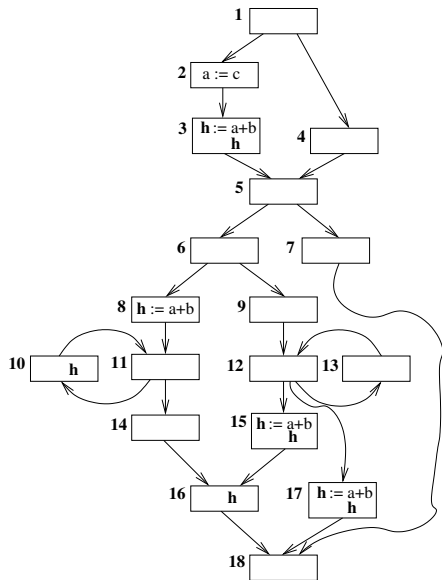
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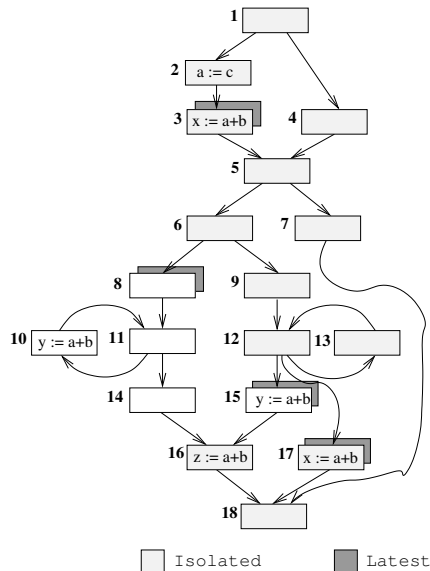
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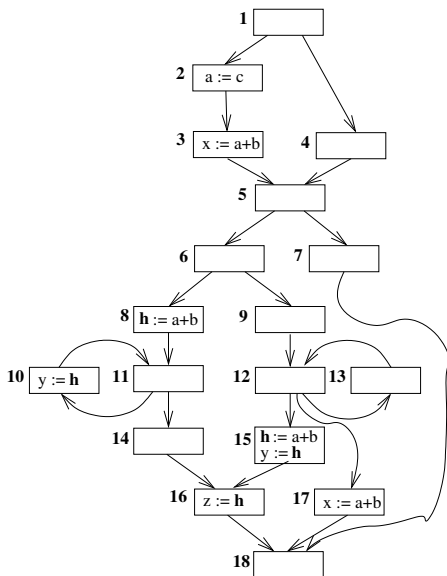
The Result of the *ALCM* Transformation






Latest and Isolated Computation Points




The Result of the *LCM* Transformation




Further Reading for Chapter 8(1)

-  Alfred V. Aho, Monica S. Lam, Ravi Sethi, Jeffrey D. Ullman. *Compilers: Principles, Techniques, & Tools*. Addison-Wesley, 2nd edition, 2007. (Chapter 9.5.3, The Lazy-Code-Motion Problem; Chapter 9.5.5, The Lazy-Code-Motion Algorithm)
-  Keith D. Cooper, Linda Torczon. *Engineering a Compiler*. Morgan Kaufman Publishers, 2004. (Chapter 10.3.2, Code Motion – Lazy Code Motion)
-  K.H. Drechsler, M. P. Stadel. *A variation of Knoop, Rüthing and Steffen's LAZY CODE MOTION*. ACM SIGPLAN Notices 28(5):29-38, 1993.

Further Reading for Chapter 8(2)

-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Lazy Code Motion*. In Proceedings of the ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI'92), ACM SIGPLAN Notices 27(7):224-234, 1992.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Optimal Code Motion: Theory and Practice*. ACM Transactions on Programming Languages and Systems 16(4):1117-1155, 1994.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Retrospective: Lazy Code Motion*. In “20 Years of the ACM SIGPLAN Conference on Programming Language Design and Implementation (1979 - 1999): A Selection”, ACM SIGPLAN Notices 39(4):460-461&462-472, 2004.

Further Reading for Chapter 8(3)

-  Stephen S. Muchnick. *Advanced Compiler Design Implementation*. Morgan Kaufman Publishers, 1997. (Chapter 13.3, Partial-Redundancy Elimination – Lazy Code Motion)

Chapter 9

Implementing Busy and Lazy Code Motion

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Implementing *BCM* and *LCM* on SI-Graphs

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Busy and Lazy Code Motion

...for node-labelled SI-graphs:

- ▶ BCM_ι transformation
- ▶ LCM_ι transformation

Convention: For the following we assume that only critical edges are split. Therefore, BCM_ι and LCM_ι require insertions at both node entries and node exits (N-insertions and X-insertions).

Local Predicates for BCM_ι and LCM_ι

Local Predicates:

- ▶ $COMP_\iota(t)$: t is computed by ι .
- ▶ $TRANSP_\iota(t)$: No operand of t is modified by ι .

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Implementing BCM_t

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Implementing BCM_ι (1)

1. Analyses for Up-Safety and Down-Safety

The *MaxFP*-Equation System for Up-Safety:

$$\text{N-USAFE}_\iota = \begin{cases} \mathbf{false} & \text{if } \iota = \mathbf{s} \\ \prod_{\hat{\iota} \in \text{pred}(\iota)} \text{X-USAFE}_{\hat{\iota}} & \text{otherwise} \end{cases}$$

$$\text{X-USAFE}_\iota = (\text{N-USAFE}_\iota + \text{COMP}_\iota) \cdot \text{TRANSP}_\iota$$

Implementing BCM_ι (2)

The *MaxFP*-Equation System for Down-Safety:

$$\text{N-DSAFE}_\iota = \text{COMP}_\iota + \text{X-DSAFE}_\iota \cdot \text{TRANSP}_\iota$$

$$\text{X-DSAFE}_\iota = \begin{cases} \mathbf{false} & \text{if } \iota = \mathbf{e} \\ \prod_{\hat{\iota} \in \text{succ}(\iota)} \text{N-DSAFE}_{\hat{\iota}} & \text{otherwise} \end{cases}$$

Implementing BCM_ι (3)

2. The Transformation: Insertion&Replacement Points

Local Predicates:

- ▶ $N\text{-USAFE}^*$, $X\text{-USAFE}^*$, $N\text{-DSAFE}^*$, $X\text{-DSAFE}^*$:
...denote the greatest solutions of the equation systems
for up-safety and down-safety of step 1.

Implementing BCM_ι (4)

Computing Earliestness (no data flow analysis!):

$$\text{N-EARLIEST}_\iota =_{df} \text{N-DSAFE}_\iota^* \cdot \prod_{\hat{\iota} \in \text{pred}(\iota)} (\overline{\text{X-USAFE}_{\hat{\iota}}^* + \text{X-DSAFE}_{\hat{\iota}}^*})$$

$$\text{X-EARLIEST}_\iota =_{df} \text{X-DSAFE}_\iota^* \cdot \overline{\text{TRANSP}_\iota}$$

Implementing BCM_ι (5)

The BCM_ι Transformation:

$N\text{-INSERT}_\iota^{BCM} =_{df} N\text{-EARLIEST}_\iota$

$X\text{-INSERT}_\iota^{BCM} =_{df} X\text{-EARLIEST}_\iota$

$REPLACE_\iota^{BCM} =_{df} COMP_\iota$

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Implementing LCM_ℓ

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Implementing LCM_{ι} (1)

3. Analyses for Delayability and Isolation

The *MaxFP*-Equation System for Delayability:

$$\text{N-DELAYED}_{\iota} = \text{N-EARLIEST}_{\iota} + \begin{cases} \text{false} & \text{if } \iota = \mathbf{s} \\ \prod_{\iota' \in \text{pred}(\iota)} \overline{\text{COMP}_{\iota'}} \cdot \text{X-DELAYED}_{\iota} & \text{otherwise} \end{cases}$$

$$\text{X-DELAYED}_{\iota} = \text{X-EARLIEST}_{\iota} + \text{N-DELAYED}_{\iota} \cdot \overline{\text{COMP}_{\iota}}$$

Implementing LCM_{ι} (2)

Computing Latestness (no data flow analysis!):

$$N\text{-LATEST}_{\iota} =_{df} N\text{-DELAYED}_{\iota}^* \cdot \text{COMP}_{\iota}$$

$$X\text{-LATEST}_{\iota} =_{df} X\text{-DELAYED}_{\iota}^* \cdot \left(\text{COMP}_{\iota} + \sum_{\iota' \in \text{succ}(\iota)} \overline{N\text{-DELAYED}_{\iota'}^*} \right)$$

where

- ▶ $N\text{-DELAYED}^*$, $X\text{-DELAYED}^*$: ...denote the greatest solutions of the equation system for delayability.

Implementing LCM_ι (3)

The $ALCM_\iota$ Transformation:

$N\text{-INSERT}_\iota^{ALCM} =_{df} N\text{-LATEST}_\iota$

$X\text{-INSERT}_\iota^{ALCM} =_{df} X\text{-LATEST}_\iota$

$REPLACE_\iota^{ALCM} =_{df} COMP_\iota$

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Implementing LCM_ι (4)

The *MaxFP*-Equation System for Isolation:

$$\text{N-ISOLATED}_\iota = \text{X-EARLIEST}_\iota + \text{X-ISOLATED}_\iota$$

$$\text{X-ISOLATED}_\iota = \prod_{\iota' \in \text{succ}(\iota)} \text{N-EARLIEST}_{\iota'} + \overline{\text{COMP}_{\iota'}} \cdot \text{N-ISOLATED}_{\iota'}$$

Implementing LCM_ι (5)

4. The Transformation: Insertion&Replacement Points

Local Predicates:

- ▶ N-ISOLATED*, X-ISOLATED*: ...denote the greatest solutions of the equation system for isolation of step 3.

Implementing LCM_ι (6)

The LCM_ι Transformation:

$$\text{N-INSERT}_\iota^{\text{LCM}} =_{df} \text{N-LATEST}_\iota \cdot \overline{\text{N-ISOLATED}_\iota^*}$$

$$\text{X-INSERT}_\iota^{\text{LCM}} =_{df} \text{X-LATEST}_\iota \cdot \overline{\text{X-ISOLATED}_\iota^*}$$

$$\text{REPLACE}_\iota^{\text{LCM}} =_{df} \text{COMP}_\iota \cdot \overline{\text{N-LATEST}_\iota \cdot \text{N-ISOLATED}_\iota^*}$$

Chapter 9.2

Implementing *BCM* and *LCM* on BB-Graphs

Chapter 9.2.1

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Implementing Busy and Lazy Code Motion

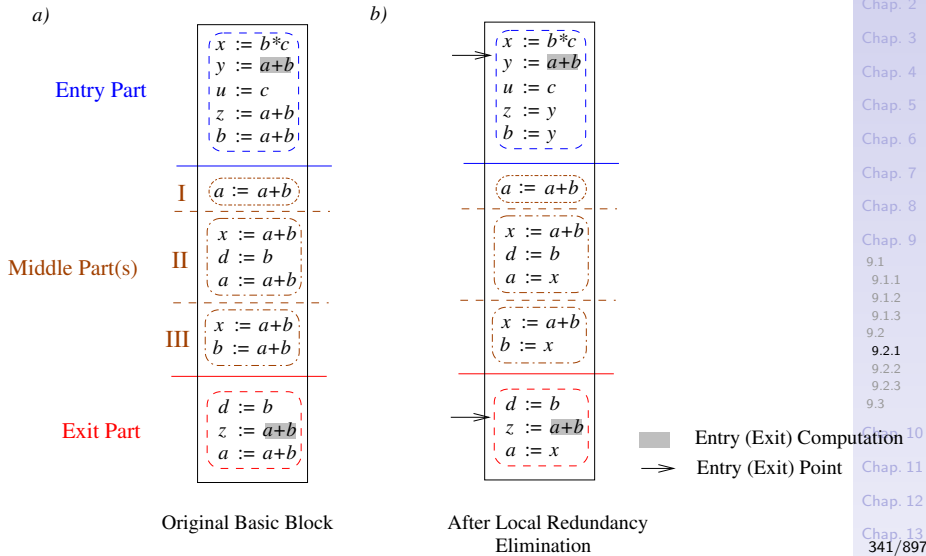
...for node-labelled BB-graphs:

- ▶ BCM_{β} Transformation
- ▶ LCM_{β} Transformation

Convention: For the following we assume that (1) only critical edges are split. Therefore, BCM_{β} and LCM_{β} require insertions at both node entries and node exits (N-insertions and X-in- sertions), and that (2) all redundancies within a basic block have been removed by a preprocess.

Conceptual Splitting of a Basic Block

...into **entry**, **middle**, and **exit** part.



Entry and Exit Parts of a Basic Block

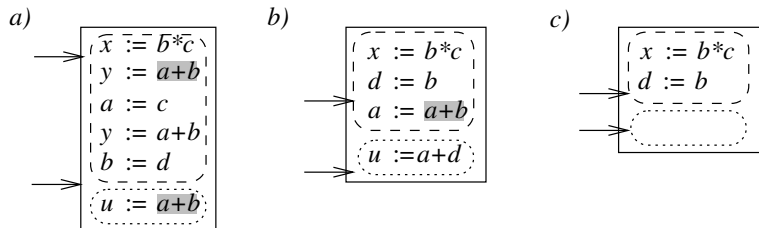
For PRE, we do not need to distinguish between entry and middle part(s), and can consider them a unit. This gives rise to the following definition:

Given a computation t , a basic block \mathbf{n} can be divided into two parts:

- ▶ an **entry part** which consists of all statements up to and including the last modification of t
- ▶ an **exit part** which consists of the remaining statements of \mathbf{n} .

Note: The entry part of a non-empty basic block is always non-empty; in distinction, the exit part of a non-empty basic block can be empty itself (this is illustrated in the following figure).

Illustrating Entry & Exit Part of a Basic Block



--- Entry Part

..... Exit Part

■ Entry (Exit) Computation

→ Entry (Exit) Point

The General Pattern of CM on BB-Graphs

1. Introducing temporary

1.1 Define a new temporary variable \mathbf{h}_{CM} for t .

2. Insertions

2.1 Insert assignments $\mathbf{h}_{CM} := t$ at the insertion point of the entry part of all $\beta \in \mathbf{N}$ satisfying N-INSERT^{CM}

2.2 Insert assignments $\mathbf{h}_{CM} := t$ at the insertion point of the exit part of all $\beta \in \mathbf{N}$ satisfying X-INSERT^{CM}

3. Replacements

3.1 Replace the (unique) entry computation of t by \mathbf{h}_{CM} in every $\beta \in \mathbf{N}$ satisfying N-REPLACE^{CM}

3.2 Replace the (unique) exit computation of t by \mathbf{h}_{CM} in every $\beta \in \mathbf{N}$ satisfying X-REPLACE^{CM}

Local Predicates for BCM_β and LCM_β

Local Predicates:

- ▶ $BB\text{-}NCOMP_\beta(t)$: β contains a statement ι that computes t , and that is not preceded by a statement that modifies an operand of t .
- ▶ $BB\text{-}XCOMP_\beta(t)$: β contains a statement ι that computes t and neither ι nor any other statement of β after ι modifies an operand of t .
- ▶ $BB\text{-}TRANSP_\beta(t)$: β contains no statement that modifies an operand of t .

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Implementing BCM_{β}

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Implementing BCM_{β} (1)

1. Analyses for Up-Safety and Down-Safety

The *MaxFP*-Equation System for Up-Safety:

$$BB-N-USAFE_{\beta} = \begin{cases} \mathbf{false} & \text{if } \beta = \mathbf{s} \\ \prod_{\hat{\beta} \in pred(\beta)} (BB-XCOMP_{\hat{\beta}} + BB-X-USAFE_{\hat{\beta}}) & \text{otherwise} \end{cases}$$

$$BB-X-USAFE_{\beta} = (BB-N-USAFE_{\beta} + BB-NCOMP_{\beta}) \cdot BB-TRANSP_{\beta}$$

Implementing BCM_{β} (2)

The *MaxFP*-Equation System for Down-Safety:

$$BB\text{-N}\text{-DSAFE}_{\beta} = BB\text{-NCOMP}_{\beta} + BB\text{-X}\text{-DSAFE}_{\beta} \cdot BB\text{-TRANSP}_{\beta}$$

$$BB\text{-X}\text{-DSAFE}_{\beta} = BB\text{-XCOMP}_{\beta} + \begin{cases} \mathbf{false} & \text{if } \beta = \mathbf{e} \\ \prod_{\hat{\beta} \in succ(\beta)} BB\text{-N}\text{-DSAFE}_{\hat{\beta}} & \text{otherwise} \end{cases}$$

Implementing BCM_{β} (3)

2. The Transformation: Insertion&Replacement Points

Local Predicates:

- ▶ $BB-N-USAFE^*$, $BB-X-USAFE^*$, $BB-N-DSAFE^*$, $BB-X-DSAFE^*$: ...denote the greatest solutions of the equation systems for up-safety and down-safety of step 1.

Implementing BCM_{β} (4)

Computing Earliestness (no data flow analysis!):

$$\text{N-EARLIEST}_{\beta} =_{df} \text{BB-N-DSAFE}_{\beta}^*$$

$$\prod_{\hat{\beta} \in \text{pred}(\beta)} (\overline{\text{BB-X-USAFE}_{\hat{\beta}}^* + \text{BB-X-DSAFE}_{\hat{\beta}}^*})$$

$$\text{X-EARLIEST}_{\beta} =_{df} \text{BB-X-DSAFE}_{\beta}^* \cdot \overline{\text{BB-TRANSP}_{\beta}}$$

Implementing BCM_{β} (5)

The BCM_{β} Transformation:

$N\text{-INSERT}_{\beta}^{BCM} =_{df} N\text{-EARLIEST}_{\beta}$

$X\text{-INSERT}_{\beta}^{BCM} =_{df} X\text{-EARLIEST}_{\beta}$

$N\text{-REPLACE}_{\beta}^{BCM} =_{df} BB\text{-NCOMP}_{\beta}$

$X\text{-REPLACE}_{\beta}^{BCM} =_{df} BB\text{-XCOMP}_{\beta}$

Chapter 9.2.3

Implementing LCM_{β}

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Implementing LCM_{β} (1)

3. Analyses for Delayability and Isolation

The *MaxFP*-Equation System for Delayability:

$$\text{N-DELAYED}_{\beta} = \text{N-EARLIEST}_{\beta} + \begin{cases} \text{false} & \text{if } \beta = \mathbf{s} \\ \prod_{\hat{\beta} \in \text{pred}(\beta)} \overline{\text{BB-XCOMP}_{\hat{\beta}}} \cdot \text{X-DELAYED}_{\hat{\beta}} & \text{otherwise} \end{cases}$$

$$\text{X-DELAYED}_{\beta} = \text{X-EARLIEST}_{\beta} + \text{N-DELAYED}_{\beta} \cdot \overline{\text{BB-NCOMP}_{\beta}}$$

Implementing LCM_{β} (2)

Computing Latestness (no data flow analysis!):

$$N\text{-LATEST}_{\beta} =_{df} N\text{-DELAYED}_{\beta}^* \cdot BB\text{-NCOMP}_{\beta}$$

$$X\text{-LATEST}_{\beta} =_{df} X\text{-DELAYED}_{\beta}^* \cdot (BB\text{-XCOMP}_{\beta} + \sum_{\hat{\beta} \in succ(\beta)} \overline{N\text{-DELAYED}_{\hat{\beta}}^*})$$

where

- ▶ $N\text{-DELAYED}_{\beta}^*$, $X\text{-DELAYED}_{\beta}^*$: ...denote the greatest solutions of the equation system for delayability.

Implementing LCM_{β} (3)

The $ALCM_{\beta}$ Transformation:

$N\text{-INSERT}_{\beta}^{ALCM} =_{df} N\text{-LATEST}_{\beta}$

$X\text{-INSERT}_{\beta}^{ALCM} =_{df} X\text{-LATEST}_{\beta}$

$N\text{-REPLACE}_{\beta}^{ALCM} =_{df} BB\text{-NCOMP}_{\beta}$

$X\text{-REPLACE}_{\beta}^{ALCM} =_{df} BB\text{-XCOMP}_{\beta}$

Implementing LCM_{β} (4)

The *MaxFP*-Equation System for Isolation:

$$\text{N-ISOLATED}_{\beta} = \text{X-EARLIEST}_{\beta} + \text{X-ISOLATED}_{\beta}$$

$$\text{X-ISOLATED}_{\beta} = \prod_{\hat{\beta} \in \text{succ}(\beta)} \text{N-EARLIEST}_{\hat{\beta}} + \overline{\text{BB-NCOMP}_{\hat{\beta}}} \cdot \text{N-ISOLATED}_{\hat{\beta}}$$

Implementing LCM_{β} (5)

4. The Transformation: Insertion&Replacement Points

Local Predicates:

- ▶ N-ISOLATED*, X-ISOLATED*: ...denote the greatest solutions of the equation system for isolation of step 3.

Implementing LCM_{β} (6)

The LCM_{β} Transformation:

$$\text{N-INSERT}_{\beta}^{\text{LCM}} =_{df} \text{N-LATEST}_{\beta} \cdot \overline{\text{N-ISOLATED}_{\beta}^*}$$

$$\text{X-INSERT}_{\beta}^{\text{LCM}} =_{df} \text{X-LATEST}_{\beta} \cdot \overline{\text{X-ISOLATED}_{\beta}^*}$$

$$\text{N-REPLACE}_{\beta}^{\text{LCM}} =_{df} \text{BB-NCOMP}_{\beta} \cdot \overline{\text{N-LATEST}_{\beta} \cdot \text{N-ISOLATED}_{\beta}^*}$$

$$\text{X-REPLACE}_{\beta}^{\text{LCM}} =_{df} \text{BB-XCOMP}_{\beta} \cdot \overline{\text{X-LATEST}_{\beta} \cdot \text{X-ISOLATED}_{\beta}^*}$$

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An Extended Example

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9.1.2

9.1.3

9.2

9.2.1

9.2.2

9.2.3

9.3

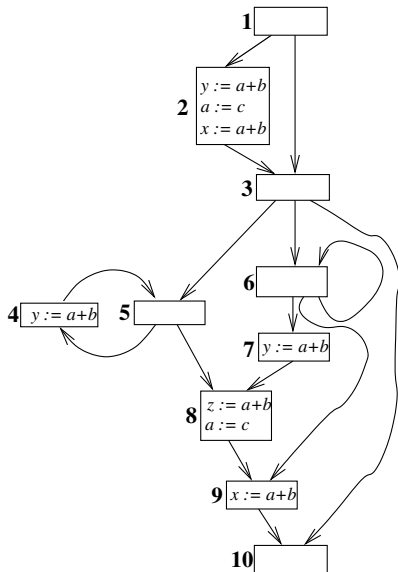
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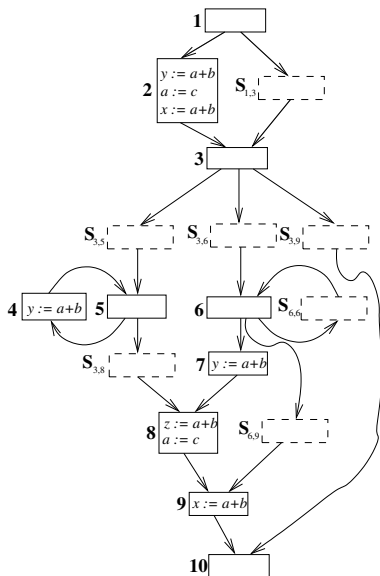
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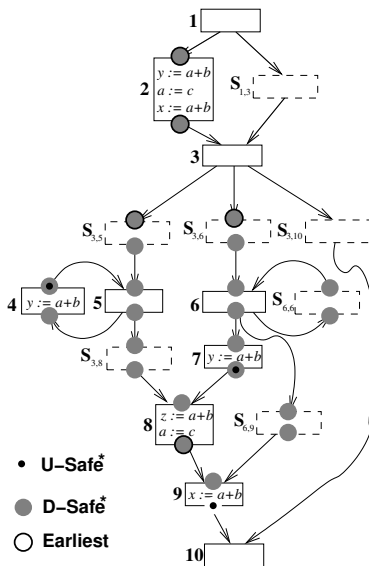
The Original Program



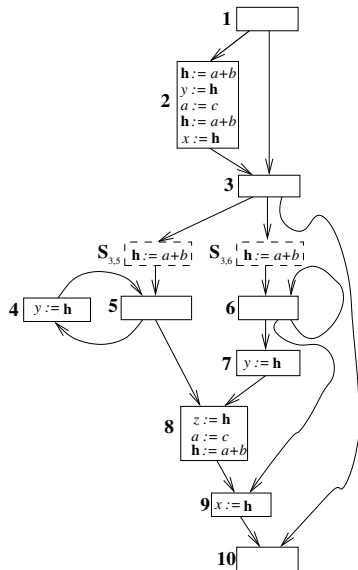
After the Splitting of Critical Edges



Up/Down-Safe, Earliest Computation Points



The Result of the BCM_{β} Transformation



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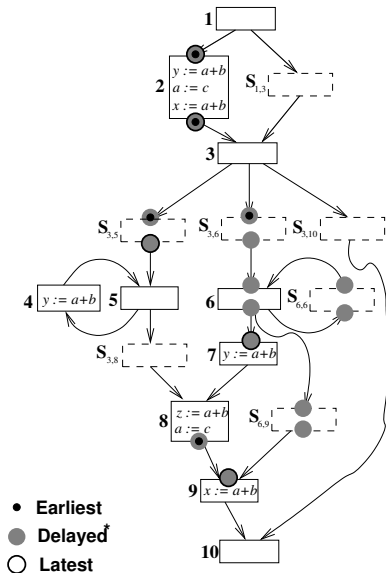
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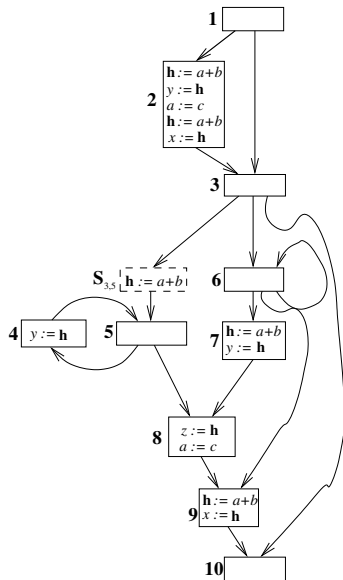
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Delayable and Latest Computation Points



The Result of the $ALCM_{\beta}$ -Transformation



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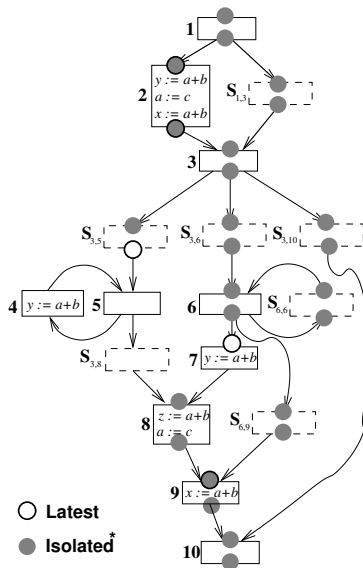
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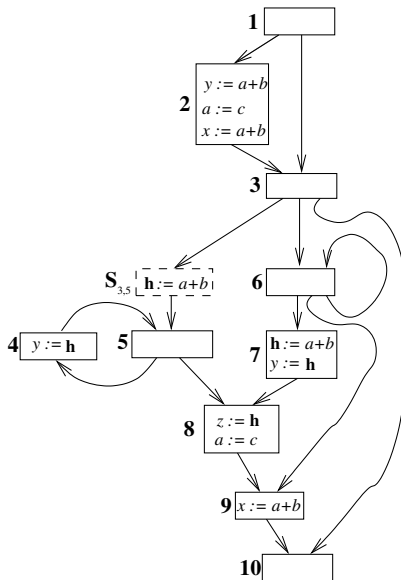
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Latest and Isolated Program Points



The Result of the LCM_{β} Transformation



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Further Reading for Chapter 9

-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Lazy Code Motion*. In Proceedings of the ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI'92), ACM SIGPLAN Notices 27(7):224-234, 1992.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Optimal Code Motion: Theory and Practice*. ACM Transactions on Programming Languages and Systems 16(4):1117-1155, 1994.

Chapter 10

Sparse Code Motion

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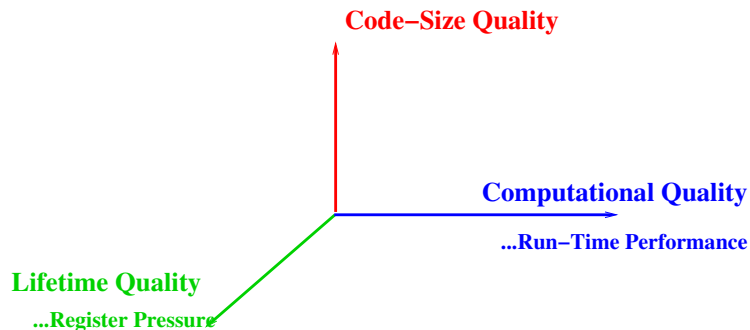
Motivation

These days [Lazy Code Motion](#) is the

- ▶ *de-facto* standard algorithm for [PRE](#) that is used in current state-of-the-art compilers
 - ▶ Gnu compiler family
 - ▶ Sun Sparc compiler family
 - ▶ ...

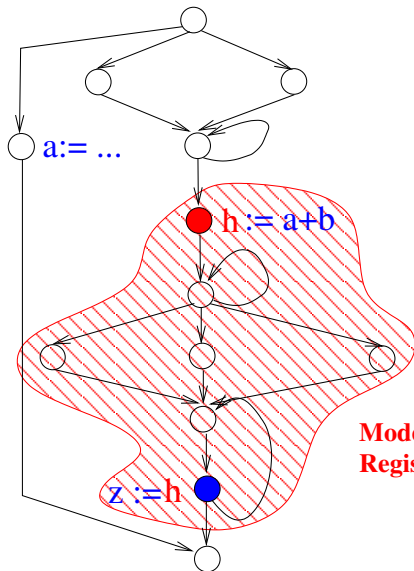
In the following

...we consider a (modular) extension of *LCM* in order to take user priorities into account!



To render possible

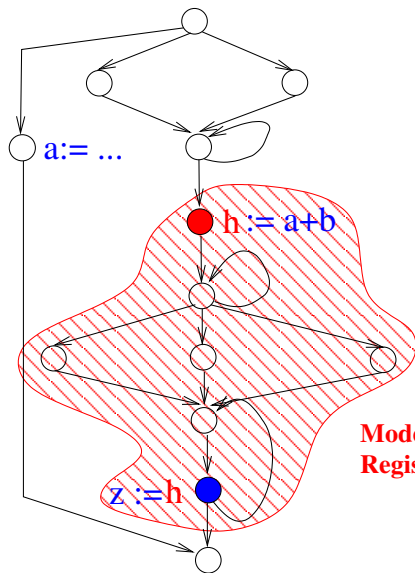
...also the below transformation:



**Moderate
Register Pressure!**

There is more than speed!

...for instance **space**!



The World Market for Microprocessors in 1999

Chip Category	Sold Processors
Embedded 4-bit	2000 Millions
Embedded 8-bit	4700 Millions
Embedded 16-bit	700 Millions
Embedded 32-bit	400 Millions
DSP	600 Millions
Desktop 32/64-bit	150 Millions

...[David Tennenhouse](#) (Intel Director of Research), key note lecture at the *20th IEEE Real-Time Systems Symposium (RTSS'99)*, Phoenix, Arizona, December 1999.

The World Market for Microprocessors in 1999

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Desktop 32/64-bit	150 Millions

~ 2%

...[David Tennenhouse](#) (Intel Director of Research), key note lecture at the *20th IEEE Real-Time Systems Symposium (RTSS'99)*, Phoenix, Arizona, December 1999.

Think about

...domain-specific processors used in embedded systems:

- ▶ Telecommunication
 - ▶ Cellular phones, pagers,...
- ▶ Consumer electronics
 - ▶ MP3-players, cameras, game consoles,...
- ▶ Automotive field
 - ▶ GPS navigation, airbags,...
- ▶ ...

Code for Embedded Systems

Demands:

- ▶ **Performance** (often real-time demands)
- ▶ **Code size** (system-on-chip, on-chip RAM/ROM)
- ▶ ...

For embedded systems:

- ▶ **Code size** is often more critical than **speed!**

Code for Embedded Systems (Cont'd)

Demands (and how they are often still addressed):

- ▶ Assembler programming
- ▶ Manual post-optimization

Shortcomings:

- ▶ Error prone
- ▶ Delayed time-to-market

...problems getting more severe with increasing complexity.

Generally, we observe:

- ▶ a trend towards high-level languages programming (C/C++)

In View of this Trend

...how do **traditional** compiler and optimizer technologies support the specific demands of code for embedded systems?



...unfortunately, only little.

W/out Doubt

Traditional Optimizations

- ▶ are almost exclusively tuned towards performance optimization
- ▶ are not code-size sensitive and in general do not provide any control on their impact on the [code size](#)

This holds especially

...for **code motion** based optimizations.

In particular, this includes:

- ▶ **Partial redundancy elimination**
- ▶ Partial dead-code elimination (cf. Lecture Course 185.A05 Analysis and Verification)
- ▶ Partial redundant-assignment elimination (cf. Lecture Course 185.A05 Analysis and Verification)
- ▶ **Strength reduction**
- ▶ ...

Recalling the Essence of PRE

PRE can conceptually be considered a two-stage process:

1. **Expression Hoisting**

...hoisting computations to “**earlier**” safe computation points

2. **Totally Redundant Expression Elimination**

...eliminating computations, which become totally redundant by expression hoisting

Recalling the Essence of *LCM*

LCM can conceptually be considered the result of a two-stage process:

1. **Hoisting Expressions**

...to their “**earliest**” safe computation points

2. **Sinking Expressions**

...to their “**latest**” safe still computationally optimal computation points

The Road to Code-size Sensitive PRE

Classical PRE

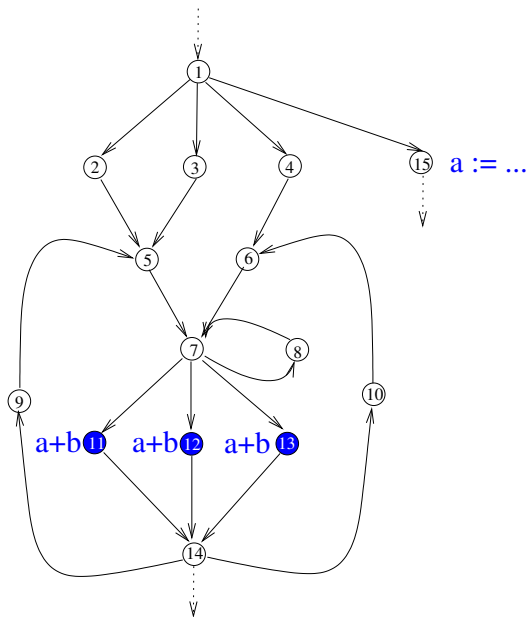
- ▶ The PRE Algorithm of Morel&Renvoise (CACM 22, 1979)
- ▶ Computationally and Lifetime Optimal Code Motion
 - ↪ Busy Code Motion (BCM) / Lazy Code Motion (LCM)
(Knoop, Rüthing, Steffen, PLDI'92)
 - ▶ Distinguished w/ the ACM SIGPLAN Most Influential PLDI Paper Award 2002 (for 1992)
 - ▶ Selected for the “20 Years of the ACM SIGPLAN PLDI: A Selection” (60 articles out of about 600 articles)

The Road to Code-size Sensitive PRE (Cont'd)

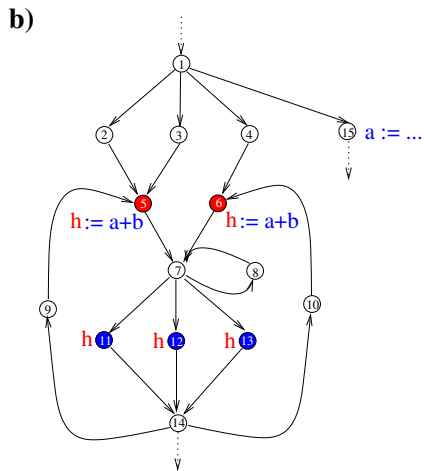
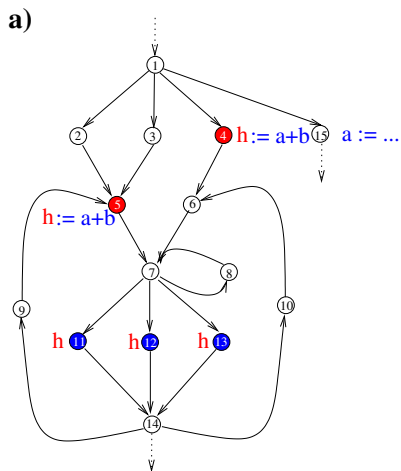
Non-Classical PRE

- ▶ Code-size Sensitive PRE
 - ↪ Sparse Code Motion (SpCM)
(Knoop, Rüthing, Steffen, POPL'00)
 - ▶ ...modular extension of BCM/LCM
 - ↪ Modelling and solving the problem:
...based on graph-theoretic means
 - ↪ Main Results:
...Correctness, Optimality

The Running Example (1)

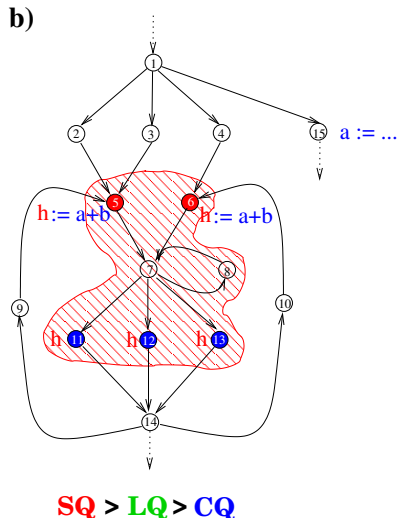
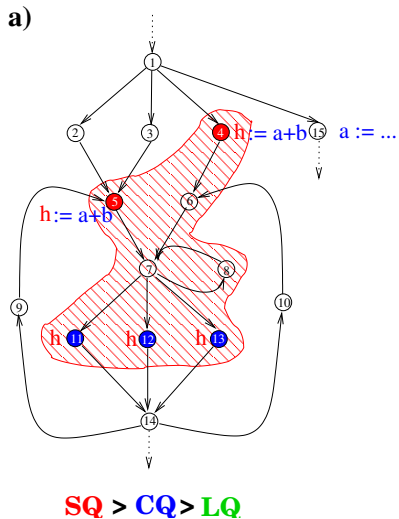


The Running Example (2)



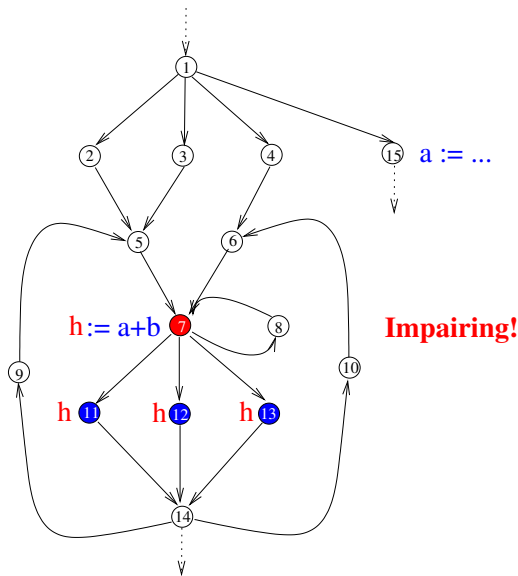
Two Code-size Optimal Programs

The Running Example (3)



The Running Example (4)

Recall: The below transformation is not desired!



Impairing!

Code-size Sensitive PRE

~> The Problem

...how do we get **code-size minimal** placement of the computations, i.e., a placement that is

- ▶ **admissible** (semantics & performance preserving)
- ▶ **code-size minimal?**

~> The Solution: A new View to PRE

...consider **PRE** as a **trade-off** problem: Exchange original computations for newly inserted ones!

~> The Clou: Use Graph Theory!

...reduce the **trade-off** problem to the computation of **tight sets** in **bipartite graphs** based on **maximum matchings**!

We postpone but keep in mind

...that we have to answer:

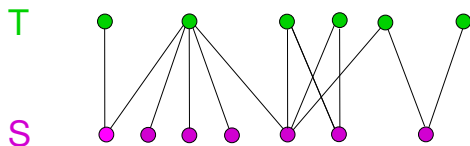
- ▶ Where are computations to be inserted and where are original computations to be replaced?

...and to prove:

- ▶ Why is this correct (i.e., semantics preserving)?
- ▶ What is the impact on the code size?
- ▶ Why is this “optimal” wrt a given prioritization of goals?

For each of these questions we will provide a specific theorem that yields the corresponding answer!

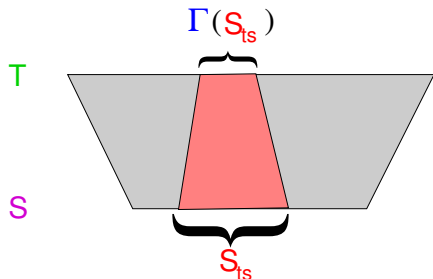
Bipartite Graphs



Tight Set

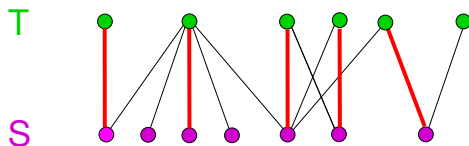
...of a bipartite graph $(S \cup T, E)$: Subset $S_{ts} \subseteq S$ w/

$$\forall S' \subseteq S. |S_{ts}| - |\Gamma(S_{ts})| \geq |S'| - |\Gamma(S')|$$



Two Variants: (1) Largest Tight Sets (2) Smallest Tight

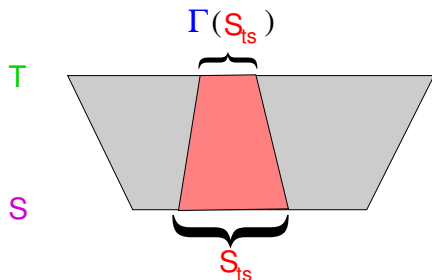
Bipartite Graphs



Tight Set

...of a bipartite graph $(S \cup T, E)$: Subset $S_{ts} \subseteq S$ w/

$$\forall S' \subseteq S. |S_{ts}| - |\Gamma(S_{ts})| \geq |S'| - |\Gamma(S')|$$



Two Variants: (1) Largest Tight Sets (2) Smallest Tight

Obviously

...we can make use of off-the-shelve algorithms from graph theory in order to compute

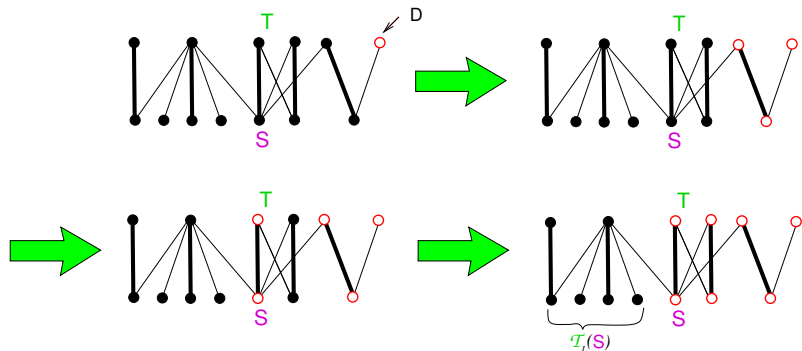
- ▶ **Maximum matchings** and
- ▶ **Tight sets**

This way the **PRE** problem boils down to

- ▶ constructing the bipartite graph that models the problem!

Computing Largest/Smallest Tight Sets

...based on **maximum matchings**:



LTS-Algorithm 10.1: Largest Tight Sets

Input: A bipartite graph $(S \dot{\cup} T, E)$, a maximum matching M .

Output: The largest tight set $\mathcal{T}_{LaTS}(S) \subseteq S$.

$S_M := S$; $D := \{t \in T \mid t \text{ is unmatched}\}$;

WHILE $D \neq \emptyset$ DO

 choose some $x \in D$; $D := D \setminus \{x\}$;

 IF $x \in S$

 THEN $S_M := S_M \setminus \{x\}$;

$D := D \cup \{y \mid \{x, y\} \in M\}$

 ELSE $D := D \cup (\Gamma(x) \cap S_M)$

 FI

OD;

$\mathcal{T}_{LaTS}(S) := S_M$

STS-Algorithmus 10.2: Smallest Tight Sets

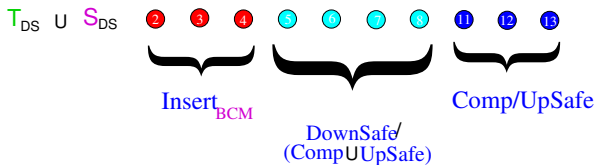
Input: A bipartite graph $(S \dot{\cup} T, E)$, a maximum matching M .

Output: The smallest tight set $\mathcal{T}_{SmTS}(S) \subseteq S$.

```
 $S_M := \emptyset; A := \{s \in S \mid s \text{ is unmatched}\};$   
WHILE  $A \neq \emptyset$  DO  
  choose some  $x \in A; A := A \setminus \{x\};$   
  IF  $x \in S$   
    THEN  $S_M := S_M \cup \{x\};$   
          $A := A \cup (\Gamma(x) \setminus S_M)$   
    ELSE  $A := A \cup \{y \mid \{x, y\} \in M\}$   
  FI  
OD;  
 $\mathcal{T}_{SmTS}(S) := S_M$ 
```

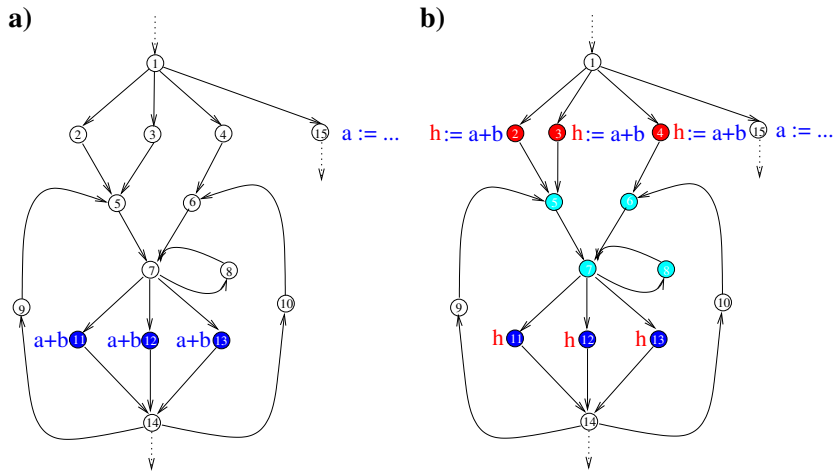
Modelling the Trade-off Problem

The Set of Nodes



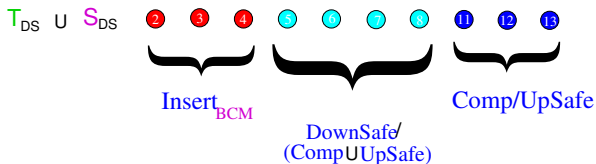
The Set of Edges...

The Set of Nodes

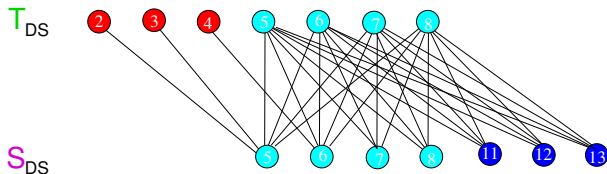


Modelling the Trade-off Problem

The Set of Nodes



The Bipartite Graph



The Set of Edges $\dots \forall n \in S_{DS} \forall m \in T_{DS}.$

$$\{n, m\} \in E_{DS} \iff_{df} m \in \mathbf{Closure}(pred(n))$$

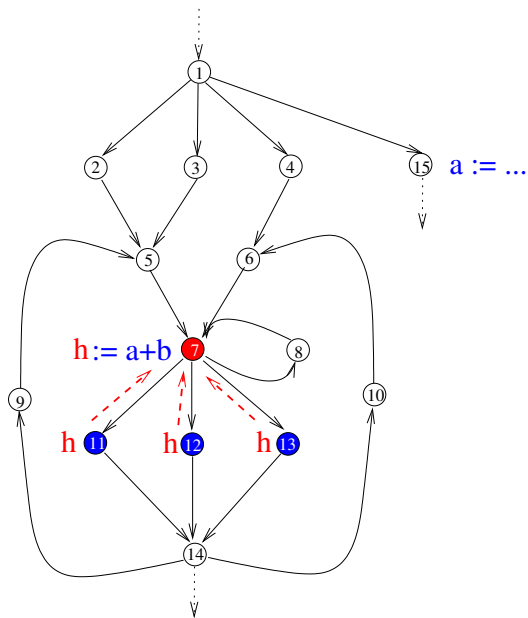
Down-Safety Closures

Definition (10.3, Down-Safety Closure)

Let $n \in \text{DownSafe}/\text{Upsafe}$. Then the **Down-Safety Closure** $\text{Closure}(n)$ is the smallest set of nodes such that

1. $n \in \text{Closure}(n)$
2. $\forall m \in \text{Closure}(n) \setminus \text{Comp. } \text{succ}(m) \subseteq \text{Closure}(n)$
3. $\forall m \in \text{Closure}(n). \text{pred}(m) \cap \text{Closure}(n) \neq \emptyset \Rightarrow \text{pred}(m) \setminus \text{UpSafe} \subseteq \text{Closure}(n)$

Down-Safety Closures: The Intuition (1)



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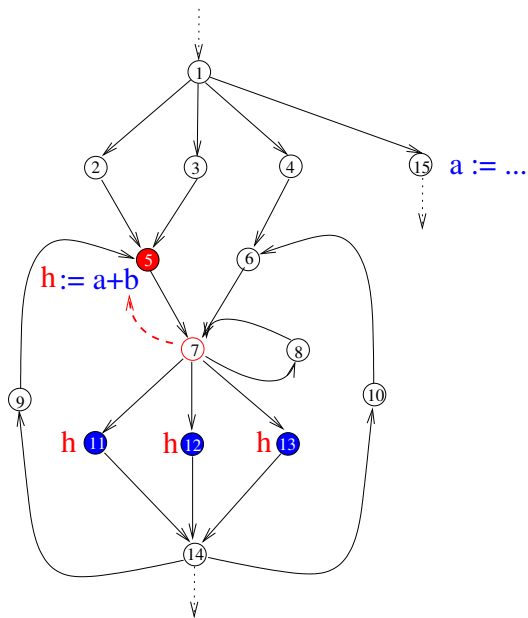
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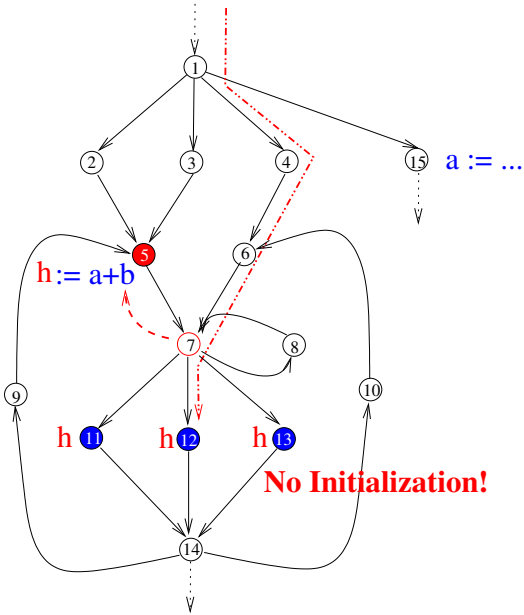
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Down-Safety Closures: The Intuition (2)

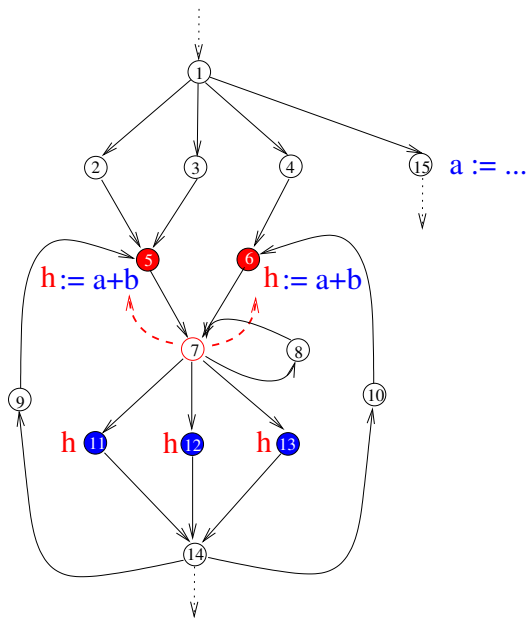


Down-Safety Closures: The Intuition (3)



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Down-Safety Closures: The Intuition (4)



This intuition

...is condensed in the notion of **down-safety closures**. Recall:

Definition (10.3, Down-Safety Closure)

Let $n \in \text{DownSafe}/\text{UpSafe}$. Then the **Down-Safety Closure** $\text{Closure}(n)$ is the smallest set of nodes such that

1. $n \in \text{Closure}(n)$
2. $\forall m \in \text{Closure}(n) \setminus \text{Comp}. \text{succ}(m) \subseteq \text{Closure}(n)$
3. $\forall m \in \text{Closure}(n). \text{pred}(m) \cap \text{Closure}(n) \neq \emptyset \Rightarrow \text{pred}(m) \setminus \text{UpSafe} \subseteq \text{Closure}(n)$

Down-Safety Regions

...lead to a characterization of semantics-preserving PRE transformations via their insertion points.

Definition (10.4, Down-Safety Region)

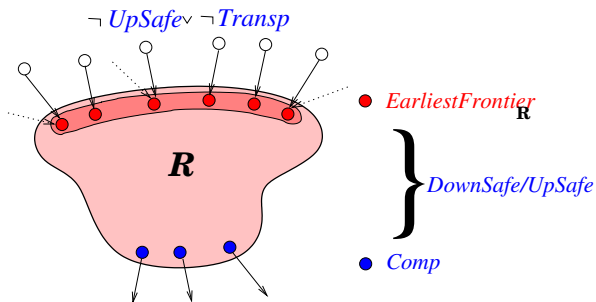
A set $\mathcal{R} \subseteq N$ of nodes is a **down-safety region** iff

1. $Comp \setminus UpSafe \subseteq \mathcal{R} \subseteq DownSafe \setminus UpSafe$
2. $Closure(\mathcal{R}) = \mathcal{R}$

Fundamental

Theorem (10.5, Initialization Theorem)

Initializations of *admissible* PRE transformationen are always at the *earliestness frontiers* of down-safety regions.



...characterizes exactly the set of semantics preserving PRE transformations.

The Key Questions

...regarding **correctness** and **optimality**:

1. Where to insert computations, why is it correct?
2. What is the impact on the code size?
3. Why is the result optimal, i.e., code-size minimal?

...three theorems will answer one of these questions each.

Main Results / Question 1

1. Where to insert computations, why is it correct?

Intuitively: At the earliestness frontier of the DS-region induced by the tight set.

Theorem (10.6, Tight Sets: Insertion Points)

Let $TS \subseteq S_{DS}$ be a *tight set*.

Then $\mathcal{R}_{TS} =_{df} \Gamma(TS) \cup (Comp \setminus UpSafe)$
is a *down-safety region* w/ $Body_{\mathcal{R}_{TS}} = TS$

Correctness

- ▶ An immediate corollary of [Theorem 10.6](#) and the [Initialization Theorem 10.5](#)

Main Results / Question 2

2. What is the impact on the code size?

Intuitively: The difference between the number of inserted and replaced computations.

Theorem (10.7, Down-Safety Regions: Space Gain)

Let \mathcal{R} be a *down-safety region* w/

$$\text{Body}_{\mathcal{R}} =_{df} \mathcal{R} \setminus \text{EarliestFrontier}_{\mathcal{R}}$$

Then

- ▶ *Space Gain by Inserting at EarliestFrontier $_{\mathcal{R}}$:*

$$\begin{aligned} |\text{Comp} \setminus \text{UpSafe}| - |\text{EarliestFrontier}_{\mathcal{R}}| = \\ |\text{Body}_{\mathcal{R}}| - |\Gamma(\text{Body}_{\mathcal{R}})| \quad df = \text{defic}(\text{Body}_{\mathcal{R}}) \end{aligned}$$

Main Results / Question 3

3. Why is the result optimal, i.e., code-size minimal?

Intuitively: Due to a property inherent to tight sets (non-negative deficiency!).

Theorem (10.8, Optimality: Transformation)

Let $TS \subseteq S_{DS}$ be a *tight set*.

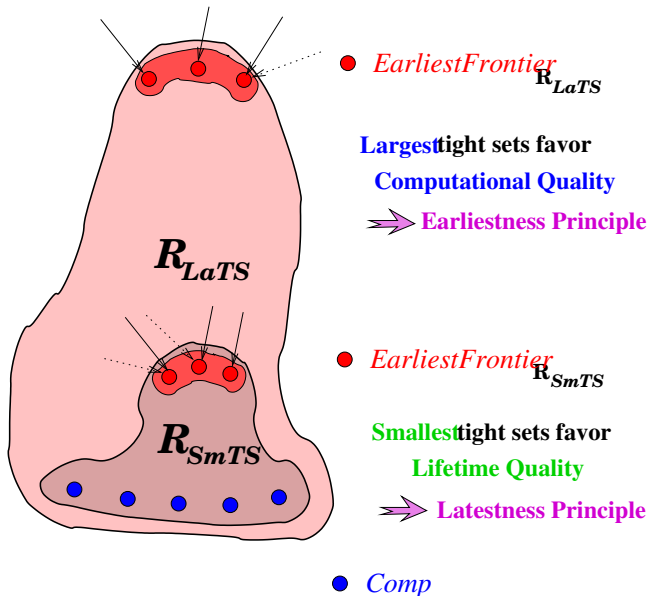
▶ **Insertion Points:**

$$\text{Insert}_{SpCM} =_{df} \text{EarliestFrontier}_{\mathcal{R}_{TS}} = \mathcal{R}_{TS} \setminus TS$$

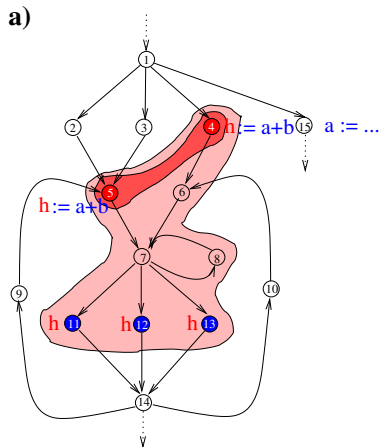
▶ **Space Gain:**

$$\text{defic}(TS) =_{df} |TS| - |\Gamma(TS)| \geq 0 \text{ max.}$$

Largest vs. Smallest Tight Sets: The Impact



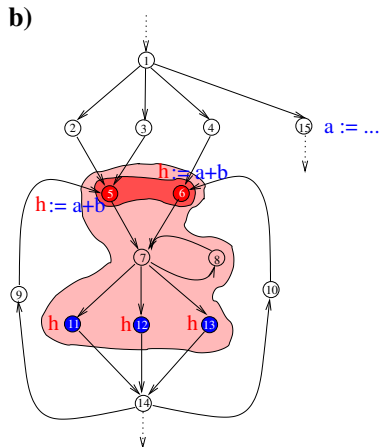
The Impact illustrated on the Running Exam.



Largest Tight Set

(SQ > CQ)

Earliestness Principle



Smallest Tight Set

(SQ > LQ)

Latestness Principle

Code-size Sensitive PRE at a Glance

Preprocess

- **Optional:** Perform **LCM** (3 GEN/KILL-DFAs)
- Compute Predicates of **LCM** for **G** resp. **LCM(G)** (2 GEN/KILL-DFAs)



Main Process

Reduction Phase

- **Construct Bipartite Graph**
- **Compute Maximum Matching**



Optimization Phase

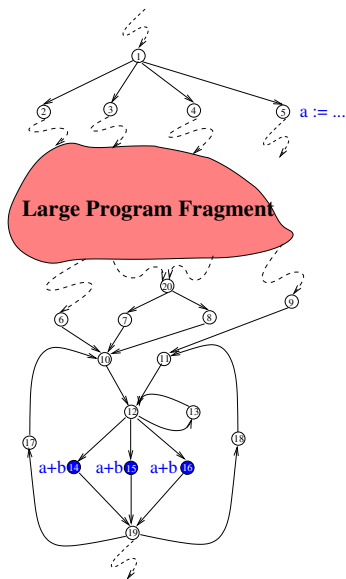
- **Compute Largest/Smallest Tight Set**
- **Determine Insertion Points**

The Cookbook of Recipes for Prioritization

Choice of Priority	Apply	To	Using	Yields	Auxiliary Information Required
\mathcal{LQ}	Not meaningful: The identity, i.e., G itself is optimal!				
SQ	Subsumed by $SQ > \mathcal{CQ}$ and $SQ > \mathcal{LQ}$!				
\mathcal{CQ}	BCM	G			UpSafe(G), DownSafe(G)
$\mathcal{CQ} > \mathcal{LQ}$	LCM	G		LCM(G)	UpSafe(G), DownSafe(G), Delay(G)
$SQ > \mathcal{CQ}$	SpCM	G	Largest tight set	SpCM _{LTS} (G)	UpSafe(G), DownSafe(G)
$SQ > \mathcal{LQ}$	SpCM	G	Smallest tight set		UpSafe(G), DownSafe(G)
$\mathcal{CQ} > SQ$	SpCM	LCM(G)	Largest tight set		UpSafe(G), DownSafe(G), Delay(G) UpSafe(LCM(G)), DownSafe(LCM(G))
$\mathcal{CQ} > SQ > \mathcal{LQ}$	SpCM	LCM(G)	Smallest tight set		UpSafe(G), DownSafe(G), Delay(G) UpSafe(LCM(G)), DownSafe(LCM(G))
$SQ > \mathcal{CQ} > \mathcal{LQ}$	SpCM	DL(SpCM _{LTS} (G))	Smallest tight set		UpSafe(G), DownSafe(G), Delay(SpCM _{LTS} (G)), UpSafe(DL(SpCM _{LTS} (G))), DownSafe(DL(SpCM _{LTS} (G)))

Flexibility as the Reward of SpCM (1)

The original program:



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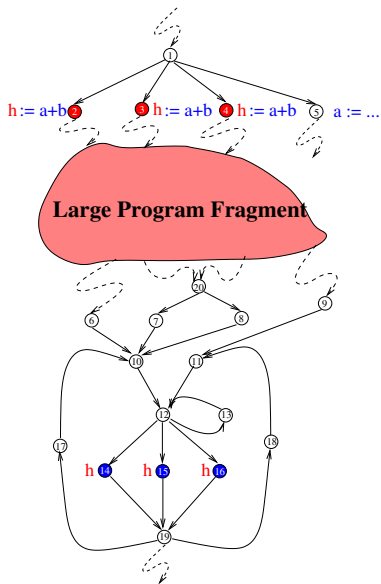
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Flexibility as the Reward of SpCM (2)

BCM: A computationally optimal program (\mathcal{CQ})



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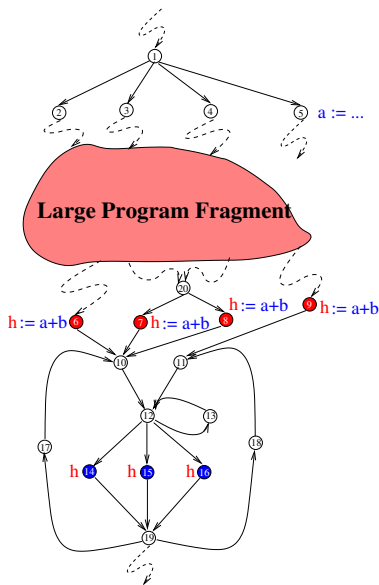
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Flexibility as the Reward of SpCM (3)

LCM: A computationally & lifetime opt. program ($\mathcal{CQ} > \mathcal{LQ}$)



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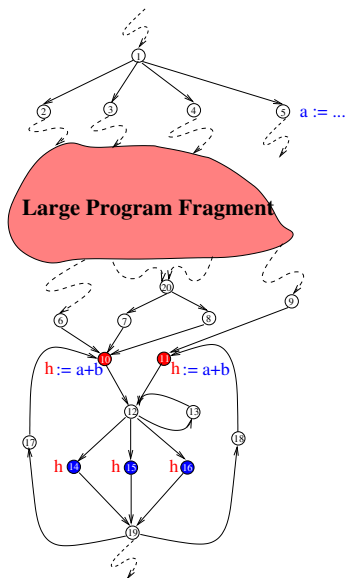
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Flexibility as the Reward of SpCM (4)

SpCM: A code-size & lifetime opt. program ($\mathcal{SQ} > \mathcal{LQ}$)



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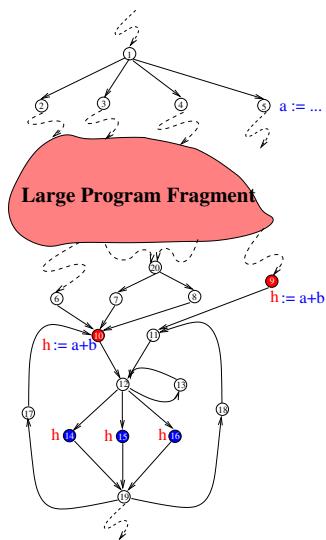
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Flexibility as the Reward of SpCM (5)

SpCM: A computationally & lifetime best code-size optimal program ($SQ > CQ > LQ$)



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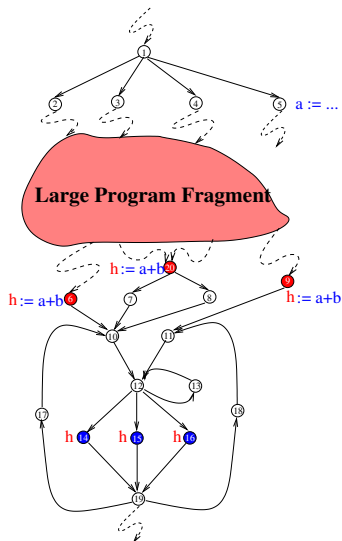
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Flexibility as the Reward of SpCM (6)

SpCM: A code-size & lifetime best computationally optimal program ($CQ > SQ > LQ$)



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On the Origin and Advancement of PRE (1)

- ▶ 1958: A first glimpse of PRE
 - ↪ Ershov's work on "On Programming of Arithmetic Operations."
- ▶ < 1979: Special techniques
 - ↪ Total redundancy elimination, loop invariant code motion
- ▶ 1979: The origin of modern PRE
 - ↪ Morel/Renvoise's seminal work on PRE
- ▶ < ca. 1992: Heuristic improvements of the PRE algorithm of Morel and Renvoise
 - ↪ Dhamdhere [1988, 1991]; Drechsler, Stadel [1988]; Sorkin [1989]; Dhamdhere, Rosen, Zadeck [1992], Briggs, Cooper [1994],...


On the Origin and Advancement of PRE (2)

- ▶ 1992: *BCM* and *LCM* [Knoop Rüthing, Steffen (PLDI'92)]
 - ↪ *BCM* first to achieve **computational optimality** based on the **earliestness principle**
 - ↪ *LCM* first to achieve **computational optimality with minimum register pressure** based on the **latestness principle**
 - ↪ first to **rigorously be proven correct and optimal**
- ▶ 2000: *SpCM*: The origin of code-size sensitive PRE [Knoop, Rüthing, Steffen (POPL 2000)]
 - ↪ first to allow **prioritization of goals**
 - ↪ **rigorously be proven correct and optimal**
 - ↪ first to bridge the gap between traditional compilation and compilation for embedded systems

On the Origin and Advancement of PRE (3)

- ▶ Since ca. 1997: A new strand of research on PRE
 - ↪ Speculative PRE: Gupta, Horspool, Soffa, Xue, Scholz, Knoop,...
- ▶ 2005: Another fresh look at PRE (as maximum flow problem)
 - ↪ Unifying PRE and Speculative PRE [Xue, Knoop (CC 2006)]

Further Reading for Chapter 10

-  Oliver Rüthing, Jens Knoop, Bernhard Steffen. *Sparse Code Motion*. In Conference Record of the 27th Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages (POPL 2000), 170-183, 2000.
-  Bernhard Scholz, R. Nigel Horspool, Jens Knoop. *Optimizing for Space and Time Usage with Speculative Partial Redundancy Elimination*. Proceedings of the ACM SIGPLAN Workshop on Languages, Compilers, and Tools for Embedded Systems (LCTES 2004), ACM SIGPLAN Notices 39(7):221-230, 2004.

Chapter 11

Lazy Strength Reduction

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Objective

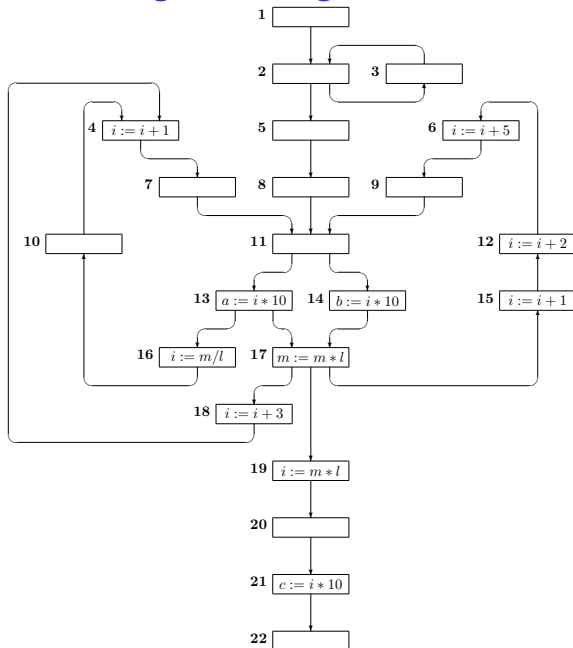
Developing a program optimization that

- ▶ uniformly covers
 - ▶ Partial Redundancy Elimination (PRE) and
 - ▶ Strength Reduction (SR)
- ▶ avoids superfluous register pressure due to unnecessary code motion
- ▶ requires only uni-directional data flow analyses

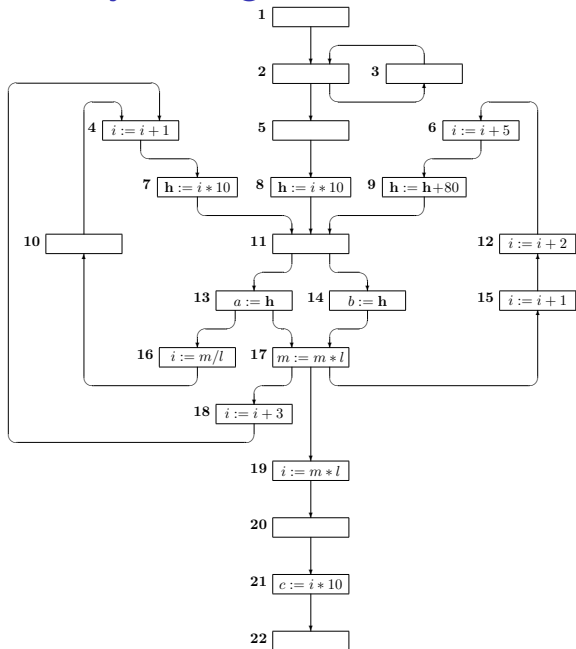
The Approach:

- ▶ Stepwise and modularly extending the *BCM* and the *LCM* to arrive at the
 - ▶ Busy (*BSR*) and Lazy Strength Reduction (*LSR*)

Illustration: The Original Program



The Result of Lazy Strength Reduction



From PRE towards LSR (1)

First, the notion of a candidate expression has to be adapted:

Candidate expressions for

- ▶ PRE: Each term t
- ▶ SR: Terms of the form $v * c$, where
 - ▶ v is a variable
 - ▶ c is a source code constant

From PRE towards LSR (2)

Second, the set of local predicates has to be extended:

- ▶ $Used(n) =_{df} \forall v * c \in SubTerms(t)$
- ▶ $Transp(n) =_{df} x \not\equiv v$
- ▶ $SR-Transp(n) =_{df} Transp(n) \vee t \equiv v + d$ with $d \in \mathbf{C}$

Intuitively

The value of a candidate expression is

- ▶ **killed** at a node n , if $\neg(Transp(n) \vee SR-Transp(n))$
- ▶ **injured** at a node n , if $\neg Transp(n) \wedge SR-Transp(n)$

Important: Injured but not killed values can be

- ▶ **cured** by inserting an **update assignment** of the form $\mathbf{h} := \mathbf{h} + Eff(n)$ where $Eff(n) =_{df} c * d$.

Note that $Eff(n)$ can be computed at compile time since both c and d are source code constants.

Extending BCM straightforward to SR

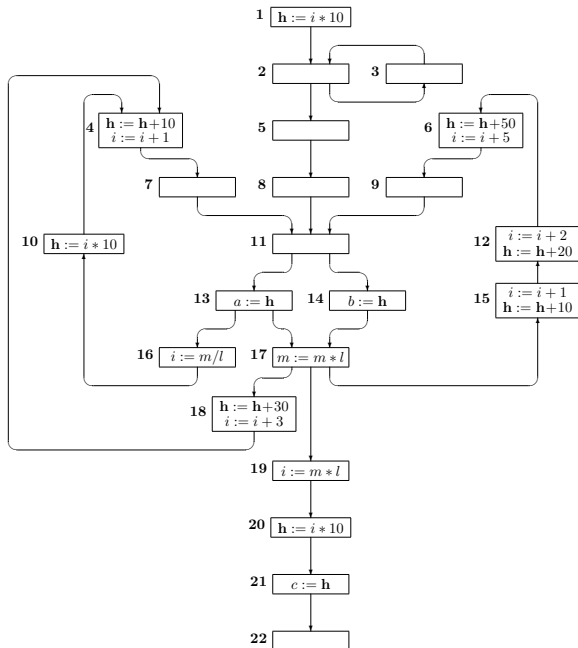
...leads to Simple Strength Reduction (*SSR*).

The *SSR*-Transformation:

1. Introduce a new auxiliary variable \mathbf{h} for $v * c$
2. Insert at the entry of every node satisfying
 - 2.1 Ins_{SSR} the assignment $\mathbf{h} := v * c$
 - 2.2 $\text{InsUpd}_{\text{SSR}}$ the assignment $\mathbf{h} := \mathbf{h} + \text{Eff}(n)$
3. Replace every (original) occurrence of $v * c$ in G by \mathbf{h}

Note: If both Ins_{SSR} and $\text{InsUpd}_{\text{SSR}}$ hold, the initialization statement $\mathbf{h} := v * c$ must precede the update assignment $\mathbf{h} := \mathbf{h} + \text{Eff}(n)$.

The Result of SSR



Discussing the Effect of *SSR*

Shortcoming

- ▶ The **multiplication-addition-deficiency**

Remedy:

- ▶ Moving **critical insertion points** in the direction of the control flow to “earliest” non-critical ones.

Intuitively:

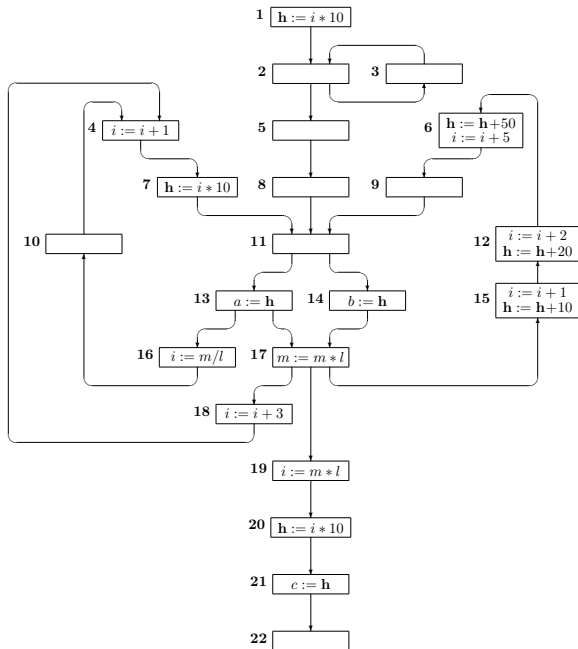
- ▶ A program point is **critical** if there is a $v * c$ -free program path from this point to a modification of v

The 1st Refinement of *SSR*

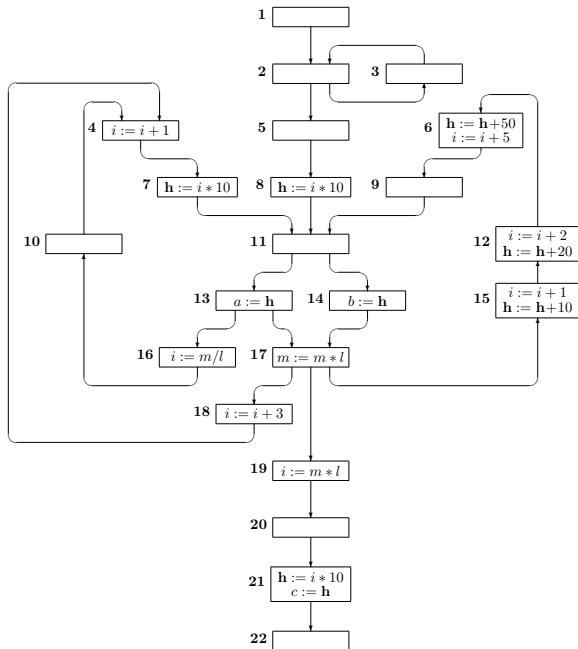
The SSR_{FstRef} -Transformation:

1. Introduce a new auxiliary variable \mathbf{h} for $v * c$
2. Insert at the entry of every node satisfying
 - 2.1 Ins_{FstRef} the assignment $\mathbf{h} := v * c$
 - 2.2 $InsUpd_{FstRef}$ the assignment $\mathbf{h} := \mathbf{h} + Eff(n)$
3. Replace every (original) occurrence of $v * c$ in G by \mathbf{h}

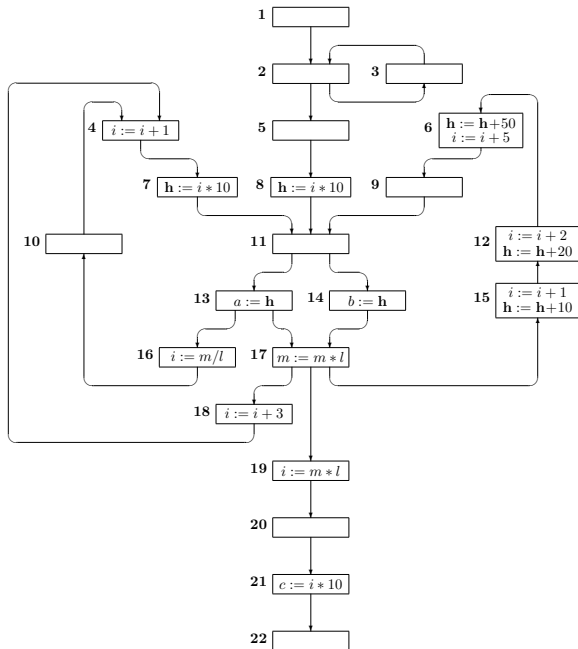
The Result of SSR_{FstRef}



Adding Laziness

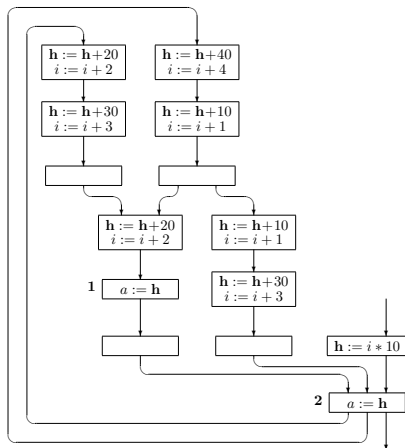


The Result of SSR_{SndRef}



The Multiple-Addition Deficiency

Illustration:



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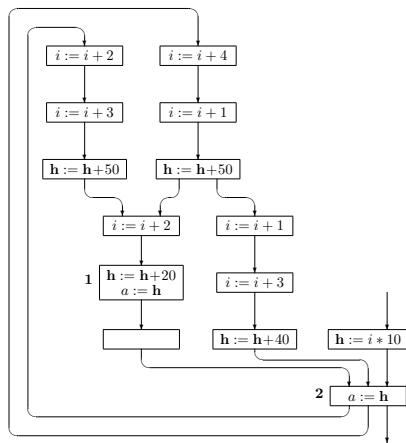
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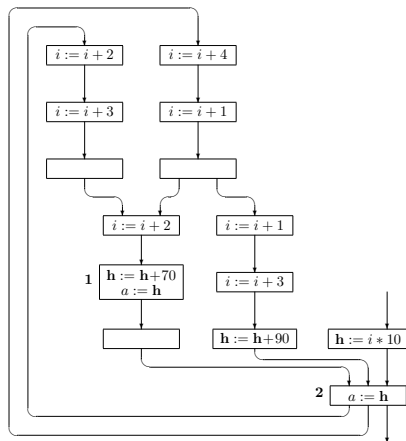
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Overcoming the Multiple-Addition Deficiency

Accumulating the effect of *cure* assignments:



Refined Accumulation of Cure Assignments



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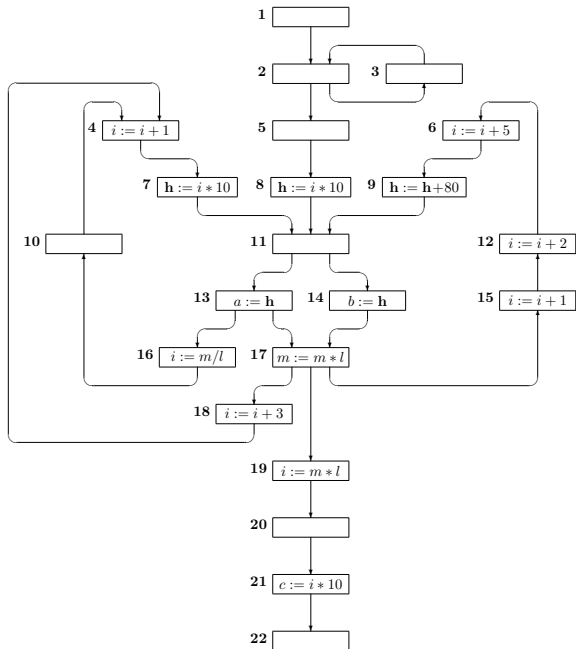
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The 3rd Refinement of *SSR*: *LSR*



Homework

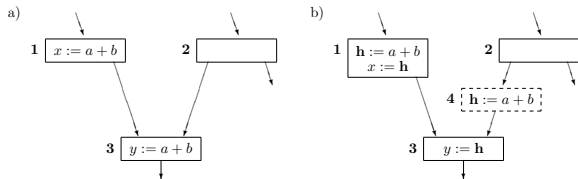
Assignment 5:

1. Specify the data flow analyses and transformations for
 - ▶ SSR
 - ▶ SSR_{FstRef} (overcoming the multiplication-addition deficiency)
 - ▶ SSR_{SndRef} (overcoming the register-pressure deficiency)
 - ▶ $SSR_{ThdRef} = LSR$ (overcoming the multiple-addition deficiency)
2. implement them in PAG, and
3. validate them on the running example of this chapter (or an example coming close to it).

Critical Edges

Like for *BCM* and *LCM* critical edges need to be split in order to get the full power of

- Lazy Strength Reduction (LSR)



Summary of Predicate Values

...of the analyses of the LSR transformation:

Predicate	Node Number																				21	22
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
<i>Safe</i> _{CM}	1	1	1	0	1	0	1	1	1	0	1	0	1	1	0	0	0	0	0	1	1	0
<i>Earliest</i> _{CM}	1	0	0	1	0	1	1	0	1	1	0	1	0	0	0	0	0	0	0	1	0	0
<i>Insert</i> _{CM}	1	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0
<i>Safe</i> _{SR}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	0
<i>Earliest</i> _{SR}	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0
<i>Insert</i> _{SSR}	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0
<i>Critical</i>	0	0	0	1	0	1	0	0	0	1	0	1	0	0	1	1	1	1	1	0	0	0
<i>Subst-Crit</i>	0	0	0	1	0	0	1	0	0	1	1	0	1	1	0	0	0	0	0	0	0	0
<i>Insert</i> _{FstRef}	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
<i>Delay</i>	1	1	1	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0
<i>Latest</i>	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>Isolated</i>	1	1	1	1	1	0	0	0	0	1	0	0	0	0	1	0	1	1	1	1	1	1
<i>Update</i> _{SndRef}	0	0	0	0	0	1	1	1	1	0	1	1	1	1	1	0	1	0	0	0	0	0
<i>Insert</i> _{SndRef}	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>Accumulating</i>	0	0	0	0	0	0	1	1	1	0	0	0	1	1	0	0	0	0	0	0	1	0
<i>Insert</i> _{LSR}	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>InsUpd</i> _{LSR}	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
<i>Delete</i> _{LSR}	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0

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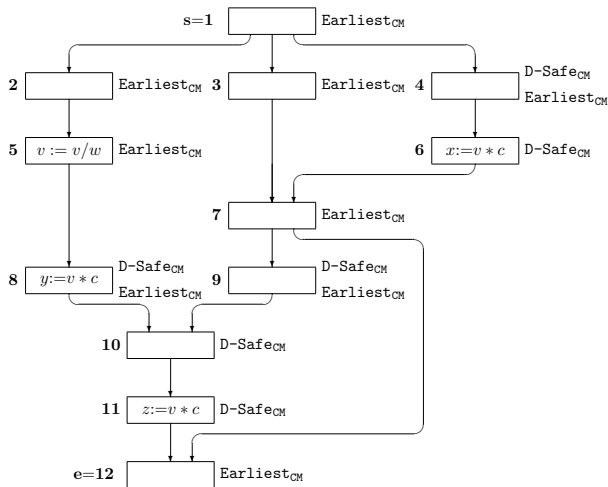
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Illustrating Down-Safety and Earliestness




Further Reading for Chapter 11 (1)

-  F. E. Allen, John Cocke, Ken Kennedy. *Reduction of Operator Strength*. In Stephen S. Muchnick, Neil D. Jones (Eds.). *Program Flow Analysis: Theory and Applications*. Prentice Hall, 1981, Chapter 3, 79-101.
-  Keith D. Cooper, Linda Torczon. *Engineering a Compiler*. Morgan Kaufman Publishers, 2004. (Chapter 10.4.2, Strength Reduction)
-  D. M. Dhamdhere. *A New Algorithm for Composite Hoisting and Strength Reduction Optimisation (+ Corrigendum)*. *International Journal of Computer Mathematics* 27:1-14,31-32, 1989.

Further Reading for Chapter 11 (2)

-  D. M. Dhamdhere, J. R. Isaac. *A Composite Algorithm for Strength Reduction and Code Movement Optimization*. International Journal of Computer and Information Sciences 9(3):243-273, 1980.
-  S. M. Joshi, D. M. Dhamdhere. *A Composite Hoisting-strength Reduction Transformation for Global Program Optimization – Part I and Part II*. International Journal of Computer Mathematics 11:21-41,111-126, 1982.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Lazy Strength Reduction*. Journal of Programming Languages 1(1):71-91, 1993.

Further Reading for Chapter 11 (2)

-  Bernhard Steffen, Jens Knoop, Oliver Rüthing. *Efficient Code Motion and an Adaption to Strength Reduction*. In Proceedings of the 4th International Joint Conference on Theory and Practice of Software Development (TAPSOFT'91), Springer-V., LNCS 494, 394-415, 1991.

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More on Code Motion

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Motivation

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Motivation

Why is it rewarding to consider PRE?

Because PRE is

- ▶ **General**: A family of optimizations rather than a single optimization
- ▶ **Well understood**: Proven correct and **optimal**
- ▶ **Relevant**: Widely used in practice because of its power
- ▶ **Truly classical**: Looks back to a long history beginning with
 - ▶ Etienne Morel, Claude Renvoise. **Global Optimization by Suppression of Partial Redundancies**. Communications of the ACM 22(2):96-103, 1979.
 - ▶ Andrei P. Ershov. **On Programming of Arithmetic Operations**. Communications of the ACM 1(8):3-6, 1958.

Motivation (Cont'd)

Last but not least, **PRE** is

- ▶ **Challenging**: Conceptually simple but exhibits a variety of thought provoking phenomena

Some of these challenges we are going to sketch next.

Code Motion Reconsidered

Traditionally:

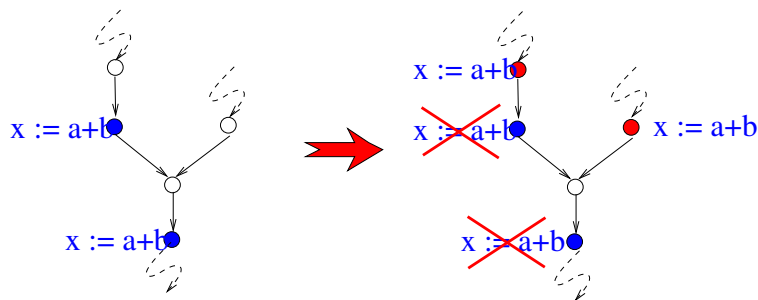
- ▶ Code (C) means expressions
- ▶ Motion (M) means hoisting

But:

- ▶ CM is more than hoisting of expressions and PR(E)E!

Obviously

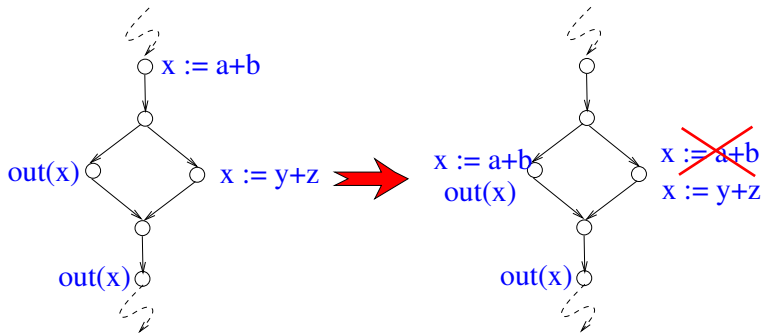
...assignments are code, too.



- ▶ Here, CM means eliminating partially redundant assignments (PRAE)

Differently from expressions...

...assignments can also be **sunk**.



- ▶ Here, **CM** means **eliminating partially dead code (PDCE)**

Design Space of CM-Algorithms (1)

This results in the following design space of CM-algorithms:

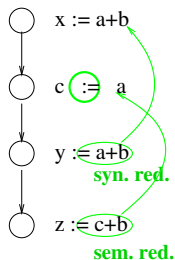
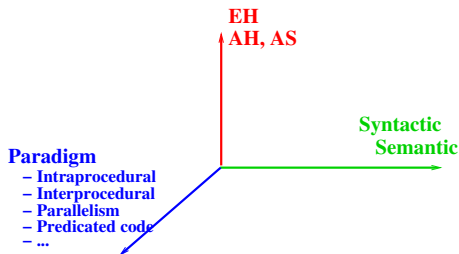
Generally:

- ▶ **Code** means **expressions/assignments**
- ▶ **Motion** means **hoisting/sinking**

Code / Motion	Hoisting	Sinking
Expressions	EH	·/·
Assignments	AH	AS

Design Space of CM-Algorithms (2)

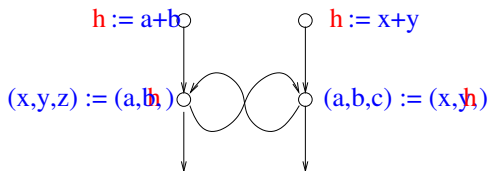
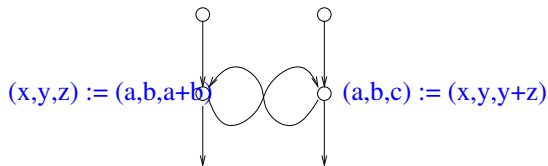
Adding further dimensions to the design space of CM-algorithms:



Introducing semantics... !

Semantic Code Motion

...enables more powerful optimizations!



(Example from B. Steffen, TAPSOFT'87)



What is the Impact on Optimality?

Optimality statements are quite sensitive towards setting changes!

Three examples shall provide evidence for this:

- ▶ **Code motion** vs. **code placement**
- ▶ **Interdependencies** of elementary transformations
- ▶ **Paradigm** dependencies

Further Reading for Chapter 12.1

-  Bernhard Steffen. *Optimal Run Time Optimization – Proved by a New Look at Abstract Interpretation*. In Proceedings of the 2nd Joint Conference on Theory and Practice of Software Development (TAPSOFT'87), Springer-V., LNCS 249, 52-68, 1987.
-  Bernhard Steffen, Jens Knoop, Oliver Rüthing. *The Value Flow Graph: A Program Representation for Optimal Program Transformations*. In Proceedings of the 3rd European Symposium on Programming (ESOP'90), Springer-V., LNCS 432, 389-405, 1990.

Chapter 12.2

Code Motion vs. Code Placement

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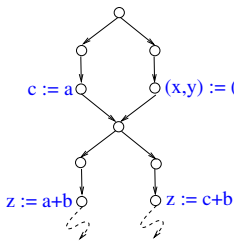
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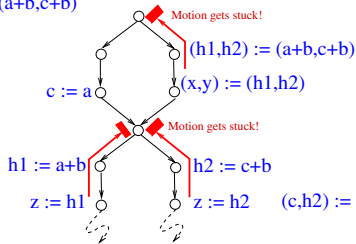
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Code Motion (CM) vs. Code Placement (CP)

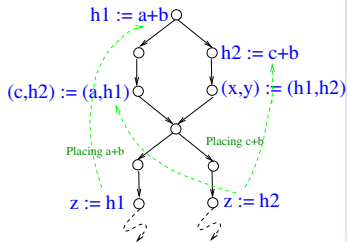
CM and CP are no synonyms!



Original Program



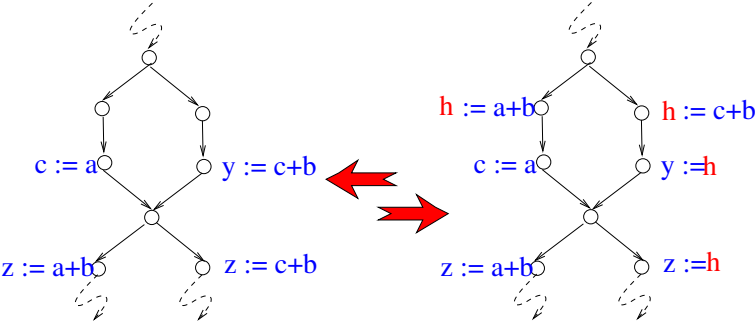
After Sem. Code Motion



After Sem. Code Placement

Even worse

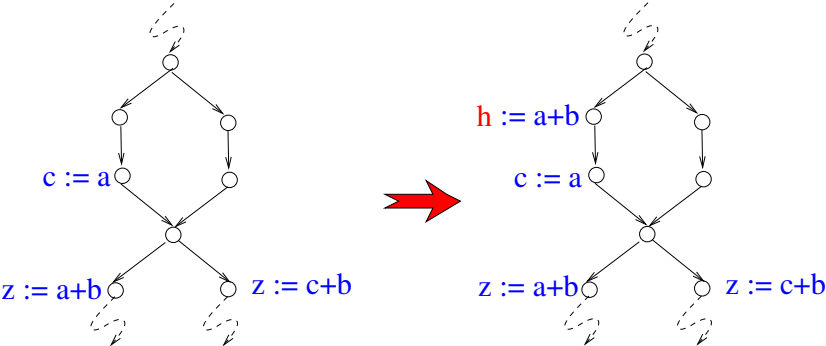
Optimality is lost!



Incomparable!

Even more worse

The **performance** can be impaired, when applied naively!



Further Reading for Chapter 12.2

-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Code Motion and Code Placement: Just Synonyms?* In Proceedings of the 7th European Symposium on Programming (ESOP'98), Springer-V., LNCS 1381, 154-169, 1998.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Expansion-based Removal of Semantic Partial Redundancies.* In Proceedings of the 8th International Conference on Compiler Construction (CC'99), Springer-V., LNCS 1575, 91-106, 1999.

Chapter 12.3

Interactions of Elementary Transformations

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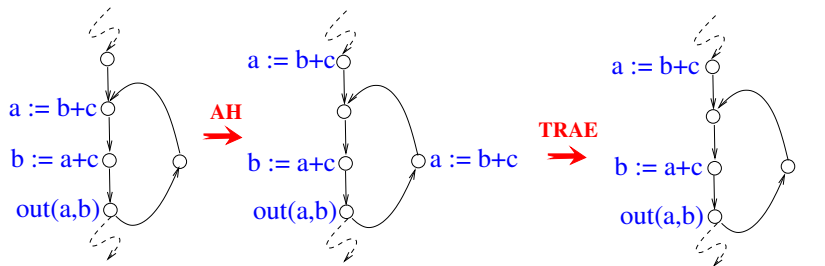
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Assignment Hoisting (AH) plus Totally Redundant Assignment Elimination (TRAЕ)

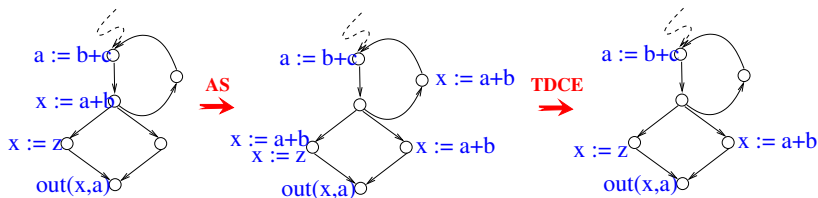
...leads to Partially Redundant Assignment Elimination (PRAE):



...2nd Order Effects!

Assignment Sinking (AS) plus Total Dead-Code Elimination (TDCE)

...leads to **Partial Dead-Code Elimination (PDCE)**:



...2nd Order Effects!

Conceptually

...we can understand **PREE**, **PRAE**, and **PDCE** as follows:

- ▶ $PREE = EH ; TREE$
- ▶ $PRAE = (AH + TRAE)^*$
- ▶ $PDCE = (AS + TDCE)^*$

Optimality Results for PREE

Theorem (12.2.1, Optimality)

1. The BCM transformation yields *computationally optimal results*.
2. The LCM transformation yields *computationally and lifetime optimal results*.
3. The SpCM transformation yields *optimal results wrt a given prioritization of the goals of redundancy avoidance, register pressure, and code size*.

Optimality Results for (Pure) PRAE/PDCE

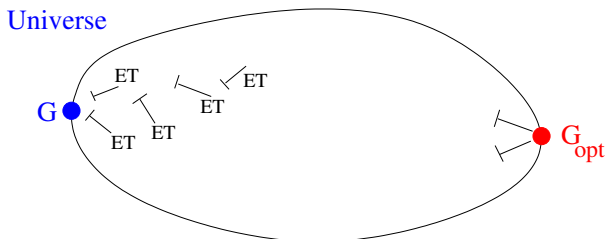
Deriving relation \vdash ...

- ▶ PRAE... $G \vdash_{AH, TRAE} G'$ (ET={AH, TRAE})
- ▶ PDCE... $G \vdash_{AS, TDCE} G'$ (ET={AS, TDCE})

We can prove:

Theorem (12.2.2, Optimality)

For *PRAE* and *PDCE* the deriving relation \vdash_{ET} is confluent and terminating.



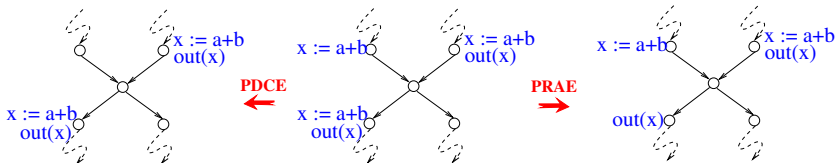
Now

...extend and amalgamate **PRAE** and **PDCE** to **Assignment Placement (AP)**:

$$\blacktriangleright AP = (AH + TRAE + AS + TDCE)^*$$

...**AP** should be more powerful than **PRAE** and **PDCE** alone!

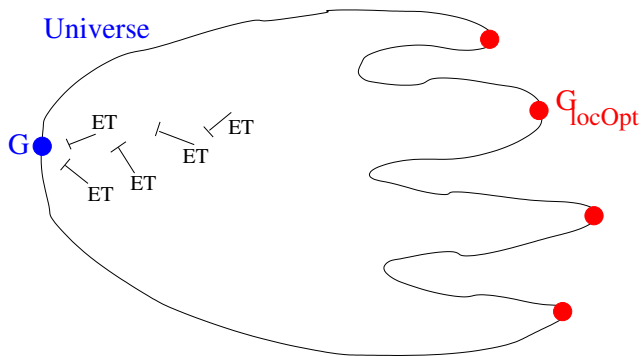
Indeed, it is but:



The resulting two programs are **incomparable**.

Confluence

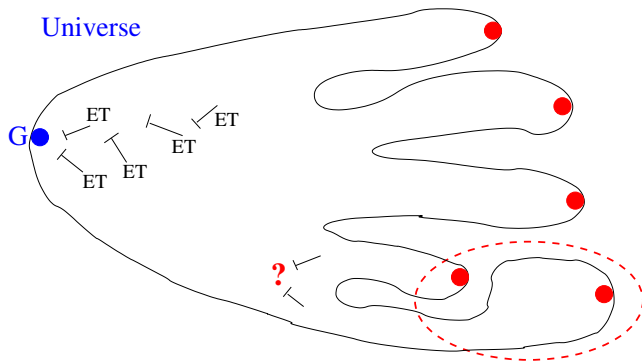
...and hence (global) optimality is lost!



Fortunately, we retain local optimality!



However

...there are settings, where we end up w/ universes like the following:



Here, even **local optimality** is lost!

Further Reading for Chapter 12.3

-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Partial Dead Code Elimination*. In Proceedings of the ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI'94), ACM SIGPLAN Notices 29(6):147-158, 1994.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *The Power of Assignment Motion*. In Proceedings of the ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI'95), ACM SIGPLAN Notices 30(6):233-245, 1995.

Chapter 12.4

Paradigm Impacts

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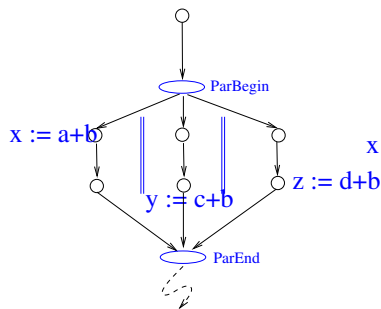
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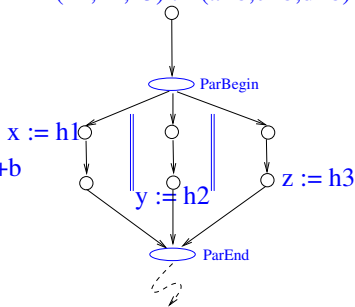
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Adding Parallelism



Original Program

$(h1,h2,h3) := (a+b,c+b,d+b)$



After Earliestness Transformation

...a naive transfer of the “place computations as early as possible” transformation strategy leads here to an essentially sequential program!

Adding Procedures

Similar phenomena are encountered when naively applying successful transformation strategies from the intraprocedural to the interprocedural setting.

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Further Reading for Chapter 12.4

-  Jens Knoop. *Optimal Interprocedural Program Optimization: A New Framework and Its Application*. Springer-V., LNCS 1428, 1998. (Chapter 10, Interprocedural Code Motion: The Transformations)
-  Jens Knoop, Bernhard Steffen. *Code Motion for Explicitly Parallel Programs*. In Proceedings of the 7th ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming (PPoPP'99), ACM SIGPLAN Notices 34(8):13-24, 1999.

Chapter 12.5

Further Code Motion Transformations

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Suggested Reading (1)

► Syntactic PRE

- Knoop, J., Rüthing, O., and Steffen, B. Retrospective: Lazy Code Motion. In “20 Years of the ACM SIGPLAN Conference on Programming Language Design and Implementation (1979 - 1999): A Selection”, ACM SIGPLAN Notices 39, 4 (2004), 460 - 461 & 462-472.
- Knoop, J., Rüthing, O., and Steffen, B. Optimal code motion: Theory and practice. ACM Transactions on Programming Languages and Systems 16, 4 (1994), 1117 - 1155.
- Rüthing, O., Knoop, J., and Steffen, B. Sparse code motion. In Conference Record of the 27th Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages (POPL 2000) (Boston, MA, Jan. 19 - 21, 2000), ACM New York, (2000), 170 - 183.

Suggested Reading (2)

- ▶ [Eliminating partially dead code](#)
 - ▶ Knoop, J., Rüthing, O., and Steffen, B. Partial dead code elimination. In Proceedings of the ACM SIGPLAN'94 Conference on Programming Language Design and Implementation (PLDI'94) (Orlando, FL, USA, June 20 - 24, 1994), ACM SIGPLAN Notices 29, 6 (1994), 147 - 158.
- ▶ [Eliminating partially redundant assignments](#)
 - ▶ Knoop, J., Rüthing, O., and Steffen, B. The power of assignment motion. In Proceedings of the ACM SIGPLAN'95 Conference on Programming Language Design and Implementation (PLDI'95) (La Jolla, CA, USA, June 18 - 21, 1995), ACM SIGPLAN Notices 30, 6 (1995), 233 - 245.

Suggested Reading (3)

▶ BB-graphs vs. SI-graphs

- ▶ Knoop, J., Koschützki, D., and Steffen, B. Basic-block graphs: Living dinosaurs? In Proceedings of the 7th International Conference on Compiler Construction (CC'98) (Lisbon, Portugal, March 30 - April 3, 1998), Springer-Verlag, Heidelberg, LNCS 1383 (1998), 65 - 79.

▶ Moving vs. placing code

- ▶ Knoop, J., Rüthing, O., and Steffen, B. Code motion and code placement: Just synonyms? In Proceedings of the 7th European Symposium On Programming (ESOP'98) (Lisbon, Portugal, March 30 - April 3, 1998), Springer-Verlag, Heidelberg, LNCS 1381 (1998), 154 - 169.

Suggested Reading (4)

► Speculative vs. classical PRE




- Scholz, B., Horspool, N. and Knoop, J. Optimizing for space and time usage with speculative partial redundancy elimination. In Proceedings of the ACM SIGPLAN/SIGBED 2004 Conference on Languages, Compilers, and Tools for Embedded Systems (LCTES 2004) (Washington, DC, June 11 - 13, 2004), ACM SIGPLAN Notices 39, 7 (2004), 221 -230.
- Xue, J., Knoop, J. A fresh look at PRE as a maximum flow problem. In Proceedings of the 15th International Conference on Compiler Construction (CC 2006) (Vienna, Austria, March 25 - April 2, 2006), Springer-Verlag, Heidelberg, LNCS 3923 (2006), 139 - 154.

Suggested Reading (5)




► Further techniques

- Geser, A., Knoop, J., Lüttgen, G., Rüthing, O., and Steffen, B. Non-monotone fixpoint iterations to resolve second order effects. In Proceedings of the 6th International Conference on Compiler Construction (CC'96) (Linköping, Sweden, April 24 - 26, 1996), Springer-V., Heidelberg, LNCS 1060 (1996), 106 - 120.
- Knoop, J., and Mehofer, E. Optimal distribution assignment placement. In Proceedings of the 3rd European Conference on Parallel Processing (Euro-Par'97) (Passau, Germany, August 26 - 29, 1997), Springer-V., Heidelberg, LNCS 1300 (1997), 364 - 373.




Further Reading for Chapter 12 (1)

-  B. Alpern, Mark N. Wegman, F. Ken Zadeck. *Detecting Equality of Variables in Programs*. In Proceedings of POPL'88, 1-11, 1988.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Partial Dead Code Elimination*. In Proceedings of the ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI'94), ACM SIGPLAN Notices 29(6):147-158, 1994.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *The Power of Assignment Motion*. In Proceedings of the ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI'95), ACM SIGPLAN Notices 30(6):233-245, 1995.

Further Reading for Chapter 12 (2)

-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Code Motion and Code Placement: Just Synonyms?* In Proceedings of the 7th European Symposium on Programming (ESOP'98), Springer-V., LNCS 1381, 154-169, 1998.
-  Jens Knoop. *Optimal Interprocedural Program Optimization: A New Framework and Its Application*. Springer-V., LNCS 1428, 1998. (Chapter 10.1, Essential Differences to the Intraprocedural Setting)
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Expansion-based Removal of Semantic Partial Redundancies*. In Proceedings of the 8th International Conference on Compiler Construction (CC'99), Springer-V., LNCS 1575, 91-106, 1999.

Further Reading for Chapter 12 (3)

-  Jens Knoop, Bernhard Steffen. *Code Motion for Explicitly Parallel Programs*. In Proceedings of the 7th ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming (PPoPP'99), ACM SIGPLAN Notices 34(8):13-24, 1999.
-  Bernhard Steffen. *Optimal Run Time Optimization – Proved by a New Look at Abstract Interpretation*. In Proceedings of the 2nd Joint Conference on Theory and Practice of Software Development (TAPSOFT'87), Springer-V., LNCS 249, 52-68, 1987.
-  Bernhard Steffen, Jens Knoop, Oliver Rüthing. *The Value Flow Graph: A Program Representation for Optimal Program Transformations*. In Proceedings of the 3rd European Symposium on Programming (ESOP'90), Springer-V., LNCS 432, 389-405, 1990.

Part III

Interprocedural Data Flow Analysis

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Outline

We consider:

- ▶ **The Context Information Approach** (cf. Chapter 13)
 - ▶ Call Strings
 - ▶ Assumption Sets Strings
- ▶ **The Functional Approach** (cf. Chapter 14)
 - ▶ **The Base Setting**
Adding Procedures but no parameters and local variables
 - ▶ **The General Setting**
Adding value parameters and local variables
 - ▶ **Extensions**
Adding reference parameters and procedural parameters

Chapter 13

The Context Information Approach

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Outline

From [intraprocedural](#) to [interprocedural data-flow analysis](#)...

To this end we extend our programming language WHILE by introducing programs with

- ▶ top-level declarations of global mutually recursive procedures and
- ▶ a [call-by-value](#) and a [call-by-result](#) parameter.

Note: Extensions to multiple call-by-value, call-by-result, and call-by-value-result parameters are straightforward.

Syntax

Extended WHILE-Language $WHILE_{\pi}$:

$$P_{\star} ::= \text{begin } D_{\star} \ S_{\star} \ \text{end}$$
$$D ::= D; D \mid \text{proc } p(\text{val } x; \text{ res } y) \text{ is}^{\ell_n} \ S \ \text{end}^{\ell_x}$$
$$S ::= \dots \mid [\text{call } p(a, z)]_{\ell_r}^{\ell_c}$$

Labeling scheme

► Procedure declarations

ℓ_n : for entering the body

ℓ_x : for exiting the body

► Procedure calls

ℓ_c : for the call

ℓ_r : for the return

Assumptions

We assume that

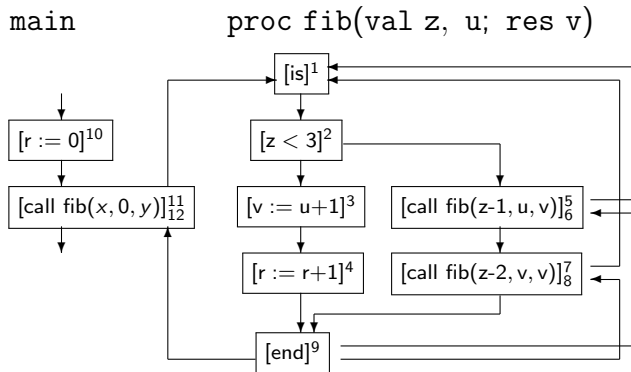
- ▶ WHILE_π is statically scoped.
- ▶ The parameter mechanism is
 - ▶ call-by-value for the first parameter
 - ▶ call-by-result for the second parameter.
- ▶ Procedures may be mutually recursive.
- ▶ Programs are uniquely labelled.
- ▶ There are no procedures of the same name.
- ▶ Only procedures may be called by a program that have been declared in it.

Example

The procedure `proc fib` computing the Fibonacci numbers:

```
begin
  proc fib(val z,u; res v) is
    if z<3 then
      (v:=u+1; r:=r+1)
    else (
      call fib (z-1,u,v);
      call fib (z-2,v,v)
    )
  end;
  r:=0;
  call fib(x,0,y)
end
```

The Flow Graph of Procedure `proc fib`



Notions and Notations for Flow Graphs (1)

...for **procedure calls** and **procedure declarations**:

	$[\text{call } p(a, z)]_{l_r}^{l_c}$	$\text{proc } p(\text{val } x; \text{ res } y) \text{ is}^{l_n} S \text{ end}^{l_x}$
init	l_c	l_n
final	$\{l_r\}$	$\{l_x\}$
blocks	$\{[\text{call } p(a, z)]_{l_r}^{l_c}\}$	$\{\text{is}^{l_n}\} \cup \text{blocks}(S) \cup \{\text{end}^{l_x}\}$
labels	$\{l_c, l_r\}$	$\{l_n, l_x\} \cup \text{labels}(S)$
flow	$\{(l_c; l_n), (l_x; l_r)\}$	$\{(l_n, \text{init}(S))\} \cup \text{flow}(S) \cup \{l, l_x \mid l \in \text{final}(S)\}$

Note: $(l_c; l_n)$ and $(l_x; l_r)$ denote a new kind of flow, **interprocedural flow**:

- ▶ $(l_c; l_n)$ is the flow corresponding to **calling** a procedure at l_c and entering the procedure body at l_n and
- ▶ $(l_x; l_r)$ is the flow corresponding to exiting a procedure body at l_x and **returning** to the call at l_r .

Remark: Intraprocedural flow uses ',' while interprocedural flow uses ';'.

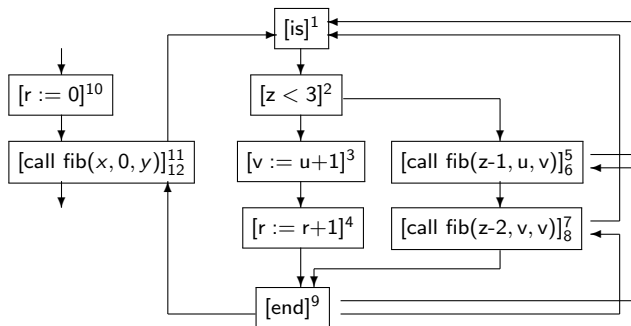
Notions and Notations for Flow Graphs (2)

...for (whole) **programs**:

	P_*
init_*	$\text{init}(S_*)$
final_*	$\text{final}(S_*)$
blocks_*	$\bigcup \{ \text{blocks}(p) \mid \text{proc } p(\text{val } x; \text{ res } y) \text{ is}^{\ell_n} S \text{ end}^{\ell_x} \text{ is in } D_* \} \cup \text{blocks}(S_*)$
labels_*	$\bigcup \{ \text{labels}(p) \mid \text{proc } p(\text{val } x; \text{ res } y) \text{ is}^{\ell_n} S \text{ end}^{\ell_x} \text{ is in } D_* \} \cup \text{labels}(S_*)$
flow_*	$\bigcup \{ \text{flow}(p) \mid \text{proc } p(\text{val } x; \text{ res } y) \text{ is}^{\ell_n} S \text{ end}^{\ell_x} \text{ is in } D_* \} \cup \text{flow}(S_*)$
Lab $_*$	labels_*

$\text{inter-flow}_* = \{ (\ell_c, \ell_n, \ell_x, \ell_r) \mid P_* \text{ contains } [\text{call } p(a, z)]_{\ell_r}^{\ell_c} \text{ as well as } \text{proc } p(\text{val } x; \text{ res } y) \text{ is}^{\ell_n} S \text{ end}^{\ell_x} \}$

Example



$$flow_{\star} = \{(1, 2), (2, 3), (3, 4), (4, 9), \\ (2, 5), (5, 1), (9, 6), (6, 7), (7, 1), (9, 8), (8, 9), \\ (11, 1), (9, 12), (10, 11)\}$$

$$inter-flow_{\star} = \{(11, 1, 9, 12), (5, 1, 9, 6), (7, 1, 9, 8)\}$$

Metavariables for Forward/Backward Analyses

Forward Analyses:

- ▶ $F = flow_*$
- ▶ $E = init_*$
- ▶ $IF = inter-flow_*$

Backward Analyses:

- ▶ $F = flow_*^R$
- ▶ $E = final_*$
- ▶ $IF = inter-flow_*^R$

Towards Interprocedural DFA

New **transfer functions** dealing with **interprocedural flow** are required:

- ▶ For each procedure call $[\text{call } p(a, z)]_{\ell_r}^{\ell_c}$ we require two transfer functions
 - ▶ f_{l_c} and f_{l_r}
corresponding to calling the procedure and returning from the call.
- ▶ For each procedure definition $\text{proc } p(\text{val } x; \text{ res } y) \text{ is }^{\ell_n} S \text{ end}^{\ell_x}$ we require two transfer functions
 - ▶ f_{l_n} and f_{l_x}
corresponding to entering and exiting the procedure body.

Interprocedural DFA: Naive Formulation (1)

- ▶ Treat the three kinds of flow, (l_1, l_2) , $(l_c; l_n)$, $(l_x; l_r)$ in the same way.
- ▶ Assume that the 4 transfer functions associated with procedure calls and procedure definitions are given by the identity functions, i.e., the parameter-passing is effectively ignored.

Then:

Naive Interprocedural *MaxFP*-Equation System:

$$\begin{aligned}A_o(l) &= \bigcap \{A_\bullet(l') \mid (l', l) \in F \vee (l'; l) \in F\} \sqcap \iota_E^l \\A_\bullet(l) &= f_l^A(A_o(l))\end{aligned}$$

where

$$\iota_E^l =_{df} \begin{cases} \iota & \text{if } l \in E \\ \perp & \text{if } l \notin E \end{cases}$$

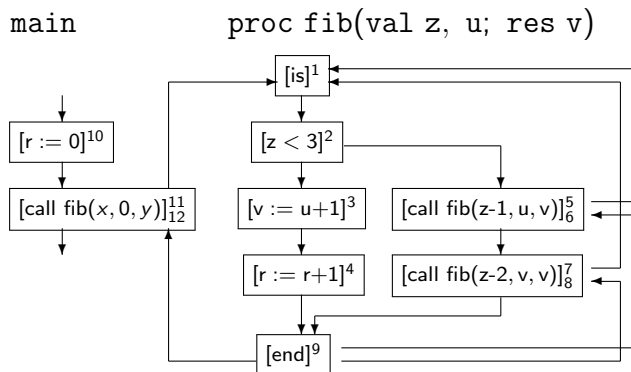
Interprocedural DFA: Naive Formulation (2)

Given the previous assumptions we have:

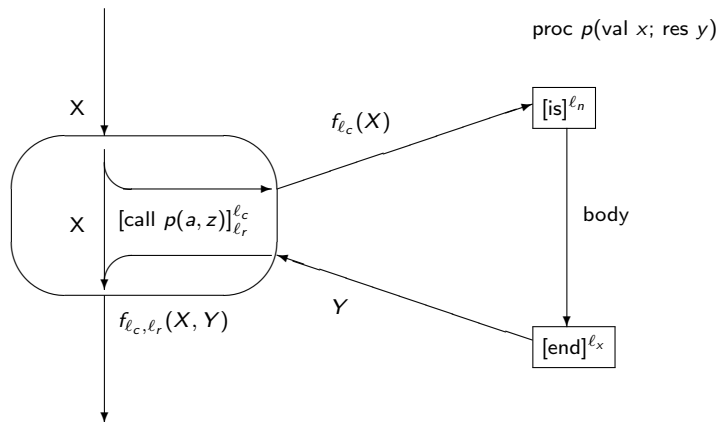
- ▶ Both procedure calls $(l_c; l_n)$ and procedure returns $(l_x; l_r)$ are treated like “goto’s”.
- ▶ There is no mechanism for ensuring that information flowing along $(l_c; l_n)$ flows back along $(l_x; l_r)$ to the *same* call
- ▶ Intuitively, the equation system considers a much too large set of “paths” through the program and hence will be grossly imprecise (although formally on the safe side)

Interprocedural DFA: Naive Formulation (3)

We want to overcome the shortcoming of the naive formulation by restricting attention to paths that have the proper nesting of procedure calls and exits. Important are the notions of **matching procedure entries and exits** and of **complete** and **valid paths**.



Matching Procedure Entries and Exits



Complete Paths

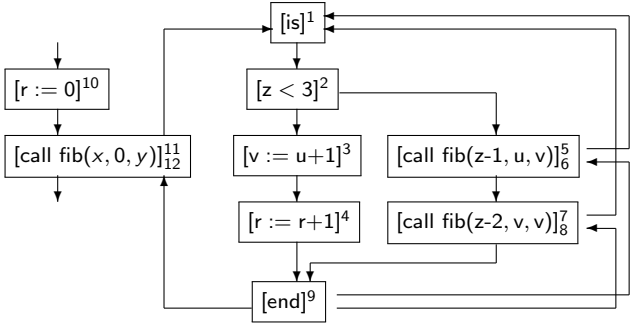
A **complete path** from l_1 to l_2 in P_\star has proper nesting of procedure entries and exits; and a procedure returns to the point where it was called:

$$\begin{array}{ll} CP_{l_1, l_2} \longrightarrow l_1 & \text{whenever } l_1 = l_2 \\ CP_{l_1, l_3} \longrightarrow l_1, CP_{l_2, l_3} & \text{whenever } (l_1, l_2) \in \text{flow}_\star \\ CP_{l_c, l} \longrightarrow l_c, CP_{l_n, l_x}, CP_{l_r, l} & \text{whenever } (l_c, l_n, l_x, l_r) \in \text{inter-flow}_\star \end{array}$$

Recall:

$(l_c, l_n, l_r, l_x) \in \text{inter-flow}_\star$, if P_\star contains $[\text{call } p(a, z)]_{l_r}^{l_c}$ as well as $\text{proc } p(\text{val } x; \text{res } y) \text{ is}^{l_n} S \text{ end}^{l_x}$.

Example: Complete Paths



- $CP_{10,12} \rightarrow 10, CP_{11,12}$
- $CP_{11,12} \rightarrow 11, CP_{1,9}, CP_{12,12}$
- $CP_{1,9} \rightarrow 1, CP_{2,9}$
- $CP_{2,9} \rightarrow 2, CP_{3,9}$
- $CP_{2,9} \rightarrow 2, CP_{5,9}$
- $CP_{3,9} \rightarrow 3, CP_{4,9}$
- $CP_{4,9} \rightarrow 4, CP_{9,9}$
- $CP_{5,9} \rightarrow 5, CP_{1,9}, CP_{6,9}, CP_{12,12}$
- $CP_{6,9} \rightarrow 6, CP_{7,9}, CP_{9,9} \rightarrow 9$
- $CP_{7,9} \rightarrow 7, CP_{1,9}, CP_{8,9}$
- $CP_{8,9} \rightarrow 8, CP_{9,9}$

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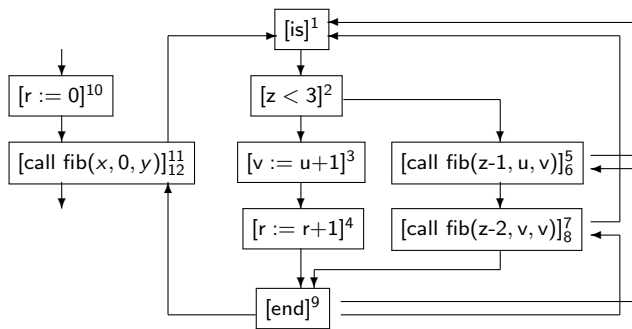
Valid Paths

A **valid path** starts at the entry node init_* of P_* , all the procedure exits match the procedure entries but some procedures might be entered but not yet exited:

$VP_* \longrightarrow VP_{\text{init}_*, l}$	whenever $l \in \text{Lab}_*$
$VP_{l_1, l_2} \longrightarrow l_1$	whenever $l_1 = l_2$
$VP_{l_1, l_3} \longrightarrow l_1, VP_{l_2, l_3}$	whenever $(l_1, l_2) \in \text{flow}_*$
$VP_{l_c, l} \longrightarrow l_c, CP_{l_n, l_x}, VP_{l_r, l}$	whenever $(l_c, l_n, l_x, l_r) \in \text{inter-flow}_*$
$VP_{l_c, l} \longrightarrow l_c, VP_{l_n, l}$	whenever $(l_c, l_n, l_x, l_r) \in \text{inter-flow}_*$

Note: The **valid paths** are generated by the non-terminal VP_* .

Example: Valid Paths



$CP_{10,12}$	\rightarrow	10,	$CP_{11,12}$	$CP_{3,9}$	\rightarrow	3,	$CP_{4,9}$
$CP_{11,12}$	\rightarrow	11,	$CP_{1,9}, CP_{12,12}$	$CP_{4,9}$	\rightarrow	4,	$CP_{9,9}$
$CP_{1,9}$	\rightarrow	1,	$CP_{2,9}$	$CP_{5,9}$	\rightarrow	5,	$CP_{1,9}, CP_{6,9}, CP_{12,12}$
$CP_{2,9}$	\rightarrow	2,	$CP_{3,9}$	$CP_{6,9}$	\rightarrow	6,	$CP_{7,9}, CP_{9,9}$
$CP_{2,9}$	\rightarrow	2,	$CP_{5,9}$	$CP_{7,9}$	\rightarrow	7,	$CP_{1,9}, CP_{8,9}$
				$CP_{8,9}$	\rightarrow	8,	$CP_{9,9}$

Some valid paths: [10,11,1,2,3,4,9,12] and [10,11,1,2,5,1,2,3,4,9,6,7,1,2,3,4,9,8,9,12]

A non-valid path: [10,11,1,2,5,1,2,3,4,9,12]

Meet over Valid Paths: The MVP Solution

$$MVP_{\circ}(\ell) = \bigsqcap \{f_{\vec{\ell}}(\iota) \mid \vec{\ell} \in vpath_{\circ}(\ell)\}$$

$$MVP_{\bullet}(\ell) = \bigsqcap \{f_{\vec{\ell}}(\iota) \mid \vec{\ell} \in vpath_{\bullet}(\ell)\}$$

where

$$vpath_{\circ}(\ell) = \{[\ell_1, \dots, \ell_{n-1}] \mid n \geq 1 \wedge \ell_n = \ell \wedge [\ell_1, \dots, \ell_n] \text{ is valid path}\}$$

$$vpath_{\bullet}(\ell) = \{[\ell_1, \dots, \ell_n] \mid n \geq 1 \wedge \ell_n = \ell \wedge [\ell_1, \dots, \ell_n] \text{ is valid path}\}$$

Discussing the MVP Solution (1)

The **MVP solution** may be undecidable (even) for lattices satisfying the Descending Chain Condition, just as was the case for the MOP solution.

Therefore, we need to reconsider the **maximal fixed point approach** and adapt it to

- ▶ avoid considering too many paths
- ▶ taking call context information into account.

Discussing the MVP Solution (2)

In more detail:

We have to

- ▶ reconsider the MFP solution and avoid taking too many invalid paths into account.

An obvious approach is to

- ▶ encode information about the paths taken into the data flow properties themselves.

This can be achieved by

- ▶ introducing **context information** $\delta \in \Delta$.

Towards the MFP Counterpart

- ▶ **Context insensitive analysis:** No context information is used.
- ▶ **Context sensitive analysis:** Context information $\delta \in \Delta$ is used.
 - ▶ **Call strings:**
 - ▶ An abstraction of the sequences of procedure calls that have been performed so far.
 - ▶ **Example:** The program point where the call was initiated.
 - ▶ **Assumption sets:**
 - ▶ An abstraction of the states in which previous calls have been performed.
 - ▶ **Example:** An abstraction of the actual parameters of the call.

Call Strings δ as Context Information Δ

- ▶ Encode the path taken.
- ▶ Only record flows of the form $(l_c; l_n)$ corresponding to a procedure call.
- ▶ we take as context $\Delta = \text{Lab}_*^*$ where the most recent label l_c of a procedure call is at the right end.
- ▶ Elements of Δ are called **call strings**.
- ▶ The sequence of labels $l_c^1, l_c^2, \dots, l_c^m$ is the call string leading to the current call which happened at l_c^m ; the previous calls where at $l_c^2 \dots l_c^1$. If $m = 0$ then no calls have been performed so far.

For the example program the following **call strings** are of interest:

$\Lambda, [11], [11, 5], [11, 7], [11, 5, 5], [11, 5, 7], [11, 7, 5], [11, 7, 7], \dots$

The Adapted MFP Equation System

The Adapted MFP-Equation System:

$$\begin{aligned}A_o(\ell) &= \sqcap \{A_\bullet(\ell') \mid (\ell', \ell) \in F \vee (\ell'; \ell) \in F\} \sqcap \hat{\iota}_E^\ell \\A_\bullet(\ell) &= \widehat{f}_\ell^A(A_o(\ell))\end{aligned}$$

where

- ▶ $\hat{L} = \Delta \rightarrow L$ maps a context to a data flow property (i.e., a data flow lattice element)
- ▶ each transfer function \hat{f}_l is given by $\hat{f}_l(\hat{l})(\delta) = f_l(\hat{l}(\delta))$ (i.e., \hat{f}_l adapts resp. specializes f_l to the call context δ)

▶

$$\hat{\iota}_E^\ell =_{df} \begin{cases} \iota_E^\ell & \text{if } \delta = \Lambda \\ \perp & \text{otherwise} \end{cases}$$

Making it Practical: Bounding Call Strings

Problem: Call strings can be arbitrarily long (recursive calls)

Solution: Truncate the call strings to have length of at most k for some fixed number k

In practice:

- ▶ $\Delta = \text{Lab}_*^{\leq k}$, i.e. call strings of bounded length k .
- ▶ $k = 0$: Context insensitive analysis
 - ▶ Λ (the call string is the empty string)
- ▶ $k = 1$: Remember the last procedure call
 - ▶ $\Lambda, [11], [5], [7]$
- ▶ $k = 2$: Remember the last two procedure calls
 - ▶ $\Lambda, [11], [11, 5], [11, 7], [5, 5], [5, 7], [7, 5], [7, 7]$
- ▶ ...

Assumption Sets δ as Context Information Δ

Instead of describing a path directly in terms of the calls being performed (as a textcolorbluecall string does), information about the state in which a call was made can be stored (as an textcolorblueassumption set does).

For a more detailed account of the textcolorblueassumption set approach refer to

- ▶ Flemming Nielson, Hanne Riis Nielson, Chris Hankin. *Principles of Program Analysis*. 2nd edition, Springer-V., 2005. (Chapter 2.5.5, Assumption Sets as Context)

Advanced Topics

... of [interprocedural program analysis](#) and a glimpse on how they can be addressed by [static program analysis](#).

- ▶ Function pointers
- ▶ Virtual function calls
- ▶ Overloaded functions

Function Pointers

Values of function pointer variables

The value of a function pointer variable is the address of a function. At run-time different values can be assigned to pointer variables.

Interprocedural Control Flow

Any function with the same signature (=parameter types) can be potentially called by using a function pointer.

Program analysis can reduce the number of functions that may be called at run-time by computing the set of possible pointer values assigned to function pointer variables in a given program.

Virtual Function Calls & Overloaded Functions

...in object-oriented programming.

Virtual function calls

By taking the class hierarchy into account, we can limit the methods that can be called to the set of overriding methods of subclasses. Program analysis can further reduce the number of methods that may be called at run-time.

Overloaded functions

Calls to overloaded functions are resolved at compile time.

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Further Reading for Chapter 13

-  Alfred V. Aho, Monica S. Lam, Ravi Sethi, Jeffrey D. Ullman. *Compilers: Principles, Techniques, & Tools*. Addison-Wesley, 2nd edition, 2007. (Chapter 12.1.3, Call Strings)
-  Micha Sharir, Amir Pnueli. *Two Approaches to Interprocedural Data Flow Analysis*. In Stephen S. Muchnick, Neil D. Jones (Eds.). *Program Flow Analysis: Theory and Applications*. Prentice Hall, 1981, Chapter 7.3, The Call-String Approach to Interprocedural Analysis, 210-217.
-  Flemming Nielson, Hanne Riis Nielson, Chris Hankin. *Principles of Program Analysis*. 2nd edition, Springer-V., 2005. (Chapter 2.5, Interprocedural Analysis)

Chapter 14

The Functional Approach

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Chapter 14.1

The Base Setting

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The Setting

We consider **programs w/ procedures** that will be represented in terms of

- ▶ Flow graph systems
- ▶ Interprocedural flow graphs

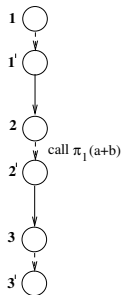
Flow Graph Systems

Definition (14.1.1, Flow Graph System)

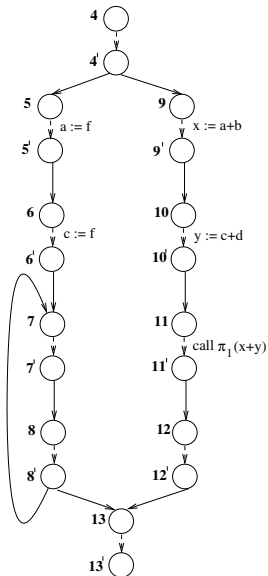
A **flow graph system** $S =_{df} \langle G_0, \dots, G_k \rangle$ is a system of (intra-procedural) flow graphs in the sense of Chapter 4, where each flow graph G_i represents a procedure of the underlying program Π . The flow graph G_0 of S represents the main procedure or the main program of Π .

Illustration: Flow Graph System

π_0 ; a, b, x, y



π_1 ; c, d



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Interprocedural Flow Graphs

Definition (14.1.2, Interprocedural Flow Graph)

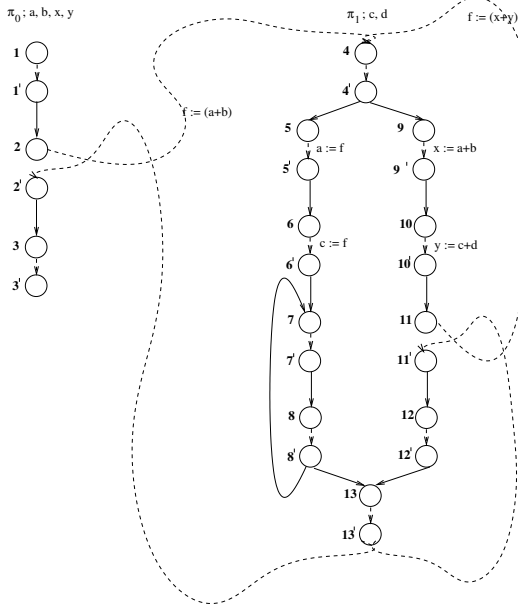
A flow graph system S induces an [interprocedural flow graph](#), where the flow graphs of S are melted to a single flow graph $G^* = (N^*, E^*, \mathbf{s}^*, \mathbf{e}^*)$.

G^* evolves from S by replacing each call edge e of a procedure π by two new edges e_c and e_r .

The edge e_c connects the source node of e w/ the start node of the called procedure.

The edge e_r connects the end node of the called procedure w/ the final node of e .

Illustration: Interprocedural Flow Graph



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Notations for Flow Graph Systems

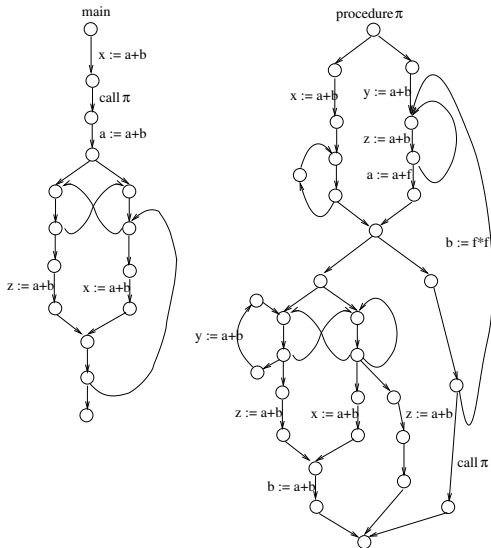
- ▶ G_0 represents the main procedure.
- ▶ The start node s_0 of G_0 is often abbreviated by s .
- ▶ The sets of nodes and edges N_i and E_i , $0 \leq i \leq k$, are assumed to be pairwise disjoint.
- ▶ $N =_{df} \bigcup \{N_i \mid i \in \{0, \dots, k\}\}$ and $E =_{df} \bigcup \{E_i \mid i \in \{0, \dots, k\}\}$ denote the set of all nodes and edges of a flow graph system.
- ▶ $E_{call} \subseteq E$ denotes the set of edges, which represent a procedure call, for short, the set of **call edges**.

Notations for Interprocedural Call Graphs

- ▶ The set of new edges in an interprocedural flow graph are called the **call edges** and **return edges** of G^* , and are denoted by E_c^* und E_r^* .
- ▶ $E_{call}^* =_{df} E_c^* \cup E_r^*$ denotes the set of call and return edges of G^* .

Streamlining Flow Graph Systems

...by removing unnecessary nodes and edges in flow graph systems:



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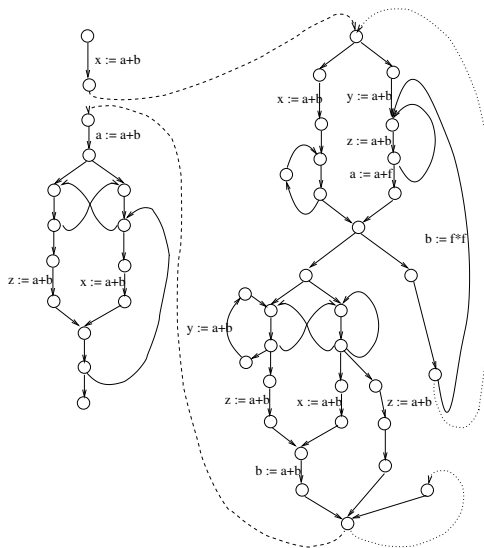
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Streamlining Interprocedural Flow Graphs

...as well as in interprocedural flow graphs:



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The Key to Interprocedural DFA (1)

The Functional *MaxFP*-Approach:

The “functional” *MaxFP* results from lifting the analysis level from elements to functions. It is the pointwise extension of the *MaxFP* approach to all DFA-lattice elements:

The Functional *MaxFP* Equation System 14.1.3

$$\llbracket n \rrbracket = \begin{cases} Id_{\mathcal{C}} & \text{if } n = \mathbf{s} \\ \bigcap \{ \llbracket (n, m) \rrbracket \circ \llbracket m \rrbracket \mid m \in \text{pred}(n) \} & \text{otherwise} \end{cases}$$

Let

$$\llbracket \rrbracket^* : N \rightarrow (\mathcal{C} \rightarrow \mathcal{C})$$

denote the **greatest solution** of the above equation system.

The Key to Interprocedural DFA (2)

Intuitively

The functional *MaxFP* approach lifts the *MaxFP* approach to the level of functions, i.e.

- ▶ it computes the *MaxFP* solution not just for a specially selected single lattice element as start information but simultaneously for all.

The Key to Interprocedural DFA (3)

MaxFP approach vs. functional *MaxFP* approach:

The [Equivalence Theorem 14.1.4](#) characterizes the relationship of the *MaxFP* approach and the functional *MaxFP* approach:

[Theorem \(14.1.4, Equivalence\)](#)

$$\forall n \in N \forall c_s \in \mathcal{C}. \text{MaxFP}_{(G, \llbracket \cdot \rrbracket)}(n)(c_s) = \llbracket n \rrbracket^*(c_s)$$

In the following we will overload the symbol $\llbracket \cdot \rrbracket$ and use it to also denote the greatest fixed point $\llbracket n \rrbracket^*$ of the functional *MaxFP* equation system 14.1.3.

Outlook

The functional variant of the *MaxFP* approach is the key to

- ▶ **interprocedural** (i.e., of programs w/ procedures)
- ▶ **object-oriented** (i.e., of programs w/ classes, objects, and methods)
- ▶ **parallel** (i.e., of programs w/ parallelism)

data flow analysis.

Chapter 14.1.1

Local Abstract Semantics

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(Local) Abstract Semantics

Two components:

- ▶ Data flow analysis lattice $\hat{\mathcal{C}} = (\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \perp, \top)$
- ▶ Data flow analysis functional $\llbracket \cdot \rrbracket' : E^* \rightarrow (\mathcal{C} \rightarrow \mathcal{C})$

Note: In the parameterless base setting call edges and return edges of E^* are given the identity function on \mathcal{C} as their semantics.

Chapter 14.1.2

The *IMOP* Approach

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Interprocedurally Valid Paths (1)

Observations:

- ▶ The notion of a **finite path** for intraprocedural flow graphs extends naturally to interprocedural flow graphs.
- ▶ Unlike, however, as in intraprocedural flow graphs, where each path connecting two nodes represents (up to non-determinism) a possible execution of the program, this does not hold for interprocedural flow graphs.
- ▶ In interprocedural DFA this is taken care of by focusing on **interprocedurally valid paths**.

Interprocedurally Valid Paths (2)

Intuitively: Interprocedurally valid paths respect the call/return behaviour of procedures.

Definition (14.1.2.1, Interprocedurally Valid Path)

Identifying call and return edges of G^* with opening and closing brackets "(" and ")", the set of **interprocedurally valid paths** is given by the set of prefix-closed expressions of the language of balanced bracket expressions.

Notation: In the following we denote the set of interprocedurally valid paths (for short: interprocedural paths) from a node m to a node n by **IP** $[m, n]$.

Interprocedurally Valid Paths (3)

Observation: If we consider the sequences of edge labelings (we suppose that each edge is uniquely marked by some label) of a path as word of a formal language, then the set of intraprocedurally valid paths is given by a **regular** language, the one of interprocedurally valid paths by a **context-free** language.

Note:

- ▶ Sharir and Pnueli gave an algorithmic definition of interprocedurally valid paths in 1981.
- ▶ An immediate definition of interprocedurally valid paths in terms of a context-free language is possible, too.
- ▶ The definition of interprocedurally valid paths as in Definition 14.1.2.1 is due to Reps, Horwitz, and Sagiv, POPL'95.

The *IMOP* Approach

The *IMOP* Solution:

$$\forall c_s \in \mathcal{C} \forall n \in N. \text{IMOP}_{c_s}(n) =_{df} \bigcap \{ \llbracket p \rrbracket'(c_s) \mid p \in \mathbf{IP}[s, n] \}$$

where $\mathbf{IP}[s, n]$ denotes the set of **interprocedurally valid paths** from s to n .

Chapter 14.1.3

The *IMaxFP* Approach

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The *IMaxFP* Approach

...is a two-stage approach:

- ▶ Stage 1: Preprocess – Computing the Semantics of Procedures
- ▶ Stage 2: Main Process – Computing the *IMaxFP* solution

Notations

The definition of the *IMaxFP approach* requires the following mappings on a flow graph system S :

- ▶ $flowGraph : N \cup E \rightarrow S$ maps the nodes and edges of S to the flow graph containing them.
- ▶ $callee : E_{call} \rightarrow S$ maps every call edge to the flow graph of the called procedure.
- ▶ $caller : S \rightarrow \mathcal{P}(E_{call})$ maps every flow graph to the set of call edges calling it.
- ▶ $start : S \rightarrow \{\mathbf{s}_0, \dots, \mathbf{s}_k\}$ and $end : S \rightarrow \{\mathbf{e}_0, \dots, \mathbf{e}_k\}$ map every flow graph of S to its start node and stop node.

The *IMaxFP* Approach (1)

Stage 1: Preprocess – Computing the Semantics of Procedures

The 2nd Order *IMaxFP* Equation System 14.1.3.1

$$\llbracket n \rrbracket = \begin{cases} Id_C & \text{if } n \in \{s_0, \dots, s_k\} \\ \bigcap \{ \llbracket (m, n) \rrbracket \circ \llbracket m \rrbracket \mid m \in \text{pred}_{\text{flowGraph}(n)}(n) \} & \text{otherwise} \end{cases}$$

and

$$\llbracket e \rrbracket = \begin{cases} \llbracket e \rrbracket' & \text{if } e \in E \setminus E_{\text{call}} \\ \llbracket \text{end}(\text{caller}(e)) \rrbracket & \text{otherwise} \end{cases}$$

The *IMaxFP* Approach (2)

Stage 2: Main Process – The “Actual” Interprocedural DFA

The 1st Order *IMaxFP* Equation System 14.1.3.2

$inf(n) =$

$$\begin{cases} c_s & \text{if } n = s_0 \\ \prod \{ inf(src(e)) \mid e \in caller(flowGraph(n)) \} & \text{if } n \in \{s_1, \dots, s_k\} \\ \prod \{ \llbracket (m, n) \rrbracket (inf(m)) \mid m \in pred_{flowGraph(n)}(n) \} & \text{otherwise} \end{cases}$$

The *IMaxFP* Solution:

$$\forall c_s \in \mathcal{C} \forall n \in N. IMaxFP_{c_s}(n) =_{df} inf^*_{c_s}(n)$$

Chapter 14.1.4

Main Results

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Main Results – 1st Stage

Safety and coincidence results of the 2nd-order 1st-stage analysis:

Theorem (14.1.4.1, 2nd Order)

For all $e \in E_{call}$ hold:

1. $\llbracket e \rrbracket \subseteq \bigcap \{ \llbracket p \rrbracket' \mid p \in \mathbf{CIP}[src(e), dst(e)] \}$, if the data flow analysis functional $\llbracket \rrbracket'$ is monotonic.
2. $\llbracket e \rrbracket = \bigcap \{ \llbracket p \rrbracket' \mid p \in \mathbf{CIP}[src(e), dst(e)] \}$, if the data flow analysis functional $\llbracket \rrbracket'$ is distributive.

where the mappings src and dst yield the start and final node of an edge.

Complete Interprocedural Paths

Definition (14.1.4.2, Complete Interproc. Path)

An interprocedural path p from the start node s_i of a procedure G_i , $i \in \{0, \dots, k\}$, to a node n within G_i is **complete**, if every procedure call, i.e., call edge, along p is completed by a corresponding procedure return, i.e., a return edge.

We denote the set of all complete interprocedural paths from s_i to n with **CIP** $[s_i, n]$.

Note:

- ▶ Intuitively, the completeness requirement states that the occurrences of s_i and n belong to the same incarnation of the procedure.
- ▶ We have that the subpaths of a complete interprocedural path that belong to a procedure call, are either disjoint or properly nested.

Main Results – 2nd Stage

Safety and coincidence results of the 1st-order 2nd-stage analysis:

Theorem (14.1.4.3, Interprocedural Safety)

The IMaxFP solution is a safe, i.e., a lower approximation of the IMOP solution, i.e.

$$\forall c_s \in \mathcal{C} \forall n \in N. \text{IMaxFP}_{c_s}(n) \sqsubseteq \text{IMOP}_{c_s}(n)$$

if the data flow analysis functional $\llbracket \cdot \rrbracket'$ is monotonic.

Theorem (14.1.4.4, Interprocedural Coincidence)

The IMaxFP solution coincides with the IMOP solution, i.e.

$$\forall c_s \in \mathcal{C} \forall n \in N. \text{IMaxFP}_{c_s}(n) = \text{IMOP}_{c_s}(n)$$

if the data flow analysis functional $\llbracket \cdot \rrbracket'$ is distributive.

Chapter 14.1.5

Algorithms

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The 2nd Order Alg. 14.1.5.1 – Preprocess

Input: (1) A flow-graph system S , and (2) an abstract semantics consisting of a data-flow lattice \mathcal{C} , and a data-flow functional $\llbracket \cdot \rrbracket' : E^* \rightarrow (\mathcal{C} \rightarrow \mathcal{C})$.

Output: Under the assumption of termination (cf. Theorem 14.1.5.4), an annotation of S with functions $\llbracket n \rrbracket : \mathcal{C} \rightarrow \mathcal{C}$ (stored in gtr , which stands for *global transformation*), and $\llbracket e \rrbracket : \mathcal{C} \rightarrow \mathcal{C}$ (stored in ltr , which stands for *local transformation*) representing the greatest solution of the 2nd order Equation System 14.1.3.1.

Remark: The variable *workset* controls the iterative process. Its elements are nodes of the flow-graph system S . Note that due to the mutual interdependence of the definitions of $\llbracket \cdot \rrbracket$ and $\llbracket \cdot \rrbracket'$ the iterative approximation of $\llbracket \cdot \rrbracket$ is superposed by an interprocedural iteration step, which updates the current approximation of the effect $\llbracket \cdot \rrbracket'$ of call edges. The temporary *meet* stores the result of the most recent meet operation.

The 2nd Order Alg. 14.1.5.1 – Preprocess

(Prologue: Initializing the annotation arrays gtr and ltr and the variable $workset$)

FORALL $n \in N$ DO

 IF $n \in \{s_0, \dots, s_k\}$ THEN $gtr[n] := Id_C$

 ELSE $gtr[n] := \top_{[C \rightarrow C]}$ FI OD;

FORALL $e \in E$ DO

 IF $e \in E_{call}$ THEN $ltr[e] := \top_{[C \rightarrow C]}$ ELSE $ltr[e] := \llbracket e \rrbracket'$ FI
OD;

$workset := \{s_0, \dots, s_k\}$;

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The 2nd Order Alg. 14.1.5.1 – Preprocess

(Main process: Iterative fixed point computation)

WHILE $workset \neq \emptyset$ DO

 CHOOSE $m \in workset$;

$workset := workset \setminus \{m\}$;

 (Update the successor-environment of node m)

 IF $m \in \{e_1, \dots, e_k\}$

 THEN

 FORALL $e \in caller(flowGraph(m))$ DO

$ltr[e] := gtr[m]$;

$meet := ltr[e] \circ gtr[src(e)] \sqcap gtr[dst(e)]$;

 IF $gtr[dst(e)] \sqsupseteq meet$

 THEN

$gtr[dst(e)] := meet$;

$workset := workset \cup \{dst(e)\}$

 FI

 OD

The 2nd Order Alg. 14.1.5.1 – Preprocess

```
ELSE (i.e.,  $m \notin \{e_1, \dots, e_k\}$ )
  FORALL  $n \in \text{succ}_{\text{flowGraph}(m)}(m)$  DO
     $\text{meet} := \text{ltr}[(m, n)] \circ \text{gtr}[m] \sqcap \text{gtr}[n]$ ;
    IF  $\text{gtr}[n] \sqsupset \text{meet}$ 
      THEN
         $\text{gtr}[n] := \text{meet}$ ;
         $\text{workset} := \text{workset} \cup \{n\}$ 
      FI
    OD
  FI
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OD.
```

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The 1st Order Alg. 14.1.5.2 – Main Process

Input: (1) A flow-graph system S , (2) an abstract semantics consisting of a data-flow lattice \mathcal{C} , and a data-flow functional $\llbracket \cdot \rrbracket$ computed by Algorithm 14.1.5.1, and (3) a context information $c_s \in \mathcal{C}$.

Output: Under the assumption of termination (cf. Theorem 14.5.1.4), the *IMaxFP*-solution. Depending on the properties of the data-flow functional, this has the following interpretation:

- (1) $\llbracket \cdot \rrbracket$ is *distributive*: variable *inf* stores for every node the strongest component information valid there wrt the context information c_s .
- (2) $\llbracket \cdot \rrbracket$ is *monotonic*: variable *inf* stores for every node a valid component information wrt the context information c_s , i.e., a lower bound of the strongest component information valid there.

Remark: The variable *workset* controls the iterative process. Its elements are nodes of the flow-graph system S . The temporary *meet* stores the result of the most recent meet operation.

The 1st Order Alg. 14.1.5.2 – Main Process

(Prologue: Initialization of the annotation array *inf* and the variable *workset*)

```
FORALL  $n \in N \setminus \{s_0\}$  DO  $inf[n] := \top$  OD;  
 $inf[s_0] := c_s$ ;  
 $workset := \{ s_0 \}$ ;
```

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The 1st Order Alg. 14.1.5.2 – Main Process

(Main process: Iterative fixed point computation)

```
WHILE  $workset \neq \emptyset$  DO
  CHOOSE  $m \in workset$ ;
   $workset := workset \setminus \{m\}$ ;
  (Update the successor-environment of node  $m$ )
  FORALL  $n \in succ_{flowGraph(m)}(m)$  DO
     $meet := \llbracket (m, n) \rrbracket (inf[m]) \sqcap inf[n]$ ;
    IF  $inf[n] \sqsupseteq meet$ 
      THEN
         $inf[n] := meet$ ;
         $workset := workset \cup \{n\}$ 
  FI;
```

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The 1st Order Alg. 14.1.5.2 – Main Process

```
IF  $(m, n) \in E_{call}$ 
  THEN
     $meet := inf[m] \sqcap inf[start(callee((m, n)))]$ ;
    IF  $inf[start(callee((m, n)))] \sqsupseteq meet$ 
      THEN
         $inf[start(callee((m, n)))] := meet$ ;
         $workset := workset \cup \{ start(callee((m, n))) \}$ 
      FI
    FI
  OD
ESOOHC
OD.
```

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A 1st Variant of the *IMaxFP*-Algorithm

- ▶ Algorithm 14.1.5.3 uses the semantics functions computed by Algorithm 14.1.5.1 more efficiently.
- ▶ Algorithm 14.1.5.1 and 14.1.5.3 constitute a pair of algorithms computing the *IMaxFP* solution, too.
- ▶ Replacing Algorithm 14.5.1.2 by Algorithm 14.1.5.3 has no impact on Algorithm 14.1.5.1.
- ▶ Unlike Algorithm 14.1.5.2, Algorithm 14.1.5.3 does not iterate over all nodes but only over procedure start nodes. After stabilization of the solution for the start nodes, a single run over all other nodes in the epilogue suffices to compute the *IMaxFP* solution at every node.

The 1st Order Algorithm 14.1.5.3 – The “Functional” Main Process

Input: (1) A flow-graph system S , (2) an abstract semantics consisting of a data-flow lattice \mathcal{C} , and the data-flow functionals $\llbracket _ \rrbracket =_{df} gtr$ and $\llbracket _ \rrbracket =_{df} ltr$ with respect to \mathcal{C} (computed by Algorithm 14.1.5.1), and (4) a context information $c_s \in \mathcal{C}$.

Output: Under the assumption of termination (cf. Theorem 14.1.5.4), the $IMaxFP$ -solution. Depending on the properties of the data-flow functional, this has the following interpretation:

- (1) $\llbracket _ \rrbracket$ is *distributive*: variable inf stores for every node the strongest component information valid there wrt the context information c_s .
- (2) $\llbracket _ \rrbracket$ is *monotonic*: variable inf stores for every node a valid component information wrt the context information c_s , i.e., a lower bound of the strongest component information valid there.

Remark: The variable $workset$ controls the iterative process, and the temporary $meet$ stores the most recent approximation.

The 1st Order Algorithm 14.1.5.3 – The “Functional” Main Process

(Prologue: Initialization of the annotation array *inf*, and the variable *workset*)

```
FORALL  $\mathbf{s} \in \{\mathbf{s}_i \mid i \in \{1, \dots, k\}\}$  DO  $inf[\mathbf{s}] := \top$  OD;  
 $inf[\mathbf{s}_0] := c_{\mathbf{s}_0}$ ;  
 $workset := \{\mathbf{s}_i \mid i \in \{1, 2, \dots, k\}\}$ ;
```

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The 1st Order Algorithm 14.1.5.3 – The “Functional” Main Process

(Main process: Iterative fixed point computation)

```
WHILE workset  $\neq \emptyset$  DO
  CHOOSE  $\mathbf{s} \in \textit{workset}$ ;
  workset := workset \ { $\mathbf{s}$ };
  meet :=  $\textit{inf}[\mathbf{s}] \sqcap$ 
     $\sqcap \{ \llbracket \textit{src}(e) \rrbracket (\textit{inf}[\textit{start}(\textit{flowGraph}(e))]) \mid e \in$ 
       $\textit{caller}(\textit{flowGraph}(\mathbf{s})) \}$ ;
  IF  $\textit{inf}[\mathbf{s}] \sqsupseteq \textit{meet}$ 
    THEN
      inf[ $\mathbf{s}$ ] := meet;
      workset := workset  $\cup \{ \textit{start}(\textit{callee}(e)) \mid e \in E_{\textit{call}}$ 
         $\textit{flowGraph}(e) = \textit{flowGraph}(\mathbf{s}) \}$ 
    FI
  ESOOHC
OD;
```

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The 1st Order *IMaxFP*-Algorithm 14.1.5.3 – The “Functional” Main Process

(Epilogue)

FORALL $n \in N \setminus \{s_i \mid i \in \{0, \dots, k\}\}$ DO
 $inf[n] := \llbracket n \rrbracket (inf[start(flowGraph(n))])$ OD.

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Termination

Theorem (14.1.5.4, Termination)

The sequential composition of Algorithm 14.1.5.1 and Algorithm 14.1.5.2 resp. Algorithm 14.1.5.3 terminates with the IMaxFP solution, if the data flow analysis functional $\llbracket \cdot \rrbracket'$ is monotonic and the function lattice $[\mathcal{C} \rightarrow \mathcal{C}]$ satisfies the descending chain condition.

Note: The descending chain condition on $[\mathcal{C} \rightarrow \mathcal{C}]$ implies the descending chain condition on \mathcal{C} .

A 2nd Variant of the *IMaxFP*-Algorithm (1)

Partial instead of total computation of the semantics of the procedures:

- ▶ Unlike to the previous two algorithm variants, the new variant allows an interleaving of preprocess and main process.
- ▶ The computation starts with the main process algorithm.
- ▶ If a procedure call is encountered during the iterative process, the preprocess algorithm is started for this procedure and the current data flow fact.
- ▶ After completion of the computation of the effect of the procedure for this data flow fact, the main process algorithm is continued with the computed result.

A 2nd Variant of the *IMaxFP*-Algorithm (2)

Note:

- ▶ The computation of the semantics of the procedures is performed demand-drivenly.
- ▶ The semantics of procedures are only computed as far as necessary.
- ▶ Overall, this results in some efficiency gain in practice, which, however, is difficult to quantify.

Chapter 14.1.6

Applications

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Applications

- ▶ For the parameterless base setting the specifications of intraprocedural DFA problems can be reused unmodified.
- ▶ In order to be effective, the descending chain condition must hold both for the data flow analysis lattice and its corresponding function lattice.
- ▶ This condition holds in particular for all bitvector problems (availability of expressions, liveness of variables, reaching definitions, etc.) but not for simple constants. Therefore, weaker and simpler classes of constants are used interprocedurally, e.g., the set of [linear constants](#).

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The General Setting

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Outline

We extend our setting by adding

- ▶ Value parameters
- ▶ Local variables

This requires to adjust our program representations towards

- ▶ Flow graph systems (FGS) w/ value parameters and local variables
- ▶ Interprocedural flow graphs (IFG) w/ value parameters and local variables

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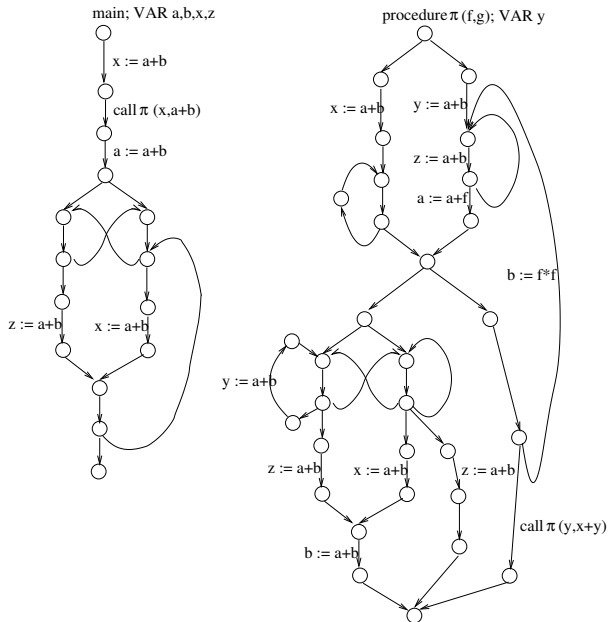
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FGS w/ Value Parameters and Local Variables



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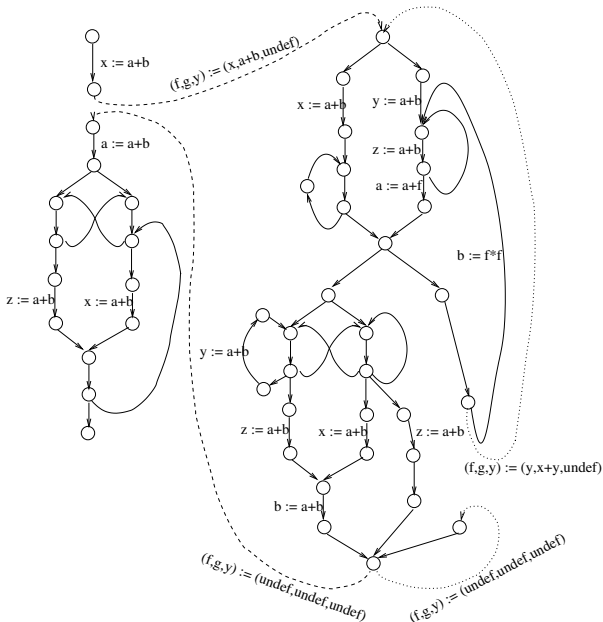
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IFG w/ Value Parameters and Local Variables



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New Phenomena

...related to procedures, value parameters, and local variables.

Conceptually most important:

- ▶ Existence of an unlimited number of copies (incarnations) of local variables and value parameters at run-time due to recursive procedures.
- ▶ After termination of a recursive procedure call the local variables and value parameters of the preceding not yet finished procedure call become valid again.
- ▶ The run-time system handles this phenomena by means of of a **run-time stack** which stores the **activation records** of the various procedure incarnations.

For program analysis, we have to take these phenomena into account and to model them properly.

Data Flow Analysis Stacks

Intuitively:

- ▶ DFA stacks are a compile-time equivalent of run-time stacks.
- ▶ Entries in DFA stacks are data flow facts of an underlying DFA lattices \mathcal{C} .
- ▶ We denote the set of all non-empty DFA stacks by *STACK*.

Manipulating DFA Stacks

DFA stacks can be manipulated by:

1. $\text{newstack} : \mathcal{C} \rightarrow \text{STACK}$
newstack(c) generates a new DFA stack with entry c .
2. $\text{push} : \text{STACK} \times \mathcal{C} \rightarrow \text{STACK}$
push stores a new entry on top of a DFA stack.
3. $\text{pop} : \text{STACK} \rightarrow \text{STACK}$
pop removes the top-most entry of a DFA stack.
4. $\text{top} : \text{STACK} \rightarrow \mathcal{C}$
top yields the contents of the top-most entry of a DFA stack w/out modifying the stack.

Remarks (1)

- ▶ The usual stack function `emptystack` : $\rightarrow STACK$ is replaced by `newstack`. Empty DFA stacks are not considered since they do not occur in interprocedural DFA.
- ▶ `push` and `pop` allow to manipulate the top-most entries of a DFA stack. This is different to and less flexible as for run-time stacks but suffices for interprocedural DFA.
- ▶ In fact, DFA stacks are only conceptually relevant, i.e., for the specifying, i.e., for the specifying *IMOP* approach but not for the algorithmic *IMaxFP* approach.

Remarks (2)

- ▶ Like run-time stacks DFA stacks store that part of the history of a program path that is relevant after finishing a procedure call.
- ▶ DFA stack entries can be considered abstractions of the activation records of procedure calls.
- ▶ The top-most entry of a DFA stack represents always the currently valid activation record (therefore, DFA stacks are never empty).
- ▶ Other than the top-most DFA stack entries represent the activation records of already started but not yet finished procedure calls.

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Local Abstract Semantics

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Basic Local Abstract Semantics

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Basic Local Abstract Semantics on DFA Lattice

1. DFA lattics $\hat{\mathcal{C}} = (\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \perp, \top)$
2. DFA functional $\llbracket \cdot \rrbracket' : E^* \rightarrow (\mathcal{C} \rightarrow \mathcal{C})$
3. Return functional $\mathcal{R} : E_{call} \rightarrow (\mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C})$

Induced Local Abstract Semantics

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Induced Local Abstract Semantics on DFA Stacks

- ▶ **DFA functional** $\llbracket \cdot \rrbracket^* : E^* \rightarrow (STACK \rightarrow STACK)$ on DFA stacks induced by a basic local abstract semantics that is defined by

$$\forall e \in E^* \forall stk \in STACK. \llbracket e \rrbracket^*(stk) =_{df}$$

$$\left\{ \begin{array}{ll} \text{push}(\text{pop}(stk), \llbracket e \rrbracket'(\text{top}(stk))) & \text{if } e \in E^* \setminus E_{call}^* \\ \text{push}(stk, \llbracket e \rrbracket'(\text{top}(stk))) & \text{if } e \in E_c^* \\ \text{push}(\text{pop}(\text{pop}(stk)), \mathcal{R}_e(\text{top}(\text{pop}(stk)), \llbracket e \rrbracket'(\text{top}(stk)))) & \text{if } e \in E_r^* \end{array} \right.$$

Notations related to DFA Stacks

- ▶ $STACK_{\geq i}$ ($STACK_{\leq i}$, etc.), $i \in \mathbb{N}$ denotes the set of all DFA Stacks w/ at last (at most, etc.) i entries (hence $STACK$ equals $STACK_{\geq 1}$).
- ▶ $STACK_i$, $i \in \mathbb{N}$, denotes the set of all DFA Stacks w/ exactly i entries.
- ▶ ϑ_{stk} denotes the number of entries of the DFA stack stk .
- ▶ stk_i , $1 \leq i \leq \vartheta_{stk}$, denotes the i th entry of the DFA stack stk .

Properties

Lemma (14.2.1.1)

Let $e \in E^*$ and $stk \in STK$. Then we have:

- $$\llbracket e \rrbracket^*(stk) \in \begin{cases} STK_{\vartheta_{stk}} & \text{if } e \in E^* \setminus E_{call}^* \\ STK_{\vartheta_{stk}+1} & \text{if } e \in E_c^* \\ STK_{\vartheta_{stk}-1} & \text{if } e \in E_r^* \wedge \vartheta_{stk} \geq 2 \end{cases}$$
- $pop(\llbracket e \rrbracket^*(stk)) = pop(stk)$, if $e \in E^* \setminus E_{call}^*$
- $pop(\llbracket e \rrbracket^*(stk)) = stk$, if $e \in E_c^*$
- $pop(\llbracket e \rrbracket_R^*(stk)) = pop(pop(stk))$, if $e \in E_r^* \wedge \vartheta_{stk} \geq 2$

The Structure of the Semantic Functions

All semantic functions occurring in interprocedural DFA are an element of the following subsets of the set of all functions $\mathcal{F} =_{df} [STACK \rightarrow STACK]$ on DFA stacks:

- ▶ \mathcal{F}_{ord}
- ▶ \mathcal{F}_{psh}
- ▶ \mathcal{F}_{pop}

These sets of functions are given by:

$$\mathcal{F}_{ord} =_{df} \{ f \in \mathcal{F} \mid \forall stk \in STACK. \text{pop}(f(stk)) = \text{pop}(stk) \}$$

$$\mathcal{F}_{psh} =_{df} \{ f \in \mathcal{F} \mid \forall stk \in STACK. \text{pop}(f(stk)) = stk \}$$

$$\mathcal{F}_{pop} =_{df} \{ f \in \mathcal{F} \mid \forall stk \in STACK_{\geq 2}. \text{pop}(f(stk)) = \text{pop}(\text{pop}(stk)) \}$$

Properties

Lemma (14.2.1.2)

$$\forall f_{pp} \in \mathcal{F}_{pop} \quad \forall f_o, f'_o \in \mathcal{F}_{ord} \quad \forall f_{ph} \in \mathcal{F}_{psh}.$$

1. $f_o \circ f'_o \in \mathcal{F}_{ord}$
2. $f_{pp} \circ f_o \circ f_{ph} \in \mathcal{F}_{ord}$

Lemma (14.2.1.3)

1. $\forall e \in E^* \setminus E_{call}^*. \llbracket e \rrbracket^* \in \mathcal{F}_{ord}$
2. $\forall e \in E_c^*. \llbracket e \rrbracket^* \in \mathcal{F}_{psh}$
3. $\forall e \in E_r^*. \llbracket e \rrbracket^* \in \mathcal{F}_{pop}$

The Significant Part of DFA Functions

Only the two top-most entries of DFA stacks are modified by DFA functions. This gives rise to the following definition:

Definition (14.2.1.4, Significant Part)

- ▶ $f \in \mathcal{F}_{ord} \cup \mathcal{F}_{psh}$: Then $f_s : \mathcal{C} \rightarrow \mathcal{C}$ is defined by:
 $f_s(c) =_{df} \text{top}(f(\text{newstack}(c)))$
- ▶ $f \in \mathcal{F}_{pop}$: Then $f_s : \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$ is defined by:
 $f_s(c_1, c_2) =_{df} \text{top}(f(\text{push}(\text{newstack}(c_1), c_2)))$
(Note that $\mathcal{C} \times \mathcal{C}$ is a lattice, if \mathcal{C} is a lattice.)

We have:

Lemma (14.2.1.5)

1. $\forall e \in E^* \setminus E_r^*. \llbracket e \rrbracket_s^* = \llbracket e \rrbracket'$
2. $\forall e \in E_r^* \forall c_1, c_2 \in \mathcal{C} \times \mathcal{C}. \llbracket e \rrbracket_s^* = \mathcal{R}_e(c_1, \llbracket e \rrbracket'(c_2))$

Properties

Lemma (14.2.1.6)

1. $\forall e \in E^* \setminus E_{call}^* \forall stk \in STK. \llbracket e \rrbracket^*(stk) = stk'$ with $\vartheta_{stk'} = \vartheta_{stk}$
and

$$\forall 1 \leq i \leq \vartheta_{stk'}. stk'_i =_{df} \begin{cases} stk_i & \text{if } i < \vartheta_{stk} \\ \llbracket e \rrbracket_s^*(stk_{\vartheta_{stk}}) & \text{if } i = \vartheta_{stk} \end{cases}$$

2. $\forall e \in E_c^* \forall stk \in STK. \llbracket e \rrbracket^*(stk) = stk'$ with $\vartheta_{stk'} = \vartheta_{stk} + 1$ and

$$\forall 1 \leq i \leq \vartheta_{stk'}. stk'_i =_{df} \begin{cases} stk_i & \text{if } i < \vartheta_{stk} + 1 \\ \llbracket e \rrbracket_s^*(stk_{\vartheta_{stk}}) & \text{if } i = \vartheta_{stk} + 1 \end{cases}$$

3. $\forall e \in E_r^* \forall stk \in STK_{\geq 2}. \llbracket e \rrbracket^*(stk) = stk'$ with $\vartheta_{stk'} = \vartheta_{stk} - 1$
and

$$\forall 1 \leq i \leq \vartheta_{stk'}. stk'_i =_{df} \begin{cases} stk_i & \text{if } i < \vartheta_{stk} - 1 \\ \llbracket e \rrbracket_s^*(stk_{\vartheta_{stk}-1}, stk_{\vartheta_{stk}}) & \text{if } i = \vartheta_{stk} - 1 \end{cases}$$

S-Monotonicity, S-Distributivity

Definition (14.2.1.7, S-Mon., S-Distrib.)

A DFA function $f \in \mathcal{F}_{ord} \cup \mathcal{F}_{psh} \cup \mathcal{F}_{pop}$ is

1. **s-monotonic** iff f_s is monotonic
2. **s-distributive** iff f_s is distributive

Properties

Lemma (14.2.1.8)

For all $e \in E^*$ the function $\llbracket e \rrbracket^*$ is s -monotonic (s -distributive), if

- ▶ $e \in E^* \setminus E_r^*$: $\llbracket e \rrbracket'$ is monotonic (distributive)
- ▶ $e \in E_r^*$: $\llbracket e \rrbracket'$ and \mathcal{R}_e are monotonic (distributive)

Conventions

In the following

- ▶ we consider s-monotonicity and s-distributivity as generalizations of the usual monotonicity and distributivity.

To this end, we

- ▶ identify lattice elements with their representation as a DFA stack with just a single entry.
- ▶ extend the meet and join operation \sqcap and \sqcup as follows to (the top most entries of) DFA stacks:

$$\sqcap STK =_{df} \text{newstack}(\sqcap \{ \text{top}(stk) \mid stk \in STK \})$$

$$\sqcup STK =_{df} \text{newstack}(\sqcup \{ \text{top}(stk) \mid stk \in STK \})$$

Chapter 14.2.2

The $IMOP_{Stk}$ Approach

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The $IMOP_{Stk}$ Approach

The $IMOP_{Stk}$ Solution:

$$\forall c_s \in \mathcal{C} \forall n \in \mathbb{N}. IMOP_{Stk\ c_s}(n) =_{df} \prod \{ \llbracket p \rrbracket^*(newstack(c_s)) \mid p \in \mathbf{IP}[s, n] \}$$

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Chapter 14.2.3

The $IMaxFP_{Stk}$ Approach

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The $IMaxFP_{Stk}$ Approach

...is a two-stage approach:

- ▶ Stage 1: Preprocess – Computing the Semantics of Procedures
- ▶ Stage 2: Main Process – Computing the $IMaxFP$ solution

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Preliminaries

Let

- ▶ Id_{STACK} denote the identity on $STACK$, and
- ▶ \sqcap the pointwise meet-operation on \mathcal{F}_{ord}

Note:

- ▶ $\forall f, f' \in \mathcal{F}_{ord}$. $f \sqcap f' =_{df} f'' \in \mathcal{F}_{ord}$ with $\forall stk \in STACK$. $topl(f''(stk)) = top(f(stk)) \sqcap top(f'(stk))$.
- ▶ “ \sqcap ” induces an inclusion relation “ \sqsubseteq ” on \mathcal{F}_{ord} by:
 $f \sqsubseteq f'$ gdw. $f \sqcap f' = f$.

The $IMaxFP_{Stk}$ Approach (1)

Stage 1: Preprocess – Computing the Semantics of Procedures:

The 2nd Order Equation System 14.2.3.1

$$\llbracket n \rrbracket = \begin{cases} Id_{STACK} & \text{if } n \in start(S) \\ \prod \{ \llbracket (m, n) \rrbracket \circ \llbracket m \rrbracket \mid m \in pred_{flowGraph}(n)(n) \} & \\ \text{otherwise} & \end{cases}$$

and

$$\llbracket e \rrbracket = \begin{cases} \llbracket e \rrbracket^* & \text{if } e \in E \setminus E_{call} \\ \llbracket e_r \rrbracket^* \circ \llbracket end(callee(e)) \rrbracket \circ \llbracket e_c \rrbracket^* & \text{otherwise} \end{cases}$$

The $IMaxFP_{Stk}$ Approach (2)

The 1st Order $IMaxFP_{Stk}$ Equation System 14.2.3.2:

$$inf(n) = \begin{cases} newstack(c_s) & \text{if } n = \mathbf{s}_0 \\ \bigcap \{ \llbracket e_c \rrbracket^*(inf(src(e))) \mid e \in caller(flowGraph(n)) \} & \text{falls } n \in start(S) \setminus \{ \mathbf{s}_0 \} \\ \bigcap \{ \llbracket (m, n) \rrbracket (inf(m)) \mid m \in pred_{flowGraph(n)}(n) \} & \text{otherwise} \end{cases}$$

The $IMaxFP_{Stk}$ Solution:

$$\forall c_s \in C \quad \forall n \in N. \quad IMaxFP_{Stk_{c_s}}(n) =_{df} inf_{c_s}^*(n)$$

Chapter 14.2.4

Main Results

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Towards the Main Results

Important:

Lemma (14.2.4.1)

For all $n \in N$ the semantic functions $\llbracket e \rrbracket^*$, $e \in E^*$, are

1. *s-monotonic*: $\llbracket n \rrbracket \sqsubseteq imop_n$
2. *s-distributive*: $\llbracket n \rrbracket = imop_n$

where $imop_n : N \rightarrow (STACK \rightarrow STACK)$ denotes a functional that is defined by:

$$\forall n \in N. imop_n =_{df} \begin{cases} Id_{STACK} & \text{if } n \in start(S) \\ \bigcap \{ \llbracket p \rrbracket^* \mid p \in \mathbf{CIP}[start(flowGraph(n)), n] \} & \text{otherwise} \end{cases}$$

Main Results – 1st Stage

Safety and coincidence results of the 2nd order 1st stage analysis:

Theorem (14.2.4.2, 2nd-Order)

For all $e \in E_{call}$ we have:

1. $\llbracket e \rrbracket \subseteq \bigcap \{ \llbracket p \rrbracket^* \mid p \in \mathbf{CIP}[src(e), dst(e)] \}$, if $\llbracket \cdot \rrbracket^*$ is *s-monotonic*.
2. $\llbracket e \rrbracket = \bigcap \{ \llbracket p \rrbracket^* \mid p \in \mathbf{CIP}[src(e), dst(e)] \}$, if $\llbracket \cdot \rrbracket^*$ is *s-distributive*.

where the mappings *src* and *dst* yield the start and final node of an edge.

Main Results – 2nd Stage

Safety and coincidence results of the 1st order 2nd stage analysis:

Theorem (14.2.4.3, Interprocedural Safety)

The $IMaxFP_{Stk}$ solution is a lower (i.e., safe) approximation of the $IMOP_{Stk}$ solution, i.e.

$$\forall c_s \in \mathcal{C} \forall n \in \mathbb{N}. IMaxFP_{Stk_{c_s}}(n) \sqsubseteq IMOP_{Stk_{c_s}}(n)$$

if $\llbracket \cdot \rrbracket^*$ is *s-monotonic*.

Theorem (14.2.4.4, Interprocedural Coincidence)

The $IMaxFP_{Stk}$ solution coincides with the $IMOP_{Stk}$ solution, i.e.

$$\forall c_s \in \mathcal{C} \forall n \in \mathbb{N}. IMaxFP_{Stk_{c_s}}(n) = IMOP_{Stk_{c_s}}(n)$$

if $\llbracket \cdot \rrbracket^*$ is *s-distributive*.

Chapter 14.2.5

Algorithms

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Algorithms

- ▶ The algorithms of Chapter 14.1.5 can straightforwardly be extended to stack-based functions.
- ▶ This way we receive
 - ▶ the standard variant of pre- and post-process
 - ▶ the more efficient variant of pre- and functional main process.
 - ▶ a demand-driven “by-need” variant.
- ▶ In the following we present another stackless variant. The clou of this variant is that stacks have at most 2 entries during analysis time.

Therefore, a single temporary storing the temporarily existing stack entry during procedure calls is sufficient for the implementation.

The Stackless 2nd Order Algorithm 14.2.5.1 – Preprocess

Input: (1) A flow-graph system S , and (2) an abstract semantics consisting of a data-flow lattice \mathcal{C} , and a data-flow functional $\llbracket \cdot \rrbracket' : E^* \rightarrow (\mathcal{C} \rightarrow \mathcal{C})$.

Output: Under the assumption of termination (cf. Theorem 14.2.5.4), an annotation of S with functions $\llbracket n \rrbracket : \mathcal{C} \rightarrow \mathcal{C}$ (stored in gtr , which stands for *global transformation*), and $\llbracket e \rrbracket : \mathcal{C} \rightarrow \mathcal{C}$ (stored in ltr , which stands for *local transformation*) representing the greatest solution of Equation System 14.2.3.1.

Remark: The variable *workset* controls the iterative process. Its elements are nodes of the flow-graph system S . Note that due to the mutual interdependence of the definitions of $\llbracket \cdot \rrbracket$ and $\llbracket \cdot \rrbracket'$ the iterative approximation of $\llbracket \cdot \rrbracket$ is superposed by an interprocedural iteration step, which updates the current approximation of the effect $\llbracket \cdot \rrbracket'$ of call edges. The temporary *meet* stores the result of the most recent meet operation.

The Stackless 2nd Order Algorithm 14.2.5.1 – Preprocess

(Prologue: Initialization of the annotation arrays gtr and ltr and the variable $workset$)

FORALL $n \in N$ DO

IF $n \in \{s_0, \dots, s_k\}$ THEN $gtr[n] := Id_C$
ELSE $gtr[n] := \top_{[C \rightarrow C]}$ FI OD;

FORALL $e \in E$ DO

IF $e \in E_{call}$ THEN $ltr[e] := \llbracket e_r \rrbracket' \circ \top_{[C \rightarrow C]} \circ \llbracket e_c \rrbracket'$
ELSE $ltr[e] := \llbracket e \rrbracket'$ FI OD; (★)

$workset := \{s_0, \dots, s_k\}$;

The Stackless 2nd Order Algorithm 14.2.5.1 – Preprocess

(Main process: Iterative fixed point computation)

WHILE $workset \neq \emptyset$ DO

 CHOOSE $m \in workset$;

$workset := workset \setminus \{m\}$;

 (Update the successor-environment of node m)

 IF $m \in \{e_1, \dots, e_k\}$

 THEN

 FORALL $e \in caller(flowGraph(m))$ DO

$ltr[e] := \mathcal{R}_e \circ (Id_C, \llbracket e_r \rrbracket' \circ gtr[m] \circ \llbracket e_c \rrbracket')$; $\langle \star \rangle$

$meet := ltr[e] \circ gtr[src(e)] \sqcap gtr[dst(e)]$;

 IF $gtr[dst(e)] \sqsupset meet$

 THEN

$gtr[dst(e)] := meet$;

$workset := workset \cup \{dst(e)\}$

 FI

 OD

The Stackless 2nd Order Algorithm 14.2.5.1 – Preprocess

```
ELSE (i.e.,  $m \notin \{e_1, \dots, e_k\}$ )
  FORALL  $n \in \text{succ}_{\text{flowGraph}(m)}(m)$  DO
     $\text{meet} := \text{ltr}[(m, n)] \circ \text{gtr}[m] \sqcap \text{gtr}[n]$ ;
    IF  $\text{gtr}[n] \sqsupseteq \text{meet}$ 
      THEN
         $\text{gtr}[n] := \text{meet}$ ;
         $\text{workset} := \text{workset} \cup \{n\}$ 
      FI
    OD
  FI
ESOOHC
OD.
```

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The Stackless 1st Order Algorithm 14.2.5.2 – Main Process

Input: (1) A flow-graph system S , (2) an abstract semantics consisting of a data-flow lattice \mathcal{C} , and a data-flow functional $\llbracket \cdot \rrbracket$ computed by Algorithm 14.5.2.1, and (3) a context information $c_s \in \mathcal{C}$.

Output: Under the assumption of termination (cf. Theorem 14.5.2.4), the $IMaxFP_{StkLSS}$ -solution. Depending on the properties of the data-flow functional, this has the following interpretation:

(1) $\llbracket \cdot \rrbracket$ is *distributive*: variable *inf* stores for every node the strongest component information valid there with respect to the context information c_s .

(2) $\llbracket \cdot \rrbracket$ is *monotonic*: variable *inf* stores for every node a valid component information with respect to the context information c_s , i.e., a lower bound of the strongest component information valid there.

Remark: The variable *workset* controls the iterative process. Its elements are nodes of the flow-graph system S . The temporary *meet* stores the result of the most recent meet operation.

The Stackless 1st Order Algorithm 14.2.5.2 – Main Process

(Prologue: Initialization of the annotation array *inf* and the variable *workset*)

FORALL $n \in N \setminus \{s_0\}$ DO $inf[n] := \top$ OD;

$inf[s_0] := c_s$;

$workset := \{s_0\}$;

(Main process: Iterative fixed point computation)

WHILE $workset \neq \emptyset$ DO

 CHOOSE $m \in workset$;

$workset := workset \setminus \{m\}$;

 (Update the successor-environment of node m)

 FORALL $n \in succ_{flowGraph(m)}(m)$ DO

$meet := \llbracket (m, n) \rrbracket (inf[m]) \sqcap inf[n]$;

 IF $inf[n] \sqsupset meet$

 THEN

$inf[n] := meet$;

$workset := workset \cup \{n\}$ FI;

The Stackless 1st Order Algorithm 14.2.5.2 – Main Process

```
IF  $(m, n) \in E_{call}$ 
  THEN
     $meet := \llbracket (m, n)_c \rrbracket'(\inf[m]) \sqcap$ 
       $\inf[start(callee((m, n)))]$ ;           $\langle \star \rangle$ 
  IF  $(m, n) \in E_{call}$ 
    THEN
       $meet := \llbracket (m, n)_c \rrbracket'(\inf[m]) \sqcap$ 
         $\inf[start(callee((m, n)))]$ ;           $\langle \star \rangle$ 
      IF  $\inf[start(callee((m, n)))] \sqsupseteq meet$ 
        THEN
           $\inf[start(callee((m, n)))] := meet$ ;
           $workset := workset \cup \{ start(callee((m, n))) \}$ 
        FI
      FI
    FI
  FI
OD
ESOOHC OD.
```

The Stackless 1st Order Algorithm 14.2.5.3 – “Functional” Main Process

Input: (1) A flow-graph system S , (2) an abstract semantics consisting of a data-flow lattice \mathcal{C} , and the data-flow functionals $\llbracket \cdot \rrbracket =_{df} gtr$ and $\llbracket \cdot \rrbracket =_{df} ltr$ with respect to \mathcal{C} (computed by Algorithm 14.5.2.1), and (4) a context information $c_s \in \mathcal{C}$.

Output: Under the assumption of termination (cf. Theorem 14.5.2.4), the $IMaxFP_{StkLSS}$ -solution. Depending on the properties of the data-flow functional, this has the following interpretation:

- (1) $\llbracket \cdot \rrbracket$ is **distributive**: variable *inf* stores for every node the strongest component information valid there with respect to the context information c_s .
- (2) $\llbracket \cdot \rrbracket$ is **monotonic**: variable *inf* stores for every node a valid component information with respect to the context information c_s , i.e., a lower bound of the strongest component information valid there.

Remark: The variable *workset* controls the iterative process, and the temporary *meet* stores the most recent approximation.

The Stackless 1st Order Algorithm 14.2.5.3 – “Functional” Main Process

(Prologue: Initialization of the annotation array *inf*, and the variable *workset*)

```
FORALL  $\mathbf{s} \in \{\mathbf{s}_i \mid i \in \{1, \dots, k\}\}$  DO  $inf[\mathbf{s}] := \top$  OD;  
 $inf[\mathbf{s}_0] := c_{\mathbf{s}}$ ;  
 $workset := \{\mathbf{s}_i \mid i \in \{1, 2, \dots, k\}\}$ ;
```

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The Stackless 1st Order Algorithm 14.2.5.3 – “Functional” Main Process

(Main process: Iterative fixed point computation)

```
WHILE  $workset \neq \emptyset$  DO
  CHOOSE  $s \in workset$ ;
   $workset := workset \setminus \{s\}$ ;
   $meet := \inf[s] \sqcap$ 
     $\sqcap \{ \llbracket e_c \rrbracket' \circ \llbracket src(e) \rrbracket (\inf[start(flowGraph(e))]) \mid$ 
       $e \in caller(flowGraph(s)) \}$ ;   $\langle \star \rangle$ 
  IF  $\inf[s] \sqsupseteq meet$ 
    THEN
       $\inf[s] := meet$ ;
       $workset := workset \cup$ 
         $\{ start(callee(e)) \mid e \in E_{call}. \}$ 
         $flowGraph(e) = flowGraph(s) \}$ 
    FI
ESOOHC
OD;
```

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The Stackless 1st Order Algorithm 14.2.5.3 – “Functional” Main Process

(Epilogue)

```
FORALL  $n \in N \setminus \{s_i \mid i \in \{0, \dots, k\}\}$  DO  
   $inf[n] := \llbracket n \rrbracket (inf[start(flowGraph(n))])$   
OD.
```

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Termination

Theorem (14.2.5.4, Termination)

The sequential composition of Algorithm 14.2.5.1 and Algorithm 14.2.5.2 resp. Algorithm 14.2.5.3 terminates with the $I\text{MaxFP}_{\text{Stk}}$ solution, if the DFA functional $\llbracket \cdot \rrbracket'$ and the return functional \mathcal{R} are monotonic and the lattice of functions $[\mathcal{C} \rightarrow \mathcal{C}]$ satisfies the descending chain condition.

Note: If $[\mathcal{C} \rightarrow \mathcal{C}]$ satisfies the descending chain condition, then \mathcal{C} does so as well.

Chapter 14.3

Extensions

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Extensions

- ▶ Further parameter transfer mechanisms
 - ▶ Reference parameters
 - ▶ Procedural parameters, for short: procedure parameters
- ▶ Static nesting of procedures

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Reference Parameters

Intuitively:

- ▶ The effect of reference parameters is encoded in the local semantic functionals of the application problems.
- ▶ Reference parameters can thus be handled and computed by suitable preprocess computing may and must aliases of variables and parameters.
- ▶ The computed alias information is then fed into the definitions of the local semantics functions of the application problems (cf. Chapter 14.4)

Procedure Parameter

Intuitively:

- ▶ A formal procedure call is replaced by the set of all ordinary procedure calls that it may call.
- ▶ This set of procedures can be computed by a suitable preprocess; dependingy on the program or programming language class this can be either a safe approximation or an exact solution.
- ▶ The computed calling information for formal procedure call reduces then the analysis of programs w/ formal procedure calls to the analysis of programs w/out formal procedure calls.

Static Procedure Nesting

Various variants are possible:

- ▶ De-nesting of procedures by a suitable preprocess; this way the analysis of programs w/ static procedure nesting is reduced to analysing programs w/out static procedure nesting.
- ▶ Taking into account the effect of relatively global variables in the definition of the local semantics functions of the application problems.

Chapter 14.4

Applications

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Preliminaries

In the following we assume:

- ▶ No static procedure nesting, no procedure parameters.
- ▶ $MstAliases_G(v)$ und $MayAliases_G(v)$ denote the sets of **must-Aliases** and **may-Aliases** different from v .

These notions can straightforward be extended to terms t :

- ▶ A term t' is a must-alias (may-alias) of t , if t' results from t by replacing of variables by variables that are must-aliases (may-aliases) of each other.

This allows us to feed alias information in a parameterized fashion into the definitions of DFA functionals and return functionals and to take their effects during the analysis into account.

Notations (1)

- ▶ $GlobVar(S)$: the set of *global variables* of S , i.e., the set of variables which are declared in the main program of S . They are accessible in each procedure of S .
- ▶ $Var(t)$: the set of variables occurring in t .
- ▶ $LhsVar(e)$: the left hand side variable of the assignment of edge e .
- ▶ $GlobId(t)$ and $LocId(t)$: abbreviations of $GlobVar(S) \cap Var(t)$ and $Var(t) \setminus GlobVar(S)$.

Notations (2)

- ▶ *NoGlobalChanges* : $E^* \rightarrow \mathbb{B}$: indicates that if a variable $v \in \text{Var}(t)$ is modified by e , then this modification will not be visible after finishing the call as the relevant memory location of v is local for the currently active call.
- ▶ *PotAccessible* : $S \rightarrow \mathbb{B}$: indicates that the memory locations of all variables $v \in \text{Var}(t)$, which are accessible immediately before entering G remain accessible after entering it, either by referring to v itself or by referring to one of its must-aliases.

Local Predicates

The definition of the preceding functions relies on the predicates $Transp_{LocId}$ and $Transp_{GlobId}$ that are defined as follows:

$$Transp_{LocId}(e) =_{df} \\ LocId(t) \cap MayAliases_{flowGraph(e)}(LhsVar(e)) = \emptyset$$

$$Transp_{GlobId}(e) =_{df} GlobId(t) \cap \\ (LhsVar(e) \cup MayAliases_{flowGraph(e)}(LhsVar(e))) = \emptyset$$

This allows us to define:

$$\forall e \in E^*. NoGlobalChanges(e) \\ =_{df} \begin{cases} \mathbf{true} & \text{if } e \in E_c^* \cup E_r^* \\ Transp_{LocId}(n) \wedge Transp_{GlobId}(n) & \text{otherwise} \end{cases}$$

Parameterized Local Predicates (1)

...parameterized wrt alias information:

$$\forall e \in E^*. A\text{-Comp}_e =_{df} \text{Comp}_e \vee \text{Comp}_e^{MstAI}$$
$$\forall e \in E^*. A\text{-Transp}_e =_{df} \text{Transp}_e \wedge \begin{cases} \mathbf{true} & \text{if } e \in E_{call}^* \\ \text{Transp}_e^{MayAI} & \text{otherwise} \end{cases}$$

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Parameterized Local Predicates (2)

Intuitively:

- ▶ $A\text{-Comp}_e$ is true for t , if t itself (i.e., Comp_e) or one of its must-aliases is computed at edge e (i.e., $\text{Comp}_e^{\text{MstAl}}$).
- ▶ $A\text{-Transp}_e$, $e \in E^* \setminus E_{\text{call}}^*$, is true, if neither an operand of t (i.e., Transp_e) nor one of its may-aliases is modified by the statement at edge e (i.e., $\text{Transp}_e^{\text{MayAl}}$).
- ▶ For call and return edges $e \in E_{\text{call}}^*$, $A\text{-Transp}_e$ is true, if no operand of t is modified (i.e., Transp_e). This makes the difference between ordinary assignments and reference parameters and parameter transfers to reference parameters; the latter are updates of pointers that leave the memory invariant except of that update.

Remark

- ▶ $\mathcal{B}_X =_{df} \{\mathbf{false}, \mathbf{true}, failure\}$

Note: The element *failure* is introduced as an artificial \top -element in \mathbb{B} in order to be prepared for reverse data flow analysis as required for demand-driven data flow analysis (cf. LVA 185.276 Analysis and Verification).

Interprocedural Availability (1)

Local Abstract Semantics:

1. Data flow lattice:

$$(\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \perp, \top) =_{df} (\mathcal{B}_X^2, \wedge, \vee, \leq, (\mathbf{false}, \mathbf{false}), (failure, failure))$$

2. Data flow functional: $\llbracket \cdot \rrbracket'_{av} : E^* \rightarrow (\mathcal{B}_X^2 \rightarrow \mathcal{B}_X^2)$ defined by

$$\forall e \in E^* \forall (b_1, b_2) \in \mathcal{B}_X^2. \llbracket e \rrbracket'_{av}(b_1, b_2) =_{df} (b'_1, b'_2)$$

where

$$b'_1 =_{df} A\text{-Transp}_e \wedge (A\text{-Comp}_e \vee b_1)$$

$$b'_2 =_{df} \begin{cases} b_2 \wedge \text{NoGlobalChanges}_e & \text{if } e \in E^* \setminus E_c \\ \mathbf{true} & \text{otherwise} \end{cases}$$

Interprocedural Availability (2)

3. **Return functional:** $\mathcal{R}_{av} : E_{call} \rightarrow (\mathcal{B}_X^2 \times \mathcal{B}_X^2 \rightarrow \mathcal{B}_X^2)$
defined by $\forall e \in E_{call} \forall ((b_1, b_2), (b_3, b_4)) \in \mathcal{B}_X^2 \times \mathcal{B}_X^2$.
 $\mathcal{R}_{av}(e)((b_1, b_2), (b_3, b_4)) =_{df} (b_5, b_6)$ where

$$b_5 =_{df} \begin{cases} b_3 & \text{if } PotAccessible(callee(e)) \\ (b_1 \vee A-Comp_e) \wedge b_4 & \text{otherwise} \end{cases}$$

$$b_6 =_{df} b_2 \wedge b_4$$

Interprocedural Availability (3)

Lemma (14.4.1)

1. *The lattice \mathcal{B}_X^2 and the induced lattice of functions satisfy the descending chain condition.*
2. *The functionals $\llbracket \rrbracket'_{av}$ and \mathcal{R}_{av} are distributive.*

\rightsquigarrow Hence, the preconditions of the Interprocedural Coincidence Theorem 14.2.4.4 and the Termination Theorem 14.2.5.4 are satisfied.

Interprocedural Simple Constants

Local Abstract Semantics:

1. Data flow lattice:

$$(\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \perp, \top) =_{df} (\Sigma_X, \sqcap, \sqcup, \sqsubseteq, \sigma_\perp, \sigma_{failure})$$

2. Data flow functional: $\llbracket \cdot \rrbracket'_{sc} : E \rightarrow (\Sigma_X \rightarrow \Sigma_X)$ defined by

$$\forall e \in E. \llbracket e \rrbracket'_{sc} =_{df} \theta_e$$

3. Return functional: $\mathcal{R}_{sc} : E_{call} \rightarrow (\Sigma_X \times \Sigma_X \rightarrow \Sigma_X)$ defined by

$$\forall e \in E_{call} \forall (\sigma_1, \sigma_2) \in \Sigma_X \times \Sigma_X. \mathcal{R}_{sc}(e)(\sigma_1, \sigma_2) =_{df} \sigma_3$$

where

$$\forall x \in Var. \sigma_3(x) =_{df} \begin{cases} \sigma_2(x) & \text{if } x \in GlobVar(S) \\ \sigma_1(x) & \text{otherwise} \end{cases}$$

Problems and Solutions/Work-Arounds

In practice

- ▶ the preceding analysis specification for simple interprocedural constants does not induce a terminating analysis since the lattice of functions does not satisfy the descending chain condition
- ▶ thus simpler constant propagation problems are considered like **copy constants** and **linear constants**.

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Copy Constants and Linear Constants

A term is a

- ▶ **copy constant** at a program point, if it is a source-code constant or an operator-less term that is itself a copy constant
- ▶ **linear constant** at a program point, if it is a source-code constant or of the form $a * x + b$ w/ a, b source-code constants and x a linear constant.

Interprocedural Copy Constants (1)

The specification of copy constants is based on the following simpler evaluation function of terms:

$$\mathcal{E}_{cc} : \mathbf{T} \rightarrow (\Sigma_X \rightarrow \mathbf{D})$$

\mathcal{E}_{cc} is undefined for the failure state $\sigma_{failure}$; otherwise it is defined as follows:

$$\forall t \in \mathbf{T} \forall \sigma \in \Sigma. \mathcal{E}_{cc}(t)(\sigma) =_{df} \begin{cases} \sigma(x) & \text{if } t = x \in \mathbf{V} \\ l_0(c) & \text{if } t = c \in \mathbf{C} \\ \perp & \text{otherwise} \end{cases}$$

Note that Σ_X is analogously to \mathcal{B}_X extended by an artificial top-element.

Interprocedural Copy Constants (2)

- ▶ Replacing θ_l in \mathcal{E} by \mathcal{E}_{cc} yields the data flow analysis functional $\llbracket \cdot \rrbracket'_{cc}$.
- ▶ Replacing of $\llbracket \cdot \rrbracket'_{sc}$ by $\llbracket \cdot \rrbracket'_{cc}$ yields the definition of the local abstract semantics of interprocedural copy constants.

Interprocedural Copy Constants (3)

Note:

- ▶ The number of source-code constants is finite.
- ▶ Hence, the lattice of functions that belongs to the relevant sublattice Σ_{cc_X} of Σ_X satisfies the descending chain condition.
- ▶ Thus, the *IMaxFP* algorithm terminates.
- ▶ Unlike as interprocedural simple constants are copy constants distributive; thus, the *IMaxFP*_{Stk} solution and the *IMOP*_{Stk} solution coincide.

Interprocedural Copy Constants (4)

Lemma (14.4.2)

1. *The lattice Σ_{ccx} and the induced lattice of functions satisfy the descending chain condition.*
2. *The functionals $\llbracket \rrbracket'_{cc}$ and \mathcal{R}_{cc} are distributive.*

Therefore, the preconditions of the Interprocedural Coincidence Theorem 14.2.4.4 and the Termination Theorem 14.2.5.4 are satisfied.

Chapter 14.5

Interprocedural DFA: Framework and Toolkit

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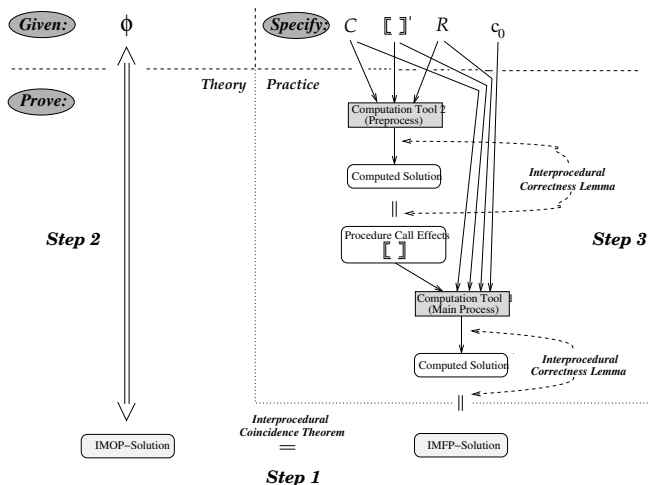
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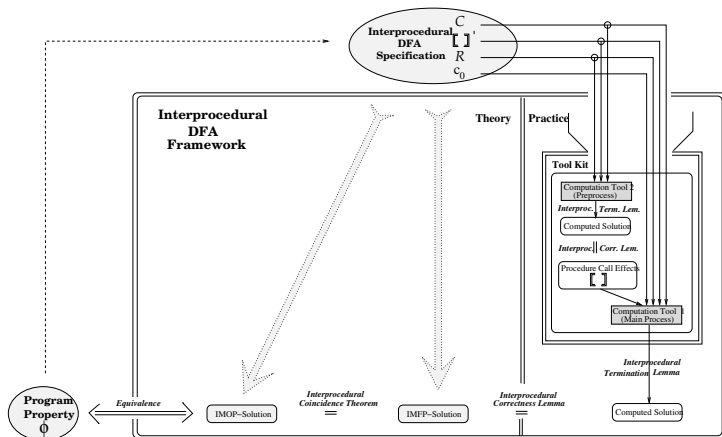
Interprocedural DFA: The Framework View

The **interprocedural** DFA Framework at a glance:



Interprocedural DFA: The Toolkit View

The Toolkit View of the **interprocedural** DFA Framework:



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Further Reading for Chapter 14 (1)

-  Alfred V. Aho, Monica S. Lam, Ravi Sethi, Jeffrey D. Ullman. *Compilers: Principles, Techniques, & Tools*. Addison-Wesley, 2nd edition, 2007. (Chapter 12, Interprocedural Analysis)
-  Randy Allen, Ken Kennedy. *Optimizing Compilers for Modern Architectures*. Morgan Kaufman Publishers, 2002. (Chapter 11, Interprocedural Analysis and Optimization)
-  Jens Knoop. *Optimal Interprocedural Program Optimization: A New Framework and Its Application*. Springer-V., LNCS 1428, 1998. (Chapter 10, Interprocedural Code Motion: The Transformations, Chapter 11, Interprocedural Code Motion: The IDFA-Algorithms)

Further Reading for Chapter 14 (2)

-  Jens Knoop. *Formal Callability and its Relevance and Application to Interprocedural Data Flow Analysis*. In Proceedings of the 6th IEEE International Conference on Computer Languages (ICCL'98), 252-261, 1998.
-  Jens Knoop. *From DFA-Frameworks to DFA-Generators: A Unifying Multiparadigm Approach*. In Proceedings of the 5th International Conference on Tools and Algorithms for the Construction and Analysis of Systems (TACAS'99), Springer-V., LNCS 1579, 360-374, 1999.
-  Jens Knoop, Bernhard Steffen. *The Interprocedural Coincidence Theorem*. In Proceedings of the 4th International Conference on Compiler Construction (CC'92), Springer-V., LNCS 641, 125-140, 1992.

Further Reading for Chapter 14 (3)

-  Stephen S. Muchnick. *Advanced Compiler Design Implementation*. Morgan Kaufman Publishers, 1997. (Chapter 19, Interprocedural Analysis and Optimization)
-  Mooly Sagiv, Tom Reps, Susan Horwitz. *Precise Interprocedural Dataflow Analysis with Applications to Constant Propagation*. In Proceedings TAPSOFT'95, Springer-V., LNCS 915, 651-665, 1995.
-  Micha Sharir, Amir Pnueli. *Two Approaches to Interprocedural Data Flow Analysis*. In Stephen S. Muchnick, Neil D. Jones (Eds.). *Program Flow Analysis: Theory and Applications*. Prentice Hall, 1981, Chapter 7.3, The Functional Approach to Interprocedural Analysis, 196-209.

Part IV

Extensions, Other Settings

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Chapter 15

Aliasing

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Chapter 15.1

Sources of Aliasing

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Aliasing Everywhere

Answers to the question “What is an alias?” in different areas:

- ▶ A short, easy to remember name created for use in place of a longer, more complicated name; commonly used in e-mail applications. Also referred to as a “nickname”.
- ▶ A hostname that replaces another hostname, such as an alias which is another name for the same Internet address. For example, `www.company.com` could be an alias for `server03.company.com`.
- ▶ A feature of UNIX shells that enables users to define program names (and parameters) and commands with abbreviations. (e.g. alias `ls 'ls -l'`)
- ▶ In MGI (Mouse Genome Informatics), an alternative symbol or name for part of the sequence of a known gene that resembles names for other anonymous DNA segments. For example, `D6Mit236` is an alias for `Cftr`.

Aliasing in Programs

In programs aliasing occurs when there exists **more than one access path** to a storage location.

An **access path** is the l-value of an expression that is constructed from variables, pointer dereference operators, and structure field operation operators.

Java (References)

```
A a,b;  
a = new A();  
b = a;  
b.val = 0;
```

C++ (Pointers)

```
A* a; A* b;  
a = new A();  
b = a;  
b->val = 0;
```

C++ (References)

```
A& a = *new A();  
A& b = a;  
b.val = 0;
```

C (Pointers)

```
A *a, *b;  
a = (A*)malloc(sizeof(A));  
b = a;  
b->val = 0;
```

Examples of Different Forms of Aliasing (1)

Fortran 77

EQUIVALENCE statement can be used to specify that two or more scalar variables, array variables, and/or contiguous portions of array variables begin at the same storage location.

Pascal, Modula 2/3, Java

- ▶ Variable of a reference type is restricted to have either the value nil/null or to refer to objects of a particular specified type.
- ▶ An object may be accessible through several references at once, but it cannot both have its own variable name *and* be accessible through a pointer.

Examples of Different Forms of Aliasing (2)

C/C++

- ▶ The union type specifier allows to create static aliases. A union type may have several fields declared, all of which overlap in (= share) storage.
- ▶ It is legal to compute the address of an object with the & operator (statically, automatically, or dynamically allocated).
- ▶ Allows arithmetic on pointers and considers it equivalent to array indexing

Chapter 15.2

Relevance of Aliasing for Program Optimization

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Relevance of Alias Analysis to Optimization

Alias analysis refers to the determination of storage locations that may be accessed in two or more ways.

- ▶ Ambiguous memory references interfere with an optimizer's ability to improve code.
- ▶ One major source of ambiguity is the use of pointer-based values.

Goal: determine for each pointer the set of memory locations to which it may refer.

Without alias analysis the compiler must assume that each pointer can refer to any addressable value, including

- ▶ any space allocated in the run-time heap
- ▶ any variable whose address is explicitly taken
- ▶ any variable passed as a call-by-reference parameter

Characterization of Aliasing

Flow-insensitive information

Binary relation on the variables in a procedure, $alias \in Var \times Var$ such that $x \text{ alias } y$ if and only if x and y

- ▶ **may** possibly at different times refer to the same memory location.
- ▶ **must** throughout the execution of the procedure refer to the same memory location.

Flow-sensitive information

A function from program points and variables to sets of abstract storage locations. $alias(p, v) = Loc$ means that at program point p variable v

- ▶ **may** refer to any of the locations in Loc .
- ▶ **must** refer to the location $l \in Loc$ with $|Loc| \leq 1$.

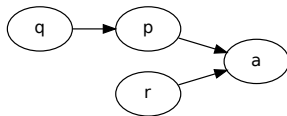
Representation of Alias Information

Representation of aliasing with pairs

	<code>q=&p; p=&a; r=&a;</code>
complete alias pairs	<code><*q,p>, <*p,a>, <*r,a>, <**q,*p>, <**q,a>, <*p,*r>, <**q,*r></code>
compact alias pairs	<code><*q,p>, <*p,a>, <*r,a></code>
points-to relations	<code>(q,p), (p,a), (r,a)</code>

Representation of aliases and shapes of data structures

- ▶ graphs
- ▶ regular expressions
- ▶ 3-valued logic



Chapter 15.3

Shape Analysis

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Questions about Heap Contents (1)

Execution State

Let *execution state* mean the set of cells in the heap, the connections between them (via pointer components of heap cells) and the values of pointer variables in the store.

NULL pointers (Question 1)

Does a pointer variable or a pointer component of a heap cell contain NULL at the entry to a statement that dereferences the pointer or component?

- ▶ Yes (for every state). Issue an error message
- ▶ No (for every state). Eliminate a check for NULL.
- ▶ Maybe. Warn about the potential NULL dereference.

Questions about Heap Contents (2)

Memory leak (Question 2)

Does a procedure or a program leave behind unreachable heap cells when it returns?

- ▶ Yes (in some state). Issue a warning.

Aliasing (Question 3)

Do two pointer expressions reference the same heap cell?

- ▶ Yes (for every state).
 - ▶ trigger a prefetch to improve cache performance
 - ▶ predict a cache hit to improve cache behavior prediction
 - ▶ increase the sets of uses and definitions for an improved liveness analysis
- ▶ No (for every state). Disambiguate memory references and improve program dependence information.

Questions about Heap Contents (3)

Sharing (Question 4)

Is a heap cell shared? (within the heap)

- ▶ Yes (for some state). Warn about explicit deallocation, because the memory manager may run into an inconsistent state.
- ▶ No (for every state). Explicitly deallocate the heap cell when the last pointer to ceases to exist.

Reachability (Question 5)

Is a heap cell reachable from a specific variable or from any pointer variable?

- ▶ Yes (for every state). Information for program verification.
- ▶ No (for every state). Insert code at compile time that collects unreachable cells at run-time.

Questions about Heap Contents (4)

Disjointness (Question 6)

Do two data structures pointed to by two distinct pointer variables ever have common elements?

- ▶ No (for every state). Distribute disjoint data structures and their computations to different processors.

Cyclicity (Question 7)

Is a heap cell part of a cycle?

- ▶ No (for every state). Perform garbage collection of data structures by reference counting. Process all elements in an acyclic linked list in a doall-parallel fashion.

Shape Analysis

Aim of Shape Analysis

The aim of shape analysis is to determine a finite representation of heap allocated data structures which can grow arbitrarily large.

It can determine the possible shapes data structures may take such as:

- ▶ lists
- ▶ trees
- ▶ directed acyclic graphs
- ▶ arbitrary graphs
- ▶ properties such as whether a data structure is or may be cyclic

As example we shall discuss a precise shape analysis (from PoPA Ch 2.6) that performs strong update and uses shape graphs to represent heap allocated data structures. It emphasises the analysis of list like data structures.

Strong Update

Here “strong” means that an update or nullification of a pointer expression allows one to *remove* (kill) the existing binding before adding a new one (gen).

We shall study a powerful analysis that achieves

- ▶ Strong nullification
- ▶ Strong update

for destructive updates that destroy (overwrite) existing values in pointer variables and in heap allocated data structures in general.

Examples:

- ▶ $[x := nil]^\ell$
- ▶ $[x.sel_1 := y.sel_2]^\ell$

Extending the WHILE Language

We extend the WHILE-language syntax with constructs that allow to create cells in the heap.

- ▶ the cells are structured and may contain values as well as pointers to other cells
- ▶ the data stored in cells is accessed via selectors; we assume that a finite and non-empty set Sel of selector names is given:

$sel \in Sel$ selector names

- ▶ we add a new syntactic category

$p \in PExp$ pointer expressions

- ▶ op_r is extended to allow for testing of equality of pointers
- ▶ unary operations op_p on pointers (e.g. is-null) are added

Abstract Syntax of Pointer Language

The syntax of the while language is extended to have:

$$\begin{aligned} p &::= x \mid x.sel \mid \text{null} \\ a &::= x \mid n \mid a_1 \text{ op}_a a_2 \\ b &::= \text{true} \mid \text{false} \mid \text{not } b \mid b_1 \text{ op}_b b_2 \mid a_1 \text{ op}_r a_2 \\ S &::= [p:=a]^\ell \mid [\text{skip}]^\ell \\ &\quad \mid \text{if } [b]^\ell \text{ then } S_1 \text{ else } S_2 \\ &\quad \mid \text{while } [b]^\ell \text{ do } S \text{ od} \\ &\quad \mid [\text{new } (p)]^\ell \\ &\quad \mid S_1; S_2 \end{aligned}$$

In the case where p contains a selector we have a **destructive update** of the heap. Statement **new** **creates** a new cell pointed to by p .

Shape Graphs

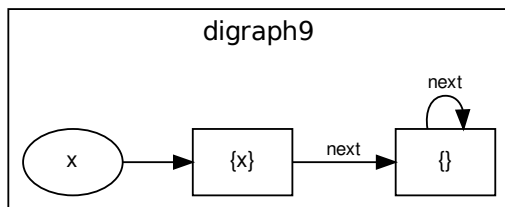
We shall introduce a method for combining the locations of the semantics into a finite number of *abstract locations*.

The analysis operates on shape graphs (S, H, is) consisting of:

- ▶ an abstract state, S (mapping variables to abstract locations)
- ▶ an abstract heap, H (specifying links between abstract locations)
- ▶ sharing information, is , for the abstract locations.

The last component allows us to recover some of the imprecision introduced by combining many locations into one abstract location.

Example



$g_9 = (S, H, is)$ where

$$S = \{(x, n_{\{x\}})\}$$

$$H = \{(n_{\{x\}}, next, n_{\emptyset}), (n_{\emptyset}, next, n_{\emptyset})\}$$

$$is = \emptyset$$

Abstract Locations

The *abstract locations* have the form n_X where X is a subset of the variables of Var_* :

$$\text{ALoc} = \{n_X \mid X \subseteq \text{Var}_*\}$$

A shape graph contains a subset of the abstract locations of ALoc

The abstract location n_\emptyset is called the *abstract summary location* and represents all the locations that cannot be reached directly from the state without consulting the heap.

Clearly n_X and n_\emptyset represent disjoint sets of locations when $X \neq \emptyset$.

Invariant 1: If two abstract locations n_X and n_Y occur in the same shape graph then either $X = Y$ or $X \cap Y = \emptyset$. (i.e. two distinct abstract locations n_X and n_Y always represent disjoint sets of locations)

Abstract State

The **abstract state**, S , maps variables to abstract locations. To maintain the naming convention for abstract locations we shall ensure that:

Invariant 2: If x is mapped to n_x by the abstract state then $x \in X$.

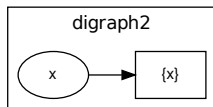
From Invariant 1 it follows that there will be at most one abstract location in the (same) shape graph containing a given variable.

We shall only be interested in the shape of heap so we shall not distinguish between integer values, nil-pointers, and uninitialized fields; hence we can view the abstract state as an element of

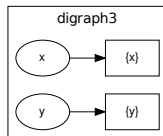
$$S \in \text{AState} = \mathcal{P}\text{Var}_* \times \text{ALoc}$$

Example: Creating Linked Data Structures

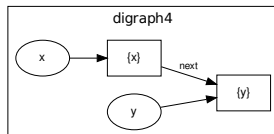
$[new(x)]^2$



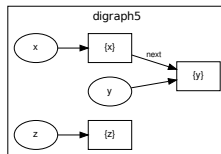
$[new(y)]^3$



$[x.next := y]^4$



$[new(z)]^5$



Abstract Heap

The **abstract heap**, H , specifies the links between the abstract locations.

The links will be specified by triples (n_V, sel, n_W) and formally we take the abstract heap as an element of

$$H \in AHeap = \mathcal{P}ALoc \times Sel \times ALoc$$

where we again not distinguish between integers, nil-pointers and uninitialized fields.

Invariant 3: Whenever (n_V, sel, n_W) and (n_V, sel, n'_W) are in the abstract heap then either $V = \emptyset$ or $W = W'$.

Thus the target of a selector field will be uniquely determined by the source unless the source is the abstract summary location n_\emptyset .

Sharing Information

The idea is to specify a subset, is , of the abstract locations that represents locations that are shared due to pointers *in* the heap:

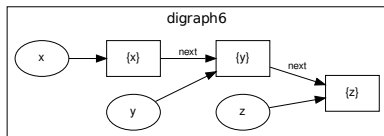
- ▶ an abstract location n_x will be included in is if it represents a location that is the target of more than one pointer in the heap.

In the case of the abstract summary location, n_\emptyset , the explicit sharing information clearly gives extra information:

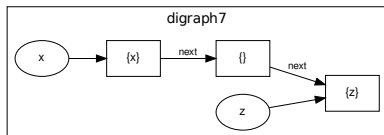
- ▶ if $n_\emptyset \in is$ then there might be a location represented by n_\emptyset that is the target of two or more heap pointers.
- ▶ if $n_\emptyset \notin is$ then all the locations of represented by n_\emptyset will be the target of at most one heap pointer.

Maintaining Sharing Information

$[y.next := z]^6$

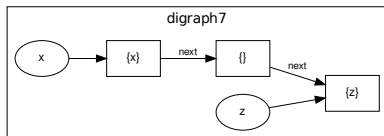


$[y := null]^7$

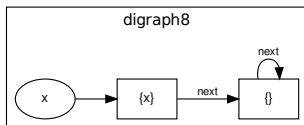


Maintaining Sharing Information

$[y := \text{null}]^7$



$[z := \text{null}]^8$



Sharing Information Invariants (1)

We shall impose two invariants to ensure that information in the sharing component is also reflected in the abstract heap.

The first ensures that information in the sharing component is also reflected in the abstract heap:

Invariant 4: If $n_X \in$ is then either

- a) (n_\emptyset, sel, n_X) is in the abstract heap for some sel , or
 - b) there exist two distinct triples (n_V, sel_1, n_X) and (n_W, sel_2, n_X) in the abstract heap (that is either $sel_1 \neq sel_2$ or $V \neq W$).
- ▶ case 4a) means that there might be several locations represented by n_\emptyset that point to n_X
 - ▶ case 4b) means that two distinct pointers (with different source or different selectors) point to n_X .

Sharing Information Invariants (2)

The second invariant ensures that sharing information present in the abstract heap is also reflected in the sharing component:

Invariant 5: Whenever there are two distinct triples (n_V, sel_1, n_X) and (n_W, sel_2, n_X) in the abstract heap and $n_X \neq n_\emptyset$ then $n_X \in is$.

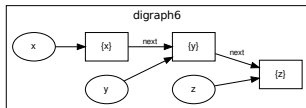
This invariant takes care of the situation where n_X represents a single location being the target of two or more heap pointers.

Note that invariant 5 is the “inverse” of invariant 4(b).

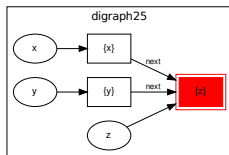
We have no “inverse” of invariant 4(a) - the presence of a pointer from n_\emptyset to n_X gives no information about sharing properties of n_X that are represented in is .

Sharing Component Example 1

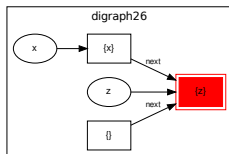
$[y.\text{next} := z]^6$



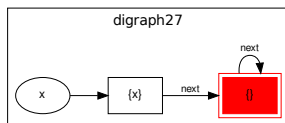
$[x.\text{next} := z]^{7'}$



$[y := \text{null}]^{8'}$

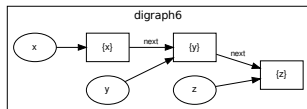


$[z := \text{null}]^{9'}$

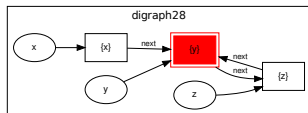


Sharing Component Example 2

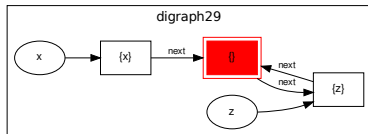
$[y.next := z]^6$



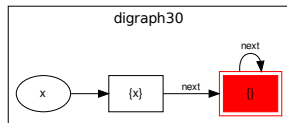
$[z.next := y]^{7''}$



$[y := null]^{8''}$



$[z := null]^{9''}$



Compatible Shape Graphs

A shape graph is a triple (S, H, is) :

$$\begin{aligned} S \in AState &= \mathcal{P}\text{Var}_* \times A\text{Loc} \\ H \in A\text{Heap} &= \mathcal{P}A\text{Loc} \times \text{Sel} \times A\text{Loc} \\ is \in \text{IsShared} &= \mathcal{P}A\text{Loc} \end{aligned}$$

where $A\text{Loc} = \{n_X \mid X \subseteq \text{Var}_*\}$.

A shape graph is a **compatible shape graph** if it fulfills the five invariants, 1-5, presented above. The set of compatible shape graphs is denoted

$$\text{SG} = \{(S, H, is) \mid (S, H, is) \text{ is compatible}\}$$

Complete Lattice of Shape Graphs

The analysis, to be called *Shape*, will operate over *sets* of compatible shape graphs, i.e. elements of \mathcal{PSG} .

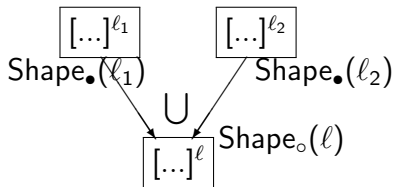
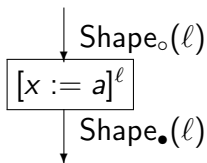
Since \mathcal{PSG} is a power set it is trivially a complete lattice with

- ▶ ordering relation \sqsubseteq being \subseteq
- ▶ combination operator \sqcap being \cup (may analysis)

\mathcal{PSG} is finite because $SG \subseteq AState \times AHeap \times IsShared$ and all of $AState$, $AHeap$, $IsShared$ are finite.

The analysis will be specified as an instance of a Monotone Framework with the complete lattice of properties being \mathcal{PSG} , and as a forward analysis.

Analysis



$$\begin{aligned} \text{Shape}_o(\ell) &= \begin{cases} \iota & : \text{ if } \ell = \text{init}(S_\star) \\ \bigcup \{ \text{Shape}_\bullet(\ell') \mid (\ell', \ell) \in \text{flow}(S_\star) \} & : \text{ otherwise} \end{cases} \\ \text{Shape}_\bullet(\ell) &= f_\ell^{SA}(\text{Shape}_o(\ell)) \end{aligned}$$

where $\iota \in \mathcal{PSG}$ is the extremal value holding at entry to S_\star .

Transfer Functions

The transfer function $f_\ell^{\text{SA}} : \mathcal{PSG} \rightarrow \mathcal{PSG}$ has the form

$$f_\ell^{\text{SA}}(\text{SG}) = \bigcup \{ \phi_\ell^{\text{SA}}((S, H, \text{is})) \mid (S, H, \text{is}) \in \text{SG} \}$$

where ϕ_ℓ^{SA} specifies how a *single* shape graph (in $\text{Shape}_\circ(\ell)$) may be transformed into a *set* of shape graphs (in $\text{Shape}_\bullet(\ell)$).

The functions ϕ_ℓ^{SA} for the statements

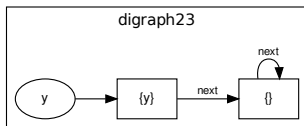
$x := a$	$x := y$	$x := y.\text{sel}$	(illustrated by
$x.\text{sel} := a$	$x.\text{sel} := y$	$x.\text{sel} := y.\text{sel}$	

example)

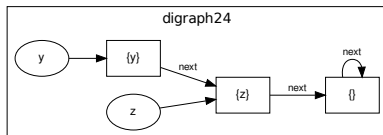
transform a shape graph into a set of different shape graphs.

The transfer functions for other statements and expressions are specified by the identity function.

Example: Materialization



$[z := y.next]^7$

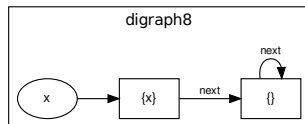


Example: Reverse List

```
[y := null]1;  
while [not isnull(x)]2 do  
  [t := y]3;  
  [y := x]4;  
  [x := x.next]5;  
  [y.next := t]6;  
od  
[t := null]7
```

The program reverses the list pointed to by x and leaves the result in y.

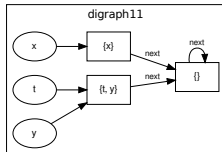
Reverse List: Extremal Value



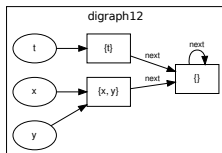
The extremal value ι is a set of graphs. The above graph is an element of this set for our example analysis of the list reversal program.

Shape Graphs in Shape.(ℓ)

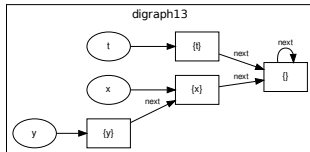
$[t := y]^3$



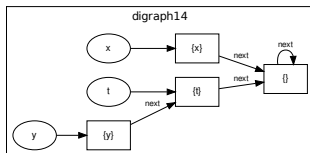
$[y := x]^4$



$[x := x.next]^5$

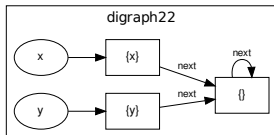


$[y.next := t]^6$

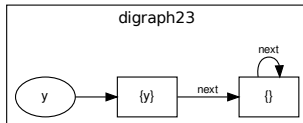


Shape Graphs in Shape.(ℓ)

$[t := \text{null}]^7$



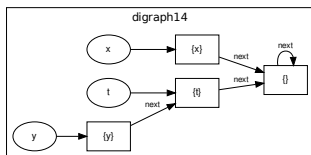
$[x := \text{null}]^7$



Reverse List: Established Properties

For the list reversal program shape analysis can detect that at the beginning of each iteration of the loop the following properties hold:

- Invariant 1:** Variable x points to an unshared, acyclic, singly linked list.
- Invariant 2:** Variable y points to an unshared, acyclic, singly linked list, and variable t may point to the second element of the y -list (if such an element exists).
- Invariant 3:** The lists pointed to by x and y are disjoint.



Drawbacks and Improvements





An improved version, on which the discussed analysis is based on, can be found in [SRW'98]:

- ▶ Operates on a single shape graph instead of sets of shape graphs
- ▶ Merges sets of compatible shape graphs in one summary shape graph
- ▶ Uses various mechanisms for extracting parts of individual compatible shape graphs
- ▶ Avoids the exponential factor in the cost of the discussed analysis




The sharing component of the shape graphs is designed to detect list-like properties:

- ▶ It can be replaced by other components detecting other shape properties [SRW'02, CDH Ch 5]




Further Reading for Chapter 15 (1)

-  D. Chase, Mark N. Wegmann, F. Ken Zadeck. *Analysis of Pointers and Structures*. In Proceedings PLDI'90, 296-310, 1990.
-  M. Emami, R. Ghiya, Laurie J. Hendren. *Context-Sensitive Interprocedural Points-to Analysis in the Presence of Function Pointers*. In Proceedings PLDI'94, 1-14, 1994.
-  R. Ghiya, Laurie J. Hendren. *Is it a Tree, a DAG, or a Cyclic Graph?* In Proceedings POPL'96, 1-15, 1996.
-  Stephen S. Muchnick. *Advanced Compiler Design Implementation*. Morgan Kaufman Publishers, 1997. (Chapter 10, Alias Analysis)

Further Reading for Chapter 15 (2)

-  Flemming Nielson, Hanne Riis Nielson, Chris Hankin. *Principles of Program Analysis*. 2nd edition, Springer-V., 2005. (Chapter 2.6, Shape Analysis)
-  Viktor Pavlu, Markus Schordan, Andreas Krall. *Computation of Alias Sets from Shape Graphs for Comparison of Shape Analysis Precision*. In Proceedings of the 11th International IEEE Working Conference on Source Code Analysis and Manipulation (SCAM 2011), 2011. [Best Paper Award SCAM 2011]
-  Mooly Sagiv, Tom Reps, Reinhard Wilhelm. *Solving Shape-Analysis Problems in Languages with Destructive Updating*. ACM Transactions on Programming 20(1):1-50, 1998.

Further Reading for Chapter 15 (3)

-  Mooly Sagiv, Tom Reps, Reinhard Wilhelm. *Parametric Shape Analysis via 3-Valued Logic*. In Proceedings POPL'99, 105-118, 1999.
-  Mooly Sagiv, Tom Reps, Reinhard Wilhelm. *Parametric Shape Analysis via 3-valued Logic*. ACM Transactions on Programming Languages and Systems 24(3), 2002.
-  Y. N. Srikant, Priti Shankar. *The Compiler Design Handbook: Optimizations & Machine Code Generation*. CRC Press, 2002. (Chapter 5, Shape Analysis and Application)

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Optimizations for Object-Oriented Languages

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Overview

- ▶ Object layout and method invocation
 - ▶ Single inheritance
 - ▶ Multiple Inheritance
- ▶ Devirtualization
 - ▶ Class hierarchy analysis
 - ▶ Rapid type analysis
 - ▶ Inlining
- ▶ Escape Analysis
 - ▶ Connection graphs
 - ▶ Intra-procedural
 - ▶ Inter-procedural

Chapter 16.1

Object Layout and Method Invocation

Object Layout and Method Invocation

The memory layout of an object and how the layout supports dynamic dispatch are crucial factors for performance.

- ▶ **Single Inheritance**

- ▶ with and without virtual dispatch table (i.e., direct calling guarded by a type test)

- ▶ **Multiple Inheritance**

...various techniques with different compromises

- ▶ embedding superclasses
- ▶ trampolines
- ▶ table compression

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Single Inheritance

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Single Inheritance Layout

```
class Point {  
    int x, y;  
}
```

```
class ColorPnt extends Point {  
    int color;  
}
```

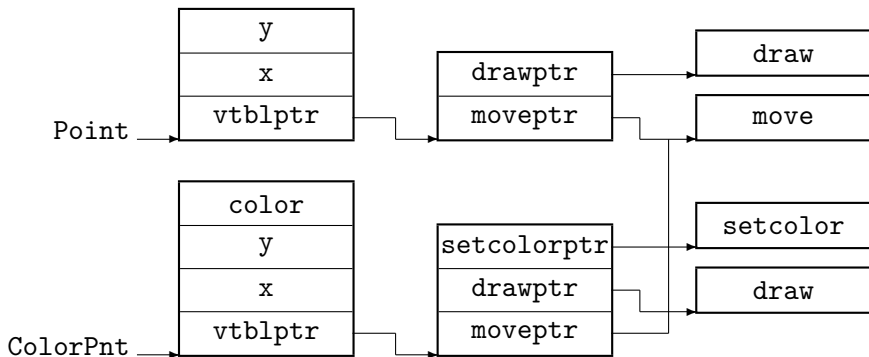


- ▶ Memory layout of an object of a superclass is a prefix of the memory layout of an object of the subclass.
- ▶ Instance variables access requires just one load or store instruction.

Single Inheritance Layout with vtbl

```
class Point {  
    int x, y;  
    void move(int x, int y) {...}  
    void draw() {...}  
}
```

```
class ColorPnt extends Point {  
    int color;  
    void draw() {...}  
    void setcolor(int c) {...}  
}
```



Invocation of Virtual Methods with vtbl

- ▶ Dynamic dispatching using a vtbl has the advantage of being fast and executing in constant time.
- ▶ It is possible to add new methods and to override methods.
- ▶ Each method is assigned a fixed offset in the virtual method table (vtbl).
- ▶ Method invocation is just three machine code instructions:
LDQ vtblptr, (obj) ; load vtbl pointer
LDQ mptr, method(vtblptr) ; load method pointer
JSR (mptr) ; call method
- ▶ One extra word of memory is needed in each object for the pointer to the virtual method table (vtbl).

Dispatch Without Virtual Method Tables

Despite the use of branch target caches, indirect branches are expensive on modern architectures.

The pointer to the class information and virtual method table is replaced by a type identifier:

- ▶ A type identifier is an integer representing the type of the object.
- ▶ It is used in a dispatch function which searches for the type of the receiver.
- ▶ Example: SmallEiffel (binary search).
- ▶ Dispatch functions are shared between calls with the same statically determined set of concrete types.
- ▶ In the dispatch function a direct branch to the dispatched method is used (or it is inlined).

Example

Let type identifiers $T_A, T_B, T_C,$ and T_D be sorted by increasing number. The dispatch code for calling $x.f$ is:

```
if  $id_x \leq T_B$  then  
    if  $id_x \leq T_A$  then  $f_A(x)$   
    else  $f_B(x)$   
else if  $id_x \leq T_C$  then  $f_C(x)$   
else  $f_D(x)$ 
```

Comparison with dispatching using a virtual method table:

- ▶ Empirical study showed that for a method invocation with three concrete types, dispatching with binary search is between 10% and 48% faster.
- ▶ For a megamorphic call with 50 concrete types, the performance is about the same.

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Multiple Inheritance

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Multiple Inheritance

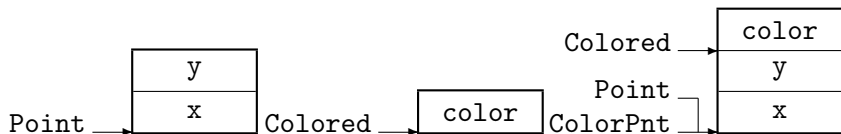
- ▶ Extending the superclasses as in single inheritance does not work anymore.
- ▶ Fields of superclass are embedded as contiguous block.
- ▶ Embedding allows fast access to instance variables exactly as in single inheritance.
- ▶ Garbage collection becomes more complex because pointers also point into the middle of objects.

Object Memory Layout (without vtbl)

```
class Point {  
    int x, y;  
}
```

```
class Colored {  
    int color;  
}
```

```
class ColorPnt extends Point, Colored {  
}
```



Dynamic Dispatching for Embedding

- ▶ Allows fast access to instance variables exactly as with single inheritance.
- ▶ For every superclass
 - ▶ virtual method tables have to be created.
 - ▶ multiple *vtbl* pointers are included in the object.
- ▶ The object pointer is adjusted to the embedded object whenever explicit or implicit pointer casting occurs (assignments, type casts, parameter and result passing).

Multiple Inheritance with vtbl (1)

```
class Point {
    int x, y;
    void move(int x, int y) {...}
    void draw() {...}
}

class Colored {
    int color;
    void setcolor(int c) {...}
}

class ColorPnt extends Point, Colored {
    void draw() {...}
}
```

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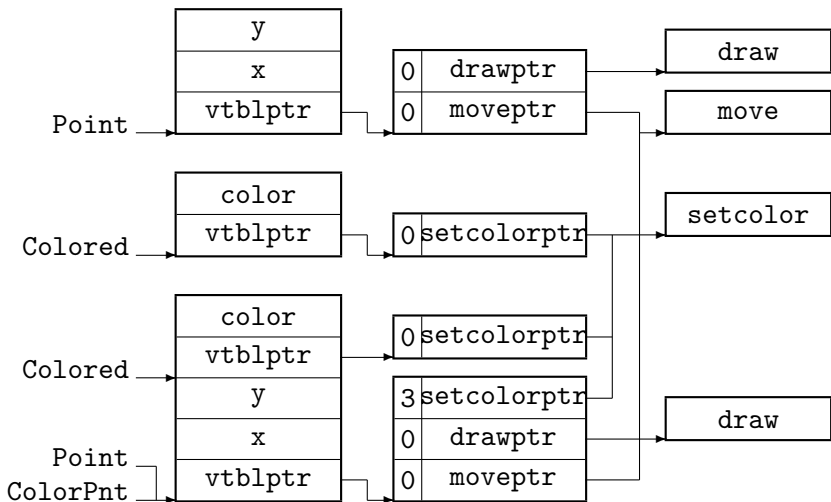
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Multiple Inheritance with vtbl (2)



Pointer Adjustment and Adjustment Offset

Pointer adjustment has to be suppressed for casts of null pointers:

```
Colored col; ColorPnt cp; ...;  
col = cp; // if (cp!=null)col=(Colored)((int*)cp+3)
```

Problem with implicit casts from actual receiver to formal receiver

- ▶ Caller has no type info of formal receiver in the callee.
- ▶ Callee has no type info of actual receiver of the caller.
- ▶ Therefore this type info has to be stored as an adjustment offset in the vtbl.

Method Invocation with vtbl

Method invocation now takes 4 to 5 machine instructions (depending on the architecture).

```
LD  vtblptr,(obj)          ; load vtbl pointer
LD  mptr,method_ptr(vtblptr) ; load method pointer
LD  off,method_off(vtblptr) ; load adjustment offset
ADD obj,off,obj            ; adjust receiver
JSR (mptr)                 ; call method
```

This overhead in table space and program code is even necessary when multiple inheritance is not used (in the code).

Furthermore, adjustments to the remaining parameters and the result are not possible.

Trampoline

To eliminate much of the overhead a small piece of code, called **trampolin** is inserted that performs the pointer adjustments and the jumps to the original code.

The advantages are

- ▶ smaller table size (no storing of an offset)
- ▶ fast method invocation when multiple inheritance is not used
 - ▶ the same dispatch code as in single inheritance

The method pointer `setColorptr` in the virtual method table of `Colorpoint` would (instead) point to code which adds 3 to the receiver before jumping to the code of method `setColor`:

```
ADD obj,3,obj          ; adjust receiver
BR  setColor          ; call method
```

Lookup at Compile-Time

Invoking a method requires looking up the address of the method and passing control to it.

In some cases, the lookup may be performed at compile-time:

- ▶ There is only one implementation of the method in the class and its subclasses.
- ▶ The language provides a declaration that forces the call to be non-virtual.
- ▶ The compiler has performed static analysis that can determine that a unique implementation is *always* called at a particular call site.

In other cases, a runtime lookup is required.

Dispatch Table

In principle the lookup can be implemented as indexing a two-dimensional table. A number is given to

- ▶ each method in the program
- ▶ each class in the program

The method call

```
result = obj.m(a1,a2);
```

can be implemented by the following three actions:

1. Fetch a pointer to the appropriate row of the dispatch table from the object `obj`.
2. Index the dispatch table row with the method number.
3. Transfer control to the address obtained.

Dispatch Table Compression (1)

▶ Virtual Tables

- ▶ Effective method for statically typed languages.
- ▶ Methods can be numbered compactly for each class hierarchy to leave no unused entries in each vtbl.

▶ Row Displacement Compression

- ▶ Idea: combine all rows into a single very large vector.
- ▶ It is possible to have rows overlapping as long as an entry in one row corresponds to empty entries in the other rows.
- ▶ Greedy algorithm: place first row; for all subsequent rows: place on top and shift right if conflicts exist.
- ▶ Unchanged: implementation of method invocation.
- ▶ Penalty: verify class of current object at the beginning of any method that can be accessed via more than one row.

Dispatch Table Compression (2)

- ▶ **Selector Coloring Compression**
 - ▶ Graph coloring: two rows can be merged if no column contains different method addresses for the two classes.
 - ▶ Graph: one node per class; an edge connects two nodes if the corresponding classes provide different implementations for the same method name.
 - ▶ Coloring: each color corresponds to the index for a row in the compressed table.
 - ▶ Each object contains a reference to a possibly shared row.
 - ▶ Unchanged: implementation of method invocation code.
 - ▶ Penalty: if classes C1 and C2 share the same row and C1 implements method m whereas C2 does not, then the code for m should begin with a check that control was reached via dispatching on an object of type C1.

Chapter 16.2

Devirtualization of Method Invocations

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Devirtualization

Devirtualization is a technique to reduce the overhead of virtual method invocation.

The aim of this technique is to statically determine which methods can be invoked by virtual method calls.

- ▶ If exactly one method is resolved for a method call, the method can be inlined or the virtual method call can be replaced by a static method call.

The analyses necessary for devirtualization also improve the accuracy of the call graph and the accuracy of subsequent interprocedural analyses.

Chapter 16.2.1

Class Hierarchy Analysis

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Class Hierarchy Analysis

The simplest devirtualization technique is [class hierarchy analysis \(CHA\)](#), which determines the class hierarchy [used](#) in a program.

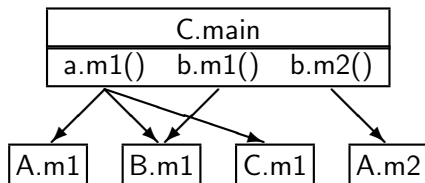
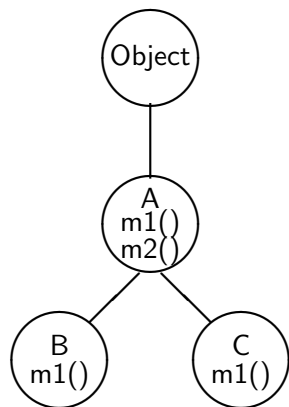
The information about all referenced classes is used to create a conservative approximation of the class hierarchy.

- ▶ The transitive closure of all classes referenced by the class containing the main method is computed.
- ▶ The declared types of the receiver of a virtual method call are used for determining all possible receivers.

Example: Class Hierarchy Analysis

```
class A extends Object {
    void m1() {...}
    void m2() {...}
}
class B extends A {
    void m1() {...}
}
class C extends A {
    void m1() {...}
    public static void main(...) {
        A a = new A();
        B b = new B();
        ...
        a.m1(); b.m1(); b.m2();
    }
}
```

Example: Class Hierarchy and Call Graph



The CHA Algorithm

main // the main method in a program
x() // call of static method *x*
type(x) // the declared type of the expression *x*
x.y() // call of virtual method *y* in expression *x*
subtype(x) // *x* and all classes which are a subtype of class *x*
method(x, y) // the method *y* which is defined for class *x*

callgraph := *main*

hierarchy := {}

for each *m* ∈ *callgraph* **do**

for each *m_{stat}*() occurring in *m* **do**

if *m_{stat}* ∉ *callgraph* **then**

add *m_{stat}* to *callgraph*

for each *e.m_{vir}*() occurring in *m* **do**

for each *c* ∈ *subtype*(*type*(*e*)) **do**

m_{def} := *method*(*c, m_{vir}*)

if *m_{def}* ∉ *callgraph* **then**

add *m_{def}* to *callgraph*

add *c* to *hierarchy*

Chapter 16.2.2

Rapid Type Analysis

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Rapid Type Analysis (1)

Rapid type analysis uses the fact that a method m of a class c can be invoked only if an object of type c is created during the execution of the program.

- ▶ It refines the class hierarchy (compared to CHA) by only including classes for which objects can be created at **runtime**.

Based on this idea

- ▶ **pessimistic**
- ▶ **optimistic**

algorithms are possible.

Rapid Type Analysis (2)

1. The pessimistic algorithm

...includes all classes in the class hierarchy for which instantiations occur in methods of the call graph from CHA.

2. The optimistic algorithm

- ▶ Initially assumes that no methods besides *main* are called and that no objects are instantiated.
- ▶ It traverses the call graph initially ignoring virtual calls (marking them in a mapping as potential calls only) following static calls only.
- ▶ When an instantiation of an object is found during analysis, all virtual methods of the corresponding objects that were left out previously are then traversed as well.
- ▶ The live part of the call graph and the set of instantiated classes grow interleaved as the algorithm proceeds.

Chapter 16.2.3

Inlining

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Using Devirtualization Information

Inlining is an important usage of devirtualization information.

If a virtual method call can be devirtualized

- ▶ it might completely be replaced by **inlining** the call (supposed it is not recursive).

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Escape Analysis

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Escape Analysis

The goal of **escape analysis** is to determine which objects have lifetimes which do not stretch outside the lifetime of their immediately enclosing scopes.

- ▶ The storage for such objects can be safely allocated as part of the current stack frame – that is, their storage can be allocated on the run-time stack.
- ▶ At method return, deallocation of the memory space used by non-escaping objects is automatic. No garbage collection is required.
- ▶ The transformation also improves the data locality of the program and, depending on the computer's cache, can significantly reduce execution time. Objects not escaping a thread can be allocated in the processor where that thread is scheduled.

Using Escape Information

Objects whose lifetimes are confined to within a single scope cannot be shared between two threads.

- ▶ Synchronization actions for these objects can be eliminated.

Escape Analysis by Abstract Interpretation

A prototype implementation of escape analysis was included in the IBM High Performance Compiler for Java.

The approach of Choi et al. (OOPSLA'99) attempts to determine whether the object

- ▶ escapes from a method (i.e. from the scope where it is allocated).
- ▶ escapes from the thread that created it
 - ▶ the object can escape a method but does not escape from the thread.

Note: The converse is not possible (if it does not escape the method then it cannot escape the thread).

Essence of Choi et al.'s Approach

- ▶ Introducing of a simple program abstraction called **connection graph**:
Intuitively, a connection graph captures the **connectivity** relationship between heap allocated objects and object references.
- ▶ Demonstrating that escape analysis boils down to a reachability problem within connections graphs:
If an object is reachable from an object that might escape, it might escape as well.

Experimental Results Reported by Choi et al.

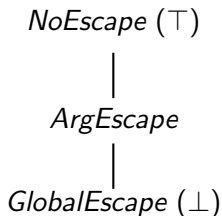
...based on 10 benchmark programs:

- ▶ **Percentage of objects that may be allocated on the stack:**
Up to 70 + %, with a median of 19%.
- ▶ **Percentage of all lock operations eliminated:**
From 11% to 92%, with a median of 51%.
- ▶ **Overall execution time reduction:**
From 2% to 23%, with a median of 7%.

These results make escape analysis and the optimizations based thereon worthwhile.

Escape States

The analysis uses a simple lattice to represent different escape states:



State	Escapes the method	Escapes the thread
NoEscape	no	no
ArgEscape	may (via args)	no
GlobalEscape	may	may

Using Escape Information

All objects which are marked

- ▶ **NoEscape**: are stack-allocatable in the method where they are created.
- ▶ **NoEscape** or **ArgEscape**: are local to the thread in which they are created; hence synchronization statements in accessing these objects can be eliminated.

Chapter 16.3.1

Connection Graphs

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Connection Graphs

We are interested only in

- ▶ following the object O from its point of allocation.
- ▶ knowing which variables reference O .
- ▶ and which other objects are referenced by O fields.

We “abstract out” the referencing information, using a graph structure where

- ▶ a circle node represents a variable.
- ▶ a square node represents objects in the heap.
- ▶ an edge from circle to square represents a reference.
- ▶ an edge from square to circle represents ownership of fields.

Example: Connection Graphs

```
A a = new A();    // line L1
a.b1 = new B();  // line L2
a.b2 = a.b1;     // line L3
```

An edge drawn as a dotted arrow is called a **deferred edge** and shows the effect of an assignment from one variable to another (example: created by the assignment in line 3) \rightsquigarrow improves efficiency of the approach.

Chapter 16.3.2

Intraprocedural Setting

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Intraprocedural Abstract Interpretation

Actions for assignments involve an update of the connection graph.

- ▶ An assignment to a variable p kills any value the variable previously had. The kill function is called *byPass*(p):

Analyzing Statements (1)

`p = new C(); // line L` The operation *byPass*(p) is applied. An object node labeled L is added to the graph - and nodes for the fields of C that have nonintrinsic types are also created and connected by edges pointing from the object node.

`p = q;` The operation *byPass*(p) is applied. A new deferred edge from p to q is created.

`p.f = q;` The operation *byPass* is *not* applied for f (no strong update!). If p does not point to any node in the graph a new (phantom) node is created. Then, for each object node connected to p by an edge, an assignment to the field f of that object is performed.

Analyzing Statements (2)

$p = q.f$; If q does not point at any object node then a phantom node is created and an edge from q to the new node is added. Then $byPass(p)$ is applied and deferred edges are added from p to all the f nodes that q is connected to by field edges.

For each statement one graph represents the state of the program at the statement.

At a point where two or more control paths converge, the connection graphs from each predecessor statements are merged.

Example: Connection Graphs (1)

Suppose that the code inside some method is as follows. The declarations of classes A, B1 and B2 are omitted.

```
A a = new A();      // line L1
if (i > 0)
    a.f1 = new B1(); // line L3
else
    a.f1 = new B2(); // line L5
a.f2 = a.f1;       // line L6
```

Example: Connection Graphs (2)

G_1 : out: A a = new A(); // line L1
 G_2 : out: a.f1 = new B1(); // line L3
 G_3 : out: a.f1 = new B2(); // line L5
 G_4 : out: $G_2 \cup G_3$
 G_5 : out: a.f2 = a.f1; // line L6

Chapter 16.3.3

Interprocedural Setting

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Interprocedural Abstract Interpretation (1)

Analyzing methods:

- ▶ It is necessary to analyze each method in the reverse order implied by the [call graph](#).
- ▶ If method A may call methods B and C, then B and C should be analyzed before A.
- ▶ Recursive edges in the call graph are ignored when determining the order.
- ▶ Java has virtual method calls – at a method call site where it is not known which method implementation is being invoked, the analysis must assume that all of the possible implementations are called, combining the effects from all the possibilities.
- ▶ The interprocedural analysis iterates over all the methods in the call graph until the results converge (fixed point).

Interprocedural Abstract Interpretation (2)

- ▶ A call to a method M is equivalent to copying the actual parameters (i.e. the arguments being passed in the method call) to the formal parameters, then executing the body of M , and finally copying any value returned by M as its result back to the caller.
- ▶ If M has already been analyzed intraprocedurally following the approach described above, the effect of M can be summarized with a connection graph. That summary information eliminates the need to re-analyze M for each call site in the program.

Analysis Results (1)

After the operation *byPass* has been used to eliminate all deferred edges, the connection graph can be partitioned into three subgraphs:

Global escape nodes: All nodes reachable from a node whose associated state is *GlobalEscape* are themselves considered to be global escape nodes (Subgraph 1)

- ▶ the nodes initially marked as *GlobalEscape* are the static fields of any classes and instances of any class that implements the *Runnable* interface.

Argument escape nodes: All nodes reachable from a node whose associated state is *ArgEscape*, but are not reachable from a *Global Escape* node. (Subgraph 2)

- ▶ the nodes initially marked as *ArgEscape* are the argument nodes a_1, \dots, a_n .




Analysis Results (2)

No escape nodes: All other nodes have *NoEscape* status.
(Subgraph 3).



The third subgraph represents the summary information for the method because it shows which objects can be reached via the arguments passed to the method.

All objects created within a method M and that have the *NoEscape* status after the three subgraphs have been determined can be **safely allocated on the stack**.

Further Reading for Chapter 16 (1)

-  J-G Choi, M. Gupta, M. Serrano, V.C Sreedar, and Sam Midkiff. *Escape Analysis for Java*. In Proceedings of OOPSLA'99, ACM Press, 1-19, 1999.
-  Flemming Nielson, Hanne Riis Nielson, Chris Hankin. *Principles of Program Analysis*. 2nd edition, Springer-V., 2005. (Chapter 1, Introduction; Chapter 2, Data Flow Analysis; Chapter 6, Algorithms)
-  H. D. Pande, Barbara Ryder. *Data-flow-based Virtual Function Resolution*. In Proceedings SAS'96, Springer-V., LNCS 1145, 238-254, 1996.

Further Reading for Chapter 16 (2)

-  Y. N. Srikant, Priti Shankar. *The Compiler Design Handbook: Optimizations and Machine Code Generation*. 1st edition, CRC Press, 2002. (Chapter 6, Optimizations for Object-Oriented Languages)
-  Y. N. Srikant, Priti Shankar. *The Compiler Design Handbook: Optimizations and Machine Code Generation*. 2nd edition, CRC Press, 2008. (Chapter 13, Optimizations for Object-Oriented Languages)

Chapter 17

Slicing

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Overview

Collection of Examples

- ▶ Definition of executable slice (liveness analysis)
- ▶ Example with scalar variables
- ▶ Example with pointers to stack-allocated variables
- ▶ Example with strong and weak update in alias analysis
- ▶ Example with non-cyclic dynamic data structures
- ▶ Example with cyclic data structures
- ▶ Comparison of slice size

Reaching Defs with/out Pointers

ReachingDefs(var, n, F) = *[without pointers]*
let
 $F = (V, A, En, Ex), A' = A \setminus \{(x, n)\}$
return $\bigcup_{(x,n) \in A}$ if var \notin def(x) then *ReachingDefs*(var, x, (V, A', En, Ex))
 else {x}

ReachingDefs(var, n, F) = *[with pointers]*
let
 $F = (V, A, En, Ex), A' = A \setminus \{(x, n)\}$
return $\bigcup_{(x,n) \in A}$ if var \notin def(x) then *ReachingDefs*(var, x, (V, A', En, Ex))
 else (if #def(x) = 1 then {x}
 else {x} \cup *ReachingDefs*(var, x, (V, A', En, Ex)))

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Program Dependence Graph

```
DataDep(P) =  
  let CFG(P) = (V, A, En, Ex)  
      D =  $\bigcup_{n \in V, var \in use(n), x \in ReachingDefs(var, n, CFG(P))} \{(n, x)\}$   
  return (V, D)
```

```
ProgramDep(P) =  
  let DataDep(P) = (V, D),  
      ControlDep(P) = (V, C, In),  
  return (V, D  $\cup$  C)
```

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Static Slice

$$\begin{aligned} \text{ReachableNodes}(v, G) = & \\ \text{let } G = (V, A) & \\ \text{return } \{v\} \cup \bigcup_{(v,x) \in A} \text{ReachableNodes}(x, (V, A \setminus \{(v,x)\})) & \end{aligned}$$
$$\begin{aligned} \text{StaticSlice}(P, n, \text{Vars}) = & \\ \text{let } F = \text{CFG}(P) & \\ D = \text{ProgramDep}(P) & \\ \text{return } \bigcup_{\text{var} \in \text{Vars}} \bigcup_{x \in \text{ReachingDefs}(\text{var}, n, F)} \text{ReachableNodes}(x, D) & \end{aligned}$$

Example: Artificial Sum (only scalar vars)

```
main() {  
0   int a,b,i,j,n,y;  
1   n=read();  
6   a=0;  
7   b=0;  
8   i=n;  
9   j=n;
```

$$y = n + \sum_{i=1}^{n-1} i = \sum_{i=1}^n i$$

```
10  while (i>0) {  
11      a=a+1;  
12      i=i-1;  
13      j=i;  
14      while (j>0) {  
15          b=b+1;  
16          j=j-1;  
17      }  
18      y=a+b;  
19      write(y);  
20  }
```

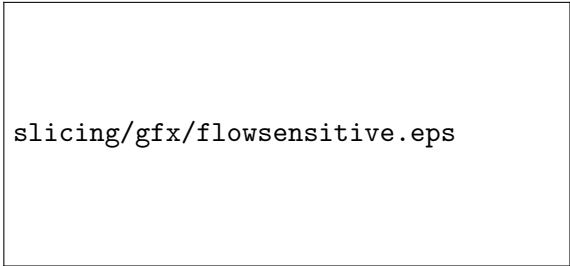

Example: With Pointers

```
main() {
0   int a,b,i,j,n, *ap,*bp,**cp;
1   n=read();
2   cp=&bp
3   ap=&a;
4   bp=ap;
5   *cp=&b;
6   a=0;
7   b=0;
8   i=n;
9   j=n;
10  while (i>0)
11     *ap=*ap+1;
12     i=i-1;
13     j=i;
14     while (j>0)
15         *bp=*bp+1;
16         j=j-1;
17  y=*ap + *bp;
18  write(y);
```

Slicing criterion 17: *ap

Example: Strong vs. Weak Update

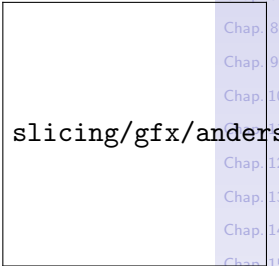
```
0    int a,b,i,j,*ap,*bp,**cp;  
2    cp=&bp  
3    ap=&a;  
4    bp=ap;  
5    *cp=&b;
```



slicing/gfx/flowsensitive.eps

strong update

update



slicing/gfx/anders

weak

Example: Reaching Definitions

slicing/gfx/prog_pointer_cfgs.eps

Example: Computation of Slice

slicing/gfx/prog_pointer_graphs_colored.eps

Example: With Pointers (Slices)

```
main()
  int a,b,i,j,n, *ap,*bp,**cp;
1  n=read();
2  cp=&bp;
3  ap=&a;
4  bp=ap;
5  *cp=&b;
6  a=0;
7  b=0;
8  i=n;
9  j=n;
10 while (i>0) }
11   *ap=*ap+1;
12   i=i-1;
13   j=i;
14   while (j>0) {
15     *bp=*bp+1;
16     j=j-1;
17   }
18   y=*ap+*bp;
19   write(y);
20 }
```

Slicing criterion 17: *ap

Example: Comparison

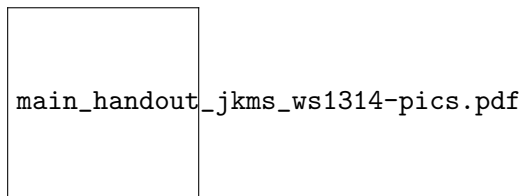
Program	Alias Analysis	Slice	Size
Sum	None	{1, 6, 8, 10, 11, 12, 17}	7
Sum (ptrs)	Flow insensitive	{1,2,3,5,6,7,8,10,11, 12,13,14,15,16,17}	15
Sum (ptrs)	Flow sensitive	{1, 3, 6, 8, 10, 11, 12, 17}	8

Example: With Dynamic Data Structures

```
main() {
List *a,*b,*ap,*bp;
int i,j,n;
1  n=read();
2  a=new List();
3  b=new List();
4  ap=a;
5  bp=b;
8  i=n;
9  j=n;
10 while (i>0) {
11     ap->next=new List();
11b    ap=ap->next;
12     i=i-1;
13     j=i;
14     while (j>0) {
15         bp->next=new List();
15b        bp=bp->next;
16         j=j-1;
17     }
17     ap->next = b; // conc
17b    y=a;
18     write(length(y)-2)
    }
```

Example: Shape Analysis

17:



Result for 17: `ap->next;`

Example: W/ Dynamic Data Structures (Slice)

```
main() {
List *a,*b,*ap,*bp;
int i,j,n;
1  n=read();
2  a=new List();
3  b=new List();
4  ap=a;
5  bp=b;
8  i=n;
9  j=n;
10 while (i>0) {
11     ap->next=new List();
11b    ap=ap->next;
12     i=i-1;
13     j=i;
14     while (j>0) {
15         bp->next=new List();
15b        bp=bp->next;
16         j=j-1;
17     }
17     ap->next = b; // conc
17b    y=a;
18     write(length(y)-2)
    }
```

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Example: Shape Analysis with Cycle

17:

slicing/srw17cyclic.eps

`cp=cyclicList();`

`ap->next;`

ADD: 6:

Result for 17:

Example: Comparison

Program	Alias Analysis	Inner Loop in Slice
Sum	None	No
Sum (ptrs)	Flow insensitive (weak update)	Yes
Sum (ptrs)	Flow sensitive (strong update)	No
Sum (dyn)	Heap represented by 1 node only	Yes
Sum (dyn)	Shape analysis (strong update)	No
Sum (dyn+cycle)	Shape analysis	Yes

Further Reading for Chapter 17

-  Flemming Nielson, Hanne Riis Nielson, Chris Hankin.
Principles of Program Analysis. 2nd edition, Springer-Verlag,
2005. (Chapter 1, Introduction; Chapter 2, Data Flow
Analysis; Chapter 6, Algorithms)

Part V

Conclusions and Prospectives

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Chapter 18

Summary and Outlook

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A Conclusion

...a question for the sense of life, or for what we did achieve resp.

- ▶ What did we consider?

The least!

Or vice versa...

- ▶ What did we not consider?

The most!

Especially not (or not in detail) (1)

- ▶ *Extensions of syntactic PRE beyond PDCE/PRAE*
 - ▶ Lazy Strength Reduction
 - ▶ ...
- ▶ *Semantic Extensions*
 - ▶ Semantic Code Motion/Code Placement
 - ▶ Semantic Strength Reduction
 - ▶ ...
- ▶ *Language Extensions*
 - ▶ Parallelität
 - ▶ ...

Especially not (or not in detail) (2)

- ▶ *Dynamic, profile-guided extensions*
 - ▶ Speculative PRE
 - ▶ ...
- ▶ ...

Hints to Further Reading (1)

- ▶ *Syntactic PRE*
 - ▶ Knoop, J., Rüthing, O., and Steffen, B. Retrospective: Lazy Code Motion. In "20 Years of the ACM SIGPLAN Conference on Programming Language Design and Implementation (1979 - 1999): A Selection", ACM SIGPLAN Notices 39, 4 (2004), 460 - 461 & 462-472.
 - ▶ Knoop, J., Rüthing, O., and Steffen, B. Optimal code motion: Theory and practice. ACM Transactions on Programming Languages and Systems 16, 4 (1994), 1117 - 1155.
 - ▶ Rüthing, O., Knoop, J., and Steffen, B. Sparse code motion. In Conference Record of the 27th Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages (POPL 2000) (Boston, MA, Jan. 19 - 21, 2000), ACM New York, (2000), 170 - 183.

Hints to Further Reading (2)

- ▶ *Eliminating partially dead code*
 - ▶ Knoop, J., Rüthing, O., and Steffen, B. Partial dead code elimination. In Proceedings of the ACM SIGPLAN'94 Conference on Programming Language Design and Implementation (PLDI'94) (Orlando, FL, USA, June 20 - 24, 1994), ACM SIGPLAN Notices 29, 6 (1994), 147 - 158.
- ▶ *Eliminating partially redundant assignments*
 - ▶ Knoop, J., Rüthing, O., and Steffen, B. The power of assignment motion. In Proceedings of the ACM SIGPLAN'95 Conference on Programming Language Design and Implementation (PLDI'95) (La Jolla, CA, USA, June 18 - 21, 1995), ACM SIGPLAN Notices 30, 6 (1995), 233 - 245.

Hints to Further Reading (3)

- ▶ *BB- vs. SI-Graphs*

- ▶ Knoop, J., Koschützki, D., and Steffen, B. Basic-block graphs: Living dinosaurs? In Proceedings of the 7th International Conference on Compiler Construction (CC'98) (Lisbon, Portugal, March 30 - April 3, 1998), Springer-Verlag, Heidelberg, LNCS 1383 (1998), 65 - 79.

- ▶ *Moving vs. Placing*

- ▶ Knoop, J., Rüthing, O., and Steffen, B. Code motion and code placement: Just synonyms? In Proceedings of the 7th European Symposium On Programming (ESOP'98) (Lisbon, Portugal, March 30 - April 3, 1998), Springer-Verlag, Heidelberg, LNCS 1381 (1998), 154 - 169.

Hints to Further Reading (4)

- ▶ *Speculative vs. classical PRE*
 - ▶ Scholz, B., Horspool, N. and Knoop, J. Optimizing for space and time usage with speculative partial redundancy elimination. In Proceedings of the ACM SIGPLAN/SIGBED 2004 Conference on Languages, Compilers, and Tools for Embedded Systems (LCTES 2004) (Washington, DC, June 11 - 13, 2004), ACM SIGPLAN Notices 39, 7 (2004), 221 -230.
 - ▶ Xue, J., Knoop, J. A fresh look at PRE as a maximum flow problem. In Proceedings of the 15th International Conference on Compiler Construction (CC 2006) (Vienna, Austria, March 25 - April 2, 2006), Springer-Verlag, Heidelberg, LNCS 3923 (2006), 139 - 154.

Hints to Further Reading (5)

- ▶ *Further Techniques and algorithms*
 - ▶ Geser, A., Knoop, J., Lüttgen, G., Rüthing, O., and Steffen, B. Non-monotone fixpoint iterations to resolve second order effects. In Proceedings of the 6th International Conference on Compiler Construction (CC'96) (Linköping, Sweden, April 24 - 26, 1996), Springer-V., Heidelberg, LNCS 1060 (1996), 106 - 120.
 - ▶ Knoop, J., and Mehofer, E. Optimal distribution assignment placement. In Proceedings of the 3rd European Conference on Parallel Processing (Euro-Par'97) (Passau, Germany, August 26 - 29, 1997), Springer-V., Heidelberg, LNCS 1300 (1997), 364 - 373.
 - ▶ Knoop, J., Rüthing, O., and Steffen, B. Lazy strength reduction. *Journal of Programming Languages* 1, 1 (1993), 71 - 91.
 - ▶ ...: siehe auch www.complang.tuwien.ac.at/knoop

Outlook

Emerging applications of (static) program analysis beyond optimization:

- ▶ Security Analysis
- ▶ Program Understanding
- ▶ Refactoring
- ▶ ...

...topics for master and PhD theses to come!

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Recommended Reading







...for deepened and independent studies.

- ▶ I Textbooks
- ▶ II Monographs
- ▶ III Volumes
- ▶ III Articles




I Textbooks (1)

-  Alfred V. Aho, Monica S. Lam, Ravi Sethi, Jeffrey D. Ullman. *Compilers: Principles, Techniques, & Tools*. Addison-Wesley, 2nd edition, 2007.
-  Randy Allen, Ken Kennedy. *Optimizing Compilers for Modern Architectures*. Morgan Kaufman Publishers, 2002.
-  A. Arnold, I. Guessarian. *Mathematics for Computer Science*. Prentice Hall, 1996.
-  Keith D. Cooper, Linda Torczon. *Engineering a Compiler*. Morgan Kaufman Publishers, 2004.
-  B. A. Davey, H. A. Priestley. *Introduction to Lattices and Order*. Cambridge Mathematical Textbooks, Cambridge University Press, 1990.


I Textbooks (2)

-  S. Even. *Graph Algorithms*. Pitman, 1979.
-  Matthew S. Hecht. *Flow Analysis of Computer Programs*. Elsevier, North-Holland, 1977.
-  Janusz Laski, William Stanley. *Software Verification and Analysis*. Springer-V., 2009.
-  Robert Morgan. *Building an Optimizing Compiler*. Digital Press, 1998.
-  Stephen S. Muchnick. *Advanced Compiler Design Implementation*. Morgan Kaufman Publishers, 1997.
-  Hanne Riis Nielson, Flemming Nielson. *Semantics with Applications: A Formal Introduction*. Wiley, 1992.

I Textbooks (3)

-  Hanne Riis Nielson, Flemming Nielson. *Semantics with Applications: An Appetizer*. Springer-V., 2007.
-  Flemming Nielson, Hanne Riis Nielson, Chris Hankin. *Principles of Program Analysis*. 2nd edition, Springer-V., 2005.
-  Peter Pepper, Petra Hofstedt. *Funktionale Programmierung: Sprachdesign und Programmieretechnik*. Springer-V., 2006.

II Monographs

-  Jens Knoop. *Optimal Interprocedural Program Optimization: A New Framework and Its Application*. Springer-V., LNCS 1428, 1998.

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III Volumes

-  Stephen S. Muchnick, Neil D. Jones. *Program Flow Analysis: Theory and Applications*. Prentice Hall, 1981.
-  Y. N. Srikant, Priti Shankar. *The Compiler Design Handbook: Optimizations and Machine Code Generation*. 1st edition, CRC Press, 2002.
-  Y. N. Srikant, Priti Shankar. *The Compiler Design Handbook: Optimizations and Machine Code Generation*. 2nd edition, CRC Press, 2008.




III Articles (1)

-  F. E. Allen, John A. Cocke. *A Program Data Flow Analysis Procedure*. Communications of the ACM 19(3):137-147, 1976.
-  F. E. Allen, John Cocke, Ken Kennedy. *Reduction of Operator Strength*. In Stephen S. Muchnick, Neil D. Jones (Eds.). *Program Flow Analysis: Theory and Applications*. Prentice Hall, 1981, Chapter 3, 79-101.
-  B. Alpern, Mark N. Wegman, F. Ken Zadeck. *Detecting Equality of Variables in Programs*. In Proceedings of POPL'88, 1-11, 1988.
-  D. Chase, Mark N. Wegmann, F. Ken Zadeck. *Analysis of Pointers and Structures*. In Proceedings PLDI'90, 296-310, 1990.





III Articles (2)

-  J-G Choi, M. Gupta, M. Serrano, V.C Sreedar, and Sam Midkiff. *Escape Analysis for Java*. In Proceedings of OOPSLA'99, ACM Press, 1-19, 1999.
-  Melvin E. Conway. *Proposal for an UNCOL*. Communications of the ACM 1(3):5, 1958.
-  D. M. Dhamdhere. *A New Algorithm for Composite Hoisting and Strength Reduction Optimisation (+ Corrigendum)*. International Journal of Computer Mathematics 27:1-14,31-32, 1989.
-  D. M. Dhamdhere. *Practical Adaptation of the Global Optimization Algorithm of Morel and Renvoise*. ACM Transactions on Programming Languages and Systems 13(2):291-294, 1991, Technical Correspondence.





III Articles (3)

-  D. M. Dhamdhere. *E-path_pre: Partial Redundancy Elimination Made Easy*. ACM SIGPLAN Notices 37(8):53-65, 2002.
-  D. M. Dhamdhere, J. R. Isaac. *A Composite Algorithm for Strength Reduction and Code Movement Optimization*. International Journal of Computer and Information Sciences 9(3):243-273, 1980.
-  K.-H. Drechsler, M. P. Stadel. *A Solution to a Problem with Morel and Renvoise's "Global Optimization by Suppression of Partial Redundancies"*. ACM Transactions on Programming Languages and Systems 10(4):635-640, 1988, Technical Correspondence.




III Articles (4)

-  K.-H. Drechsler, M. P. Stadel. *A variation of Knoop, Rütting and Steffen's LAZY CODE MOTION*. ACM SIGPLAN Notices 28(5):29-38, 1993.
-  M. Emami, R. Ghiya, Laurie J. Hendren. *Context-Sensitive Interprocedural Points-to Analysis in the Presence of Function Pointers*. In Proceedings PLDI'94, 1-14, 1994.
-  Andrei P. Ershov. *On Programming of Arithmetic Operations*. Communications of the ACM 1(8):3-6, 1958. (Three figures from this article are in CACM 1(9):16).
-  C. Fecht, Helmut Seidl. *An Even Faster Solver for General Systems of Equations*. In Proceedings SAS'96, LNCS 1145, 189-204, 1996.


III Articles (5)

-  C. Fecht, Helmut Seidl. *Propagating Differences: An Efficient New Fixpoint Algorithm for Distributive Constraint Systems*. In Proceedings ESOP'98, LNCS 1381, 90-104, 1998.
-  C. Fecht, Helmut Seidl. *A Faster Solver for General Systems of Equations*. *Science of Computer Programming* 35(2):137-161, 1999.
-  R. Ghiya, Laurie J. Hendren. *Is it a Tree, a DAG, or a Cyclic Graph?* In Proceedings POPL'96, 1-15, 1996.
-  R. Nigel Horspool, H. C. Ho. *Partial Redundancy Elimination Driven by a Cost-benefit Analysis*. In Proceedings of the 8th Israeli Conference on Computer Systems and Software Engineering (CSSE'97), 111-118, 1997.




III Articles (6)

-  Susan Horwitz, A. Demers, T. Teitelbaum. *An Efficient General Iterative Algorithm for Dataflow Analysis*. *Acta Informatica* 24:679-694, 1987.
-  S. M. Joshi, D. M. Dhamdhere. *A Composite Hoisting-strength Reduction Transformation for Global Program Optimization – Part I and Part II*. *International Journal of Computer Mathematics* 11:21-41,111-126, 1982.
-  John B. Kam, Jeffrey D. Ullman. *Global Data Flow Analysis and Iterative Algorithms*. *Journal of the ACM* 23:158-171, 1976.
-  John B. Kam, Jeffrey D. Ullman. *Monotone Data Flow Analysis Frameworks*. *Acta Informatica* 7:305-317, 1977.



III Articles (7)

-  Gary A. Kildall. *A Unified Approach to Global Program Optimization*. In Conference Record of the 1st Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages (POPL'73), 194-206, 1973.
-  Marion Klein, Jens Knoop, Dirk Koschützki, Bernhard Steffen. *DFA&OPT-METAFrame: A Toolkit for Program Analysis and Optimization*. In Proceedings TACAS'96, Springer-V., LNCS 1055, 422-426, 1996.
-  Jens Knoop. *Formal Callability and its Relevance and Application to Interprocedural Data Flow Analysis*. In Proceedings of the 6th IEEE International Conference on Computer Languages (ICCL'98), 252-261, 1998.




III Articles (8)

-  Jens Knoop. *From DFA-Frameworks to DFA-Generators: A Unifying Multiparadigm Approach*. In Proceedings of the 5th International Conference on Tools and Algorithms for the Construction and Analysis of Systems (TACAS'99), Springer-V., LNCS 1579, 360-374, 1999.
-  Jens Knoop, Dirk Koschützki, Bernhard Steffen. *Basic- block Graphs: Living Dinosaurs?* In Proceedings of the 7th International Conference on Compiler Construction (CC'98), Springer-V., LNCS 1383, 65-79, 1998.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Lazy Code Motion*. In Proceedings of the ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI'92), ACM SIGPLAN Notices 27(7):224-234, 1992.




III Articles (9)

-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Lazy Strength Reduction*. *Journal of Programming Languages* 1(1):71-91, 1993.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Optimal Code Motion: Theory and Practice*. *ACM Transactions on Programming Languages and Systems* 16(4):1117-1155, 1994.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Code Motion and Code Placement: Just Synonyms?* In *Proceedings of the 7th European Symposium on Programming (ESOP'98)*, Springer-V., LNCS 1381, 154-169, 1998.





III Articles (10)

-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Expansion-based Removal of Semantic Partial Redundancies*. In Proceedings of the 8th International Conference on Compiler Construction (CC'99), Springer-V., LNCS 1575, 91-106, 1999.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Partial Dead Code Elimination*. In Proceedings of the ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI'94), ACM SIGPLAN Notices 29(6):147-158, 1994.
-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *The Power of Assignment Motion*. In Proceedings of the ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI'95), ACM SIGPLAN Notices 30(6):233-245, 1995.

III Articles (11)

-  Jens Knoop, Oliver Rüthing, Bernhard Steffen. *Retro-spective: Lazy Code Motion*. In “20 Years of the ACM SIGPLAN Conference on Programming Language Design and Implementation (1979 - 1999): A Selection”, ACM SIGPLAN Notices 39(4):460-461&462-472, 2004.
-  Jens Knoop, Bernhard Steffen. *The Interprocedural Coincidence Theorem*. In Proceedings of the 4th International Conference on Compiler Construction (CC'92), Springer-V., LNCS 641, 125-140, 1992.
-  Jens Knoop, Bernhard Steffen. *Code Motion for Explicitly Parallel Programs*. In Proceedings of the 7th ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming (PPoPP'99), ACM SIGPLAN Notices 34(8):13-24, 1999.




III Articles (12)

-  Donald E. Knuth. *An Empirical Study of Fortran Programs*. *Software – Practice and Experience* 1:105-13, 1971.
-  Thomas J. Marlowe, Barbara G. Ryder. *Properties of Data Flow Frameworks*. *Acta Informatica* 28(2):121-163, 1990.
-  Florian Martin. *PAG - An Efficient Program Analyzer Generator*. *Journal of Software Tools for Technology Transfer* 2(1):46-67, 1998.
-  Etienne Morel, Claude Renvoise. *Global Optimization by Suppression of Partial Redundancies*. *Communications of the ACM* 22(2):96-103, 1979.




III Articles (13)

-  Flemming Nielson. *Semantics-directed Program Analysis: A Tool-maker's Perspective*. In Proceedings SAS'96, Springer-V., LNCS 1145, 2-21, 1996.
-  Flemming Nielson, Hanne Riis Nielson. *Finiteness Conditions for Fixed Point Iteration*. In Proceedings LFP'92, ACM Press, 96-108, 1992.
-  H. D. Pande, Barbara Ryder. *Data-flow-based Virtual Function Resolution*. In Proceedings SAS'96, Springer-V., LNCS 1145, 238-254, 1996.



III Articles (14)

-  Viktor Pavlu, Markus Schordan, Andreas Krall. *Computation of Alias Sets from Shape Graphs for Comparison of Shape Analysis Precision*. In Proceedings of the 11th International IEEE Working Conference on Source Code Analysis and Manipulation (SCAM 2011), 2011. [Best Paper Award SCAM 2011]
-  Barry K. Rosen. *High-level Data Flow Analysis*. Communications of the ACM 20(10):141-156, 1977.
-  Oliver R uthing, Jens Knoop, Bernhard Steffen. *Sparse Code Motion*. In Conference Record of the 27th Annual ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages (POPL 2000), 170-183, 2000.




III Articles (15)

-  Mooly Sagiv, Tom Reps, Reinhard Wilhelm. *Solving Shape-Analysis Problems in Languages with Destructive Updating*. *ACM Transactions on Programming* 20(1):1-50, 1998.
-  Mooly Sagiv, Tom Reps, Reinhard Wilhelm. *Parametric Shape Analysis via 3-Valued Logic*. In *Proceedings POPL'99*, 105-118, 1999.
-  Mooly Sagiv, Tom Reps, Reinhard Wilhelm. *Parametric Shape Analysis via 3-valued Logic*. *ACM Transactions on Programming Languages and Systems* 24(3), 2002.

III Articles (16)

-  Micha Sharir, Amir Pnueli. *Two Approaches to Interprocedural Data Flow Analysis*. In Stephen S. Muchnick, Neil D. Jones (Eds.). *Program Flow Analysis: Theory and Applications*. Prentice Hall, 1981, Chapter 7.3, The Functional Approach to Interprocedural Analysis, 196-209.
-  Micha Sharir, Amir Pnueli. *Two Approaches to Interprocedural Data Flow Analysis*. In Stephen S. Muchnick, Neil D. Jones (Eds.). *Program Flow Analysis: Theory and Applications*. Prentice Hall, 1981, Chapter 7.3, The Call-String Approach to Interprocedural Analysis, 210-217.


III Articles (17)

-  Bernhard Scholz, R. Nigel Horspool, Jens Knoop. *Optimizing for Space and Time Usage with Speculative Partial Redundancy Elimination*. Proceedings of the ACM SIGPLAN Workshop on Languages, Compilers, and Tools for Embedded Systems (LCTES 2004), ACM SIGPLAN Notices 39(7):221-230, 2004.
-  Bernhard Steffen. *Optimal Run Time Optimization – Proved by a New Look at Abstract Interpretation*. In Proceedings of the 2nd Joint Conference on Theory and Practice of Software Development (TAPSOFT'87), Springer-V., LNCS 249, 52-68, 1987.
-  Bernhard Steffen. *Property-Oriented Expansion*. In Proceedings of the 3rd Static Analysis Symposium (SAS'96), Springer-V., LNCS 1145, 22-41, 1996.

III Articles (18)

-  Bernhard Steffen, Jens Knoop, Oliver Rüthing. *The Value Flow Graph: A Program Representation for Optimal Program Transformations*. In Proceedings of the 3rd European Symposium on Programming (ESOP'90), Springer-V., LNCS 432, 389-405, 1990.
-  Bernhard Steffen, Jens Knoop, Oliver Rüthing. *Efficient Code Motion and an Adaption to Strength Reduction*. In Proceedings of the 4th International Joint Conference on Theory and Practice of Software Development (TAPSOFT'91), Springer-V., LNCS 494, 394-415, 1991.
-  Mooly Sagiv, Tom Reps, Susan Horwitz. *Precise Interprocedural Dataflow Analysis with Applications to Constant Propagation*. In Proceedings TAPSOFT'95, Springer-V., LNCS 915, 651-665, 1995.

III Articles (19)

-  Mooly Sagiv, Tom Reps, Reinhard Wilhelm. *Parametric Shape Analysis via 3-Valued Logic*. In Proceedings POPL'99, 105-118, 1999.

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A

Mathematical Foundations

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A.1

Sets and Relations

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Sets and Relations (1)

Definition (A.1.1)

Let M be a set and R a relation on M , i.e. $R \subseteq M \times M$.

Then R is called

- ▶ **reflexive** iff $\forall m \in M. m R m$
- ▶ **transitive** iff $\forall m, n, p \in M. m R n \wedge n R p \Rightarrow m R p$
- ▶ **anti-symmetric** iff $\forall m, n \in M. m R n \wedge n R m \Rightarrow m = n$

Sets and Relations (2)

Related notions (though less important for us here):

Definition (A.1.2)

Let M be a set and $R \subseteq M \times M$ a relation on M . Then R is called

- ▶ **symmetric** iff $\forall m, n \in M. m R n \iff n R m$
- ▶ **total** iff $\forall m, n \in M. m R n \vee n R m$

A.2

Partially Ordered Sets

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Partially Ordered Sets

Definition (A.2.1, Quasi-Order, Partial Order)

A relation R on M is called a

- ▶ **quasi-order** iff R is reflexive and transitive
- ▶ **partial order** iff R is reflexive, transitive, and anti-symmetric

For the sake of completeness we recall:

Definition (A.2.2, Equivalence Relation)

A relation R on M is called an

- ▶ **equivalence relation** iff R is reflexive, transitive, and symmetric

Remark

...a partial order is an anti-symmetric quasi-order, an equivalence relation a symmetric quasi-order.

Note: We here use terms like “partial order” as a short hand for the more accurate term “partially ordered set.”

Bounds, least and greatest Elements

Definition (A.2.3, Bounds, least/greatest Elements)

Let (Q, \sqsubseteq) be a quasi-order, let $q \in Q$ and $Q' \subseteq Q$.

Then q is called

- ▶ **upper (lower) bound** of Q' , in signs: $Q' \sqsubseteq q$ ($q \sqsubseteq Q'$), if for all $q' \in Q'$ holds: $q' \sqsubseteq q$ ($q \sqsubseteq q'$)
- ▶ **least upper (greatest lower) bound** of Q' , if q is an upper (lower) bound of Q' and for every other upper (lower) bound \hat{q} of Q' holds: $q \sqsubseteq \hat{q}$ ($\hat{q} \sqsubseteq q$)
- ▶ **greatest (least) element** of Q , if holds: $Q \sqsubseteq q$ ($q \sqsubseteq Q$)

Existence and Uniqueness of Bounds

We have:

- ▶ Given a partial order, least upper and greatest lower bounds are uniquely determined, if they exist.
- ▶ Given existence (and thus uniqueness), the least upper (greatest lower) bound of a set $P' \subseteq P$ of the basic set of a partial order (P, \sqsubseteq) is denoted by $\bigsqcup P'$ ($\bigsqcap P'$). These elements are also called **supremum** and **infimum** of P' .
- ▶ Analogously this holds for least and greatest elements. Given existence, these elements are usually denoted by \perp and \top .

A.3

Lattices

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Lattices and Complete Lattices

Definition (A.3.1, (Complete) Lattice)

Let (P, \sqsubseteq) be a partial order.

Then (P, \sqsubseteq) is called a

- ▶ **lattice**, if each **finite** subset P' of P contains a least upper and a greatest lower bound in P .
- ▶ **complete lattice**, if **each** subset P' of P contains a least upper and a greatest lower bound in P .

Hence:

...(complete) lattices are special partial orders.

Properties of Complete Lattices

Lemma (A.3.2)

Let (P, \sqsubseteq) be a complete lattice. Then we have:

1. $\perp = \bigsqcup \emptyset = \prod P$ is the least element of P .
2. $\top = \prod \emptyset = \bigsqcup P$ is the greatest element of P .

Lemma (A.3.3)

Let (P, \sqsubseteq) be a partial order. Then the following claims are equivalent:

1. (P, \sqsubseteq) is a complete lattice.
2. Every subset of P has a least upper bound.
3. Every subset of P has a greatest lower upper bound.

A.4

Complete Partially Ordered Sets

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Complete Partial Orders

...a slightly weaker notion than a lattice that, however, is often sufficient in computer science and thus often a more adequate notion:

Definition (A.4.1, Complete Partial Order)

Let (P, \sqsubseteq) be a partial order.

Then (P, \sqsubseteq) is called

- ▶ **complete**, or shorter a **CPO** (complete partial order), if each ascending chain $C \subseteq P$ has a least upper bound in P .

Remark

We have:

- ▶ A CPO (C, \sqsubseteq) (more accurate would be: “chain-complete partially ordered set (CCPO)”) has always a least element. This element is uniquely determined as the supremum of the empty chain and usually denoted by \perp : $\perp =_{df} \bigsqcup \emptyset$.

Definition (A.4.2, Chain)

Let (P, \sqsubseteq) be a partial order.

A subset $C \subseteq P$ is called

- ▶ **chain** of P , if the elements of C are totally ordered. For $C = \{c_0 \sqsubseteq c_1 \sqsubseteq c_2 \sqsubseteq \dots\}$ ($\{c_0 \supseteq c_1 \supseteq c_2 \supseteq \dots\}$) we also speak more precisely of an **ascending (descending)** chain of P .

A chain C is called

- ▶ **finite**, if C is finite; **infinite** otherwise.

Finite Chains, finite Elements

Definition (A.4.3, Chain-finite)

A partial order (P, \sqsubseteq) is called

- ▶ **chain-finite** (German: kettenendlich) iff P does not contain infinite chains

Definition (A.4.4, Finite Elements)

An element $p \in P$ is called

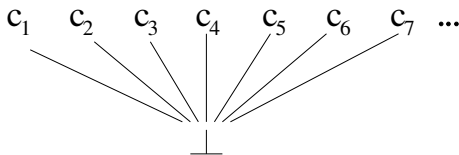
- ▶ **finite** iff the set $Q =_{df} \{q \in P \mid q \sqsubseteq p\}$ is free of infinite chains
- ▶ **finite relative to $r \in P$** iff the set $Q =_{df} \{q \in P \mid r \sqsubseteq q \sqsubseteq p\}$ does not contain infinite chains

(Standard) CPO Constructions (1)

Flat CPOs.

Let (C, \sqsubseteq) be a CPO. Then (C, \sqsubseteq) is called

- ▶ **flat**, if for all $c, d \in C$ holds: $c \sqsubseteq d \Leftrightarrow c = \perp \vee c = d$



(Standard) CPO Constructions (2)

Product construction.

Let $(P_1, \sqsubseteq_1), (P_2, \sqsubseteq_2), \dots, (P_n, \sqsubseteq_n)$ be CPOs. Then

- ▶ the **non-strict (direct) product** $(\times P_i, \sqsubseteq)$ with
 - ▶ $(\times P_i, \sqsubseteq) = (P_1 \times P_2 \times \dots \times P_n, \sqsubseteq)$ with
 $\forall (p_1, p_2, \dots, p_n),$
 $(q_1, q_2, \dots, q_n) \in \times P_i. (p_1, p_2, \dots, p_n) \sqsubseteq$
 $(q_1, q_2, \dots, q_n) \Leftrightarrow \forall i \in \{1, \dots, n\}. p_i \sqsubseteq_i q_i$
- ▶ and the **strict (direct) product (smash product)** with
 - ▶ $(\otimes P_i, \sqsubseteq) = (P_1 \otimes P_2 \otimes \dots \otimes P_n, \sqsubseteq)$, where \sqsubseteq is defined as above under the additional constraint:

$$(p_1, p_2, \dots, p_n) = \perp \Leftrightarrow \exists i \in \{1, \dots, n\}. p_i = \perp_i$$

are CPOs, too.

(Standard) CPO Constructions (3)

Sum construction.

Let $(P_1, \sqsubseteq_1), (P_2, \sqsubseteq_2), \dots, (P_n, \sqsubseteq_n)$ CPOs. Then

- ▶ the **direct sum** $(\bigoplus P_i, \sqsubseteq)$ with
 - ▶ $(\bigoplus P_i, \sqsubseteq) = (P_1 \dot{\cup} P_2 \dot{\cup} \dots \dot{\cup} P_n, \sqsubseteq)$ disjoint union of $P_i, i \in \{1, \dots, n\}$ and $\forall p, q \in \bigoplus P_i. p \sqsubseteq q \Leftrightarrow \exists i \in \{1, \dots, n\}. p, q \in P_i \wedge p \sqsubseteq_i q$

is a CPO.

Note: The least elements of $(P_i, \sqsubseteq_i), i \in \{1, \dots, n\}$, are usually identified, i.e., $\perp =_{df} \perp_i, i \in \{1, \dots, n\}$

(Standard) CPO Constructions (4)

Function-space construction.

Let (C, \sqsubseteq_C) and (D, \sqsubseteq_D) be two CPOs and $[C \rightarrow D] =_{df} \{f : C \rightarrow D \mid f \text{ continuous}\}$ the set of continuous functions from C to D .

Then

- ▶ the **continuous function space** $([C \rightarrow D], \sqsubseteq)$ is a CPO where
 - ▶ $\forall f, g \in [C \rightarrow D]. f \sqsubseteq g \iff \forall c \in C. f(c) \sqsubseteq_D g(c)$

Monotonic, Continuous Functions on CPOs

Definition (A.4.5, Monotonic, Continuous Function)

Let (C, \sqsubseteq_C) and (D, \sqsubseteq_D) be two CPOs and let $f : C \rightarrow D$ be a function from C to D .

Then f is called

- ▶ **monotonic** iff $\forall c, c' \in C. c \sqsubseteq_C c' \Rightarrow f(c) \sqsubseteq_D f(c')$
(Preservation of the ordering of elements)
- ▶ **continuous** iff $\forall C' \subseteq C. f(\bigsqcup_C C') =_D \bigsqcup_D f(C')$
(Preservation of least upper bounds)

Properties

Using the notations introduced before, we have:

Lemma (A.4.6)

f is monotonic iff $\forall C' \subseteq C. f(\bigsqcup_C C') \sqsupseteq_D \bigsqcup_D f(C')$

Corollary (A.4.7)

A continuous function is always monotonic, i.e. f continuous implies f monotonic.

Inflationary Functions on CPOs

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Definition (A.4.8, Inflationary Function)

Let (C, \sqsubseteq) be a CPO and let $f : C \rightarrow C$ be a function on C . Then f is called

- ▶ **inflationary (increasing)** iff $\forall c \in C. c \sqsubseteq f(c)$

A.5

Fixed Point Theorems

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Least and Greatest Fixed Points

Definition (A.5.1, (Least/Greatest) Fixed Point)

Let (C, \sqsubseteq) be a CPO, $f : C \rightarrow C$ be a function on C and let c be an element of C , i.e., $c \in C$.

Then c is called

- ▶ **fixed point** of f iff $f(c) = c$

A fixed point c of f is called

- ▶ **least fixed point** of f iff $\forall d \in C. f(d) = d \Rightarrow c \sqsubseteq d$
- ▶ **greatest fixed point** of f iff $\forall d \in C. f(d) = d \Rightarrow d \sqsubseteq c$

Notation:

- ▶ The **least** resp. **greatest fixed point** of a function f is usually denoted by μf resp. νf .

Conditional Fixed Points (2)

Definition (A.5.2, Conditional Fixed Point)

Let (C, \sqsubseteq) be a CPO, $f : C \rightarrow C$ be a function on C and let $d, c_d \in C$.

Then c_d is called

- ▶ **conditional (German: bedingter) least fixed point** of f wrt d iff c_d is the least fixed point of C with $d \sqsubseteq c_d$, i.e. for all other fixed points x of f with $d \sqsubseteq x$ holds: $c_d \sqsubseteq x$.

Fixed Point Theorem

Theorem (A.5.3, Knaster/Tarski, Kleene)

Let (C, \sqsubseteq) be a CPO and let $f : C \rightarrow C$ be a continuous function on C .

Then f has a least fixed point μf , which equals the least upper bound of the chain (so-called *Kleene-Chain*) $\{\perp, f(\perp), f^2(\perp), \dots\}$, i.e.

$$\mu f = \bigsqcup_{i \in \mathbb{N}_0} f^i(\perp) = \bigsqcup \{\perp, f(\perp), f^2(\perp), \dots\}$$

Proof of Fixed Point Theorem A.5.3 (1)

We have to prove:

μf

1. exists
2. is a fixed point
3. is the least fixed point

of f .

Proof of Fixed Point Theorem A.5.3 (2)

1. Existence

- ▶ It holds $f^0 \perp = \perp$ and $\perp \sqsubseteq c$ for all $c \in C$.
- ▶ By means of (natural) induction we can show: $f^n \perp \sqsubseteq f^n c$ for all $c \in C$.
- ▶ Thus we have $f^n \perp \sqsubseteq f^m \perp$ for all n, m with $n \leq m$. Hence, $\{f^n \perp \mid n \geq 0\}$ is a (non-finite) chain of C .
- ▶ The existence of $\bigsqcup_{i \in \mathbb{N}_0} f^i(\perp)$ is thus an immediate consequence of the CPO properties of (C, \sqsubseteq) .

Proof of Fixed Point Theorem A.5.3 (3)

2. Fixed point property

$$\begin{aligned} (f \text{ continuous}) &= f(\bigsqcup_{i \in \mathbb{N}_0} f^i(\perp)) \\ &= \bigsqcup_{i \in \mathbb{N}_0} f(f^i \perp) \\ &= \bigsqcup_{i \in \mathbb{N}_1} f^i \perp \\ (K \text{ chain} \Rightarrow \bigsqcup K = \perp \sqcup \bigsqcup K) &= (\bigsqcup_{i \in \mathbb{N}_1} f^i \perp) \sqcup \perp \\ (f^0 \perp = \perp) &= \bigsqcup_{i \in \mathbb{N}_0} f^i \perp \end{aligned}$$

Proof of Fixed Point Theorem A.5.3 (4)

3. Least fixed point

- ▶ Let c be an arbitrarily chosen fixed point of f . Then we have $\perp \sqsubseteq c$, and hence also $f^n \perp \sqsubseteq f^n c$ for all $n \geq 0$.
- ▶ Thus, we have $f^n \perp \sqsubseteq c$ because of our choice of c as fixed point of f .
- ▶ Thus, we also have that c is an upper bound of $\{f^i(\perp) \mid i \in \mathbb{N}_0\}$.
- ▶ Since $\bigsqcup_{i \in \mathbb{N}_0} f^i(\perp)$ is the least upper bound of this chain by definition, we obtain as desired $\bigsqcup_{i \in \mathbb{N}_0} f^i(\perp) \sqsubseteq c$.

□

Conditional Fixed Points

Theorem (A.5.4, Conditional Fixed Points)

Let (C, \sqsubseteq) be a CPO, let $f : C \rightarrow C$ be a continuous, inflationary function on C , and let $d \in C$.

Then f has a unique conditional fixed point μf_d . This fixed point equals the least upper bound of the chain $\{d, f(d), f^2(d), \dots\}$, i.e.

$$\mu f_d = \bigsqcup_{i \in \mathbb{N}_0} f^i(d) = \bigsqcup \{d, f(d), f^2(d), \dots\}$$

Finite Fixed Points

Theorem (A.5.5, Finite Fixed Points)

Let (C, \sqsubseteq) be a CPO and let $f : C \rightarrow C$ be a continuous function on C .





Then we have: If two elements in a row occurring in the Kleene-chain of f are equal, e.g. $f^i(\perp) = f^{i+1}(\perp)$, then we have: $\mu f = f^i(\perp)$.

Existence of Finite Fixed Points




Sufficient conditions for the existence of finite fixed points
e.g. are

- ▶ Finiteness of domain and range of f
- ▶ f is of the form $f(c) = c \sqcup g(c)$ for monotone g on some chain-complete domain

Appendix A: Further Reading (1)

-  A. Arnold, I. Guessarian. *Mathematics for Computer Science*. Prentice Hall, 1996.
-  B. A. Davey, H. A. Priestley. *Introduction to Lattices and Order*. Cambridge Mathematical Textbooks, Cambridge University Press, 1990.
-  Flemming Nielson, Hanne Riis Nielson. *Finiteness Conditions for Fixed Point Iteration*. In Proceedings LFP'92, ACM Press, 96-108, 1992.
-  Hanne Riis Nielson, Flemming Nielson. *Semantics with Applications: A Formal Introduction*. Wiley, 1992. (Chapter 4, Denotational Semantics)

Appendix A: Further Reading (2)

-  Hanne Riis Nielson, Flemming Nielson. *Semantics with Applications: An Appetizer*. Springer-V., 2007. (Chapter 5, Denotational Semantics)
-  Flemming Nielson, Hanne Riis Nielson, Chris Hankin. *Principles of Program Analysis*. 2nd edition, Springer-V., 2005. (Appendix A, Partially Ordered Sets)
-  Peter Pepper, Petra Hofstedt. *Funktionale Programmierung: Sprachdesign und Programmieretechnik*. Springer-V., 2006. (Chapter 10, Beispiel: Berechnung von Fixpunkten)

B

Intricacies of Basis Block Graphs

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Chapter B.1

Motivation

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Basic Block vs. Single Instruction Graphs

In this chapter we investigate the **adequacy** of different program representations.

To this end we will consider and compare programs in form of **node and edge-labelled flow graphs** with **basic blocks** and **single instructions** and investigate their

- ▶ **advantages and disadvantages for program analysis**

...thereby addressing the question:

- ▶ **Basic Block vs. Single Instruction Graphs: Just a Matter of Taste?**

On the fly we will learn:

- ▶ **Some further examples of real world data flow analysis problems and data flow analyses.**

Basic Blocks: Supposed Advantages

Advantages of basic blocks in applications that are commonly attributed to them (“folk knowledge”):

Better scalability because

- ▶ less nodes are involved in the (potentially) computationally expensive fixed point iteration and thus
- ▶ larger programs fit into the main memory.

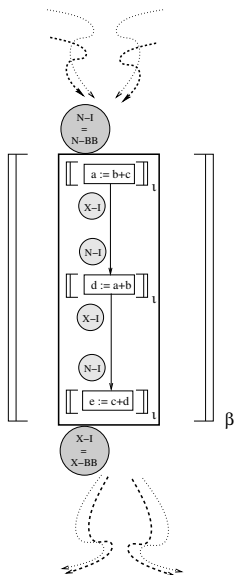
Basic Blocks: Definite Disadvantages

Definite disadvantages of basic blocks in applications:

- ▶ **Higher conceptual complexity:** Basic blocks introduce an undesired **hierarchy** into flow graphs that makes both theoretical reasoning and practical implementations more difficult.
- ▶ **Necessity of pre- and post-processes:** These are usually required in order to cope with the additional problems introduced by the hierarchical structure of basic block flow graph (e.g. in **dead code elimination**, **constant propagation**,...); or that require “tricky” formulations to avoid them (e.g. in **partial redundancy elimination**).
- ▶ **Limited generality:** Some practically relevant program analyses and optimizations are difficult or not at all expressible on the level of basic block flow graphs (e.g. **faint variable elimination**).

Hierarchy by Basic Blocks

Illustration:



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In the following

Investigating the

- ▶ advantages and disadvantages of **basic block (BB)** flow graphs compared to **single instructions (SI)** flow graphs.

by means of examples

- ▶ of some data flow analysis problems we already considered
 - ▶ **Availability of expressions**
 - ▶ **Simple constants**

and some new ones:

- ▶ **Faint variables**

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Edge-labelled Single Instruction Approach

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The MOP_{SI} Approach

...for edge-labelled single instruction flow graphs.

The MOP Solution:

$$\forall c_s \in \mathcal{C} \forall n \in N. MOP_{(\llbracket \cdot \rrbracket_l, c_s)}(n) =_{df} \bigsqcap \{ \llbracket p \rrbracket_l(c_s) \mid p \in \mathbf{P}_G[s, n] \}$$

The $MaxFP_{SI}$ Approach

...for edge-labelled single instruction flow graphs.

The $MaxFP$ Solution:

$$\forall c_s \in \mathcal{C} \forall n \in N. MaxFP_{(\llbracket \cdot \rrbracket_l, c_s)}(n) =_{df} \text{inf}_{c_s}^*(n)$$

where $\text{inf}_{c_s}^*$ denotes the **greatest solution** of the $MaxFP$ Equation System:

$$\text{inf}(n) = \begin{cases} c_s & \text{if } n = s \\ \prod \{ \llbracket (m, n) \rrbracket_l(\text{inf}(m)) \mid m \in \text{pred}_G(n) \} & \text{otherwise} \end{cases}$$

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Node-labelled Basic Block Approach

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Notations

In the following we denote

- ▶ **basic block nodes** by boldface letters ($\mathbf{m}, \mathbf{n}, \dots$)
- ▶ **single instruction nodes** by normalface letters (m, n, \dots)

Furthermore we denote by

- ▶ $\llbracket \cdot \rrbracket_\beta$ and
- ▶ $\llbracket \cdot \rrbracket_\iota$

(local) abstract data flow analysis functionals on the level of **basic blocks** and **single instructions**, respectively.

The MOP_{BB} Approach (1)

...for node-labelled basic block flow graphs.

The MOP Solution on the BB-Level:

$$\forall c_s \in \mathcal{C} \quad \forall \mathbf{n} \in \mathbf{N}. \quad MOP_{(\llbracket \cdot \rrbracket_\beta, c_s)}(\mathbf{n}) =_{df} \\ (N-MOP_{(\llbracket \cdot \rrbracket_\beta, c_s)}(\mathbf{n}), X-MOP_{(\llbracket \cdot \rrbracket_\beta, c_s)}(\mathbf{n}))$$

with

$$N-MOP_{(\llbracket \cdot \rrbracket_\beta, c_s)}(\mathbf{n}) =_{df} \bigcap \{ \llbracket p \rrbracket_\beta(c_s) \mid p \in \mathbf{P}_G[\mathbf{s}, \mathbf{n}] \}$$

$$X-MOP_{(\llbracket \cdot \rrbracket_\beta, c_s)}(\mathbf{n}) =_{df} \bigcap \{ \llbracket p \rrbracket_\beta(c_s) \mid p \in \mathbf{P}_G[\mathbf{s}, \mathbf{n}] \}$$

The MOP_{BB} Approach (2)

...and its continuation on the SI-Level:

$$\forall c_s \in \mathcal{C} \forall n \in \mathbb{N}. MOP_{(\mathbb{I}_l, c_s)}(n) =_{df} \\ (N-MOP_{(\mathbb{I}_l, c_s)}(n), X-MOP_{(\mathbb{I}_l, c_s)}(n))$$

The MOP_{BB} Approach (3)

...with

$$N-MOP_{(\llbracket \cdot \rrbracket_\ell, c_s)}(n) =_{df} \begin{cases} N-MOP_{(\llbracket \cdot \rrbracket_\beta, c_s)}(\text{block}(n)) \\ \quad \text{if } n = \text{start}(\text{block}(n)) \\ \\ \llbracket p \rrbracket_\ell(N-MOP_{(\llbracket \cdot \rrbracket_\beta, c_s)}(\text{block}(n))) \\ \quad \text{otherwise } (p \text{ prefix path from} \\ \quad \text{start}(\text{block}(n)) \text{ to (exclusively) } n) \end{cases}$$

$$X-MOP_{(\llbracket \cdot \rrbracket_\ell, c_s)}(n) =_{df} \llbracket p \rrbracket_\ell(N-MOP_{(\llbracket \cdot \rrbracket_\beta, c_s)}(\text{block}(n))) \\ (p \text{ prefix path from } \text{start}(\text{block}(n)) \\ \text{up to (inclusively) } n)$$

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The $MaxFP_{BB}$ Approach (1)

...for node-labelled basic block flow graphs:

The $MaxFP$ Solution on the BB-Level:

$$\forall c_s \in \mathcal{C} \forall \mathbf{n} \in \mathbf{N}. MaxFP_{(\llbracket \cdot \rrbracket_{\beta}, c_s)}(\mathbf{n}) =_{df} \\ (N-MFP_{(\llbracket \cdot \rrbracket_{\beta}, c_s)}(\mathbf{n}), X-MFP_{(\llbracket \cdot \rrbracket_{\beta}, c_s)}(\mathbf{n}))$$

with

$$N-MFP_{(\llbracket \cdot \rrbracket_{\beta}, c_s)}(\mathbf{n}) =_{df} pre_{c_s}^{\beta}(\mathbf{n}) \quad \text{and}$$

$$X-MFP_{(\llbracket \cdot \rrbracket_{\beta}, c_s)}(\mathbf{n}) =_{df} post_{c_s}^{\beta}(\mathbf{n})$$

The $MaxFP_{BB}$ Approach (2)

...where $pre_{c_s}^\beta$ and $post_{c_s}^\beta$ denote the **greatest solution** of the equation system

$$pre(\mathbf{n}) = \begin{cases} c_s & \text{if } \mathbf{n} = \mathbf{s} \\ \prod \{ post(\mathbf{m}) \mid \mathbf{m} \in pred_G(\mathbf{n}) \} & \text{otherwise} \end{cases}$$

$$post(\mathbf{n}) = \llbracket \mathbf{n} \rrbracket_\beta(pre(\mathbf{n}))$$

The $MaxFP_{BB}$ Approach (3)

...and its continuation on the SI-Level:

$$\forall c_s \in \mathcal{C} \forall n \in N. MaxFP_{(\llbracket \cdot \rrbracket_{\ell}, c_s)}(n) =_{df} \\ (N-MFP_{(\llbracket \cdot \rrbracket_{\ell}, c_s)}(n), X-MFP_{(\llbracket \cdot \rrbracket_{\ell}, c_s)}(n))$$

with

$$N-MFP_{(\llbracket \cdot \rrbracket_{\ell}, c_s)}(n) =_{df} \text{pre}_{c_s}^{\ell}(n) \quad \text{and}$$

$$X-MFP_{(\llbracket \cdot \rrbracket_{\ell}, c_s)}(n) =_{df} \text{post}_{c_s}^{\ell}(n)$$

The $MaxFP_{BB}$ Approach (4)

...where $pre_{c_s}^l$ and $post_{c_s}^l$ denote the **greatest solution** of the equation system

$$pre(n) = \begin{cases} pre_{c_s}^\beta(\text{block}(n)) \\ \quad \text{if } n = \text{start}(\text{block}(n)) \\ \\ post(m) \\ \quad \text{otherwise (} m \text{ is here the uniquely} \\ \quad \text{determined predecessor of } n \\ \quad \text{in } \text{block}(n) \text{)} \end{cases}$$

$$post(n) = \llbracket n \rrbracket_l(pre(n))$$

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Availability of Expressions

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Availability of Expressions (1)

...for node-labelled basic-block flow graphs.

Stage I: The Basic-block Level

Local Predicates (associated with BB-nodes):

- ▶ $\text{BB-XCOMP}_{\beta}(t)$: β contains a statement ι that computes t , and neither ι nor a statement following ι in β modifies an operand of t .
- ▶ $\text{BB-TRANSP}_{\beta}(t)$: β does not contain a statement that modifies an operand of t .

Availability of Expressions (2)

The BB-Equation System of Stage I:

$$\text{BB-N-AVAIL}_\beta = \begin{cases} \mathbf{false} & \text{if } \beta = \mathbf{s} \\ \prod_{\hat{\beta} \in \text{pred}(\beta)} \text{BB-X-AVAIL}_{\hat{\beta}} & \text{otherwise} \end{cases}$$

$$\text{BB-X-AVAIL}_\beta = \text{BB-N-AVAIL}_\beta \cdot \text{BB-TRANSP}_\beta + \text{BB-XCOMP}_\beta$$

Availability of Expressions (3)

Stage II: The Instruction Level

Lokale Prädikate (associated with SI-nodes):

- ▶ $COMP_{\iota}(t)$: ι computes t .
- ▶ $TRANSP_{\iota}(t)$: ι does not modify an operand of t .
- ▶ $BB-N-AVAIL^*$, $BB-X-AVAIL^*$: the greatest solution of the equation system of Stage I.

The SI-Equation System of Stage II:

$$N-AVAIL_{\iota} = \begin{cases} BB-N-AVAIL_{\text{block}(\iota)}^* & \text{if } \iota = \text{start}(\text{block}(\iota)) \\ X-AVAIL_{\text{pred}(\iota)} & \text{otherwise} \end{cases}$$

(note: $|\text{pred}(\iota)| = 1$)

$$X-AVAIL_{\iota} = \begin{cases} BB-X-AVAIL_{\text{block}(\iota)}^* & \text{if } \iota = \text{end}(\text{block}(\iota)) \\ (N-AVAIL_{\iota} + COMP_{\iota}) \cdot TRANSP_{\iota} & \text{otherwise} \end{cases}$$

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Availability of Expressions

...for node-labelled single instruction flow graphs.

Local Predicates (associated with SI-nodes):

- ▶ $\text{COMP}_\iota(t)$: ι computes t .
- ▶ $\text{TRANSP}_\iota(t)$: ι does not modify an operand of t .

The EA-Equation System:

$$\text{N-AVAIL}_\iota = \begin{cases} \text{false} & \text{if } \iota = s \\ \prod_{\hat{\iota} \in \text{pred}(\iota)} \text{X-AVAIL}_{\hat{\iota}} & \text{otherwise} \end{cases}$$

$$\text{X-AVAIL}_\iota = (\text{N-AVAIL}_\iota + \text{COMP}_\iota) \cdot \text{TRANSP}_\iota$$

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Edge-labelled Single Instruction Approach

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Availability of Expressions

...for edge-labelled single-instruction flow graphs.

Locale Predicates (associated with SI-edges):

- ▶ $\text{COMP}_\varepsilon(t)$: Statement ι of edge ε computes t .
- ▶ $\text{TRANSP}_\varepsilon(t)$: Statement ι of edge ε does not modify an operand of t .

The SI-Equation System:

$$\text{Avail}_n = \begin{cases} \mathbf{false} & \text{if } n = s \\ \prod_{m \in \text{pred}(n)} (\text{Avail}_m + \text{COMP}_{(m,n)}) \cdot \text{TRANSP}_{(m,n)} & \\ \text{otherwise} & \end{cases}$$

Outlook

Next we consider two further examples in order to illustrate the impact of the chosen flow graph representation variant on the conceptual and practical complexity of data flow analysis:

- ▶ Constant propagation and folding
- ▶ Faint variable elimination

To this end we consider formulations of these problems for:

- ▶ node-labelled basic-block flow graphs
- ▶ edge-labelled single instruction flow graphs

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Constant Propagation and Folding

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Constant Propagation and Folding

...considering the example of so-called [simple constants](#).

To this end we need two auxiliary functions:

- ▶ [Backward substitution](#)
- ▶ [State transformation](#)

Backward Substitution and State Transformation for Assignments

Let $\iota \equiv (x := t)$ be a statement. Then we define:

- ▶ **Backward substitution**

$\delta_\iota : \mathbf{T} \rightarrow \mathbf{T}$ by $\delta_\iota(s) =_{df} s[t/x]$ for all $s \in \mathbf{T}$, where $s[t/x]$ denotes the simultaneous replacement of all occurrences of x by t in s .

- ▶ **State transformation**

$$\theta_\iota(\sigma)(y) =_{df} \begin{cases} \mathcal{E}(t)(\sigma) & \text{if } y = x \\ \sigma(y) & \text{otherwise} \end{cases}$$

The Relationship of δ and θ

Let \mathcal{I} denote the set of all statements.

Lemma (B.3.1, Substitution Lemma)

$$\forall t \in \mathbf{T} \forall \sigma \in \Sigma \forall \iota \in \mathcal{I}. \mathcal{E}(\delta_\iota(t))(\sigma) = \mathcal{E}(t)(\theta_\iota(\sigma))$$

Proof by induction on the structure of t .

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Simple Constants

...for edge-labelled single instruction flow graphs.

- ▶ $CP_n \in \Sigma$
- ▶ $\sigma_0 \in \Sigma$ start information

The SI-Equation System:

$$\forall v \in \mathbf{V}. CP_n = \begin{cases} \sigma_0(v) & \text{if } n = s \\ \prod \{ \mathcal{E}(\delta_{(m,n)}(v))(CP_m) \mid m \in \text{pred}(n) \} & \text{otherwise} \end{cases}$$

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Backward Substitution and State Transformation on Paths

Extending δ and θ to paths (and hence to basic blocks, too):

- ▶ $\Delta_p : \mathbf{T} \rightarrow \mathbf{T}$ defined by $\Delta_p =_{df} \delta_{n_q}$ for $q = 1$ and by $\Delta_{(n_1, \dots, n_{q-1})} \circ \delta_{n_q}$ for $q > 1$
- ▶ $\Theta_p : \Sigma \rightarrow \Sigma$ defined by $\Theta_p =_{df} \theta_{n_1}$ for $q = 1$ and by $\Theta_{(n_2, \dots, n_q)} \circ \theta_{n_1}$ for $q > 1$.

The Relationship of Δ and Θ

Let \mathcal{B} denote the set of all basic blocks.

Lemma (B.3.1.1, Generalized Substitution Lemma)

$$\forall t \in \mathbf{T} \forall \sigma \in \Sigma \forall \beta \in \mathcal{B}. \mathcal{E}(\Delta_\beta(t))(\sigma) = \mathcal{E}(t)(\Theta_\beta(\sigma))$$

Proof by induction on the length of p .

Simple Constants (1)

...for node-labelled basic-block flow graphs.

Stage I: Basic-block Level

Remark:

- ▶ $\Delta_\beta(v) =_{df} \delta_{\iota_1} \circ \dots \circ \delta_{\iota_q}(v)$, where $\beta \equiv \iota_1; \dots; \iota_q$.
- ▶ $\text{BB-N-CP}_\beta, \text{BB-X-CP}_\beta, \text{N-CP}_\iota, \text{X-CP}_\iota \in \Sigma$
- ▶ $\sigma_0 \in \Sigma$ start information

Simple Constants (2)

The BB-Equation System of Stage I:

$$\text{BB-N-CP}_\beta = \begin{cases} \sigma_0 & \text{if } \beta = \mathbf{s} \\ \prod\{\text{BB-X-CP}_{\hat{\beta}} \mid \hat{\beta} \in \text{pred}(\beta)\} & \\ \text{otherwise} & \end{cases}$$

$$\forall v \in \mathbf{V}. \text{BB-X-CP}_\beta(v) = \mathcal{E}(\Delta_\beta(v))(\text{BB-N-CP}_\beta)$$

Simple Constants (3)

Stage II: Instruction Level

Pre-computed Results (of Stage I):

- ▶ $BB-N-CP^*$, $BB-X-CP^*$: the greatest solution of the equation system of Stage I.

Simple Constants (4)

The SI-Equation System of Stage II:

$$\text{N-CP}_\iota = \begin{cases} \text{BB-N-CP}_{\text{block}(\iota)}^* & \text{if } \iota = \text{start}(\text{block}(\iota)) \\ \text{X-CP}_{\text{pred}(\iota)} & \text{otherwise (note: } |\text{pred}(\iota)| = 1) \end{cases}$$

$$\forall v \in \mathbf{V}. \text{X-CP}_\iota(v) = \begin{cases} \text{BB-X-CP}_{\text{block}(\iota)}^*(v) & \text{if } \iota = \text{end}(\text{block}(\iota)) \\ \mathcal{E}(\delta_\iota(v))(\text{N-CP}_\iota) & \text{otherwise} \end{cases}$$

Chapter B.4

Faint Variables

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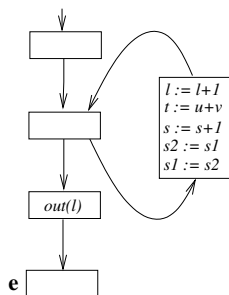
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Motivation

Statement

- ▶ $l := l + 1$ is **live**.
- ▶ $t := u + v$ is **dead**.
- ▶ $s := s + 1$ as well as $s1 := s2; s2 := s1$ are live but **faint** (schwach, kraftlos, ohnmächtig, schattenhaft).



Preliminaries

...for edge-labelled single instruction flow graphs.

Local Predicates (associated with single instruction edges):

- ▶ $USED_{\varepsilon}(v)$: Statement ι of edge ε uses v .
- ▶ $MOD_{\varepsilon}(v)$: Statement ι of edge ε modifies v .
- ▶ $REL-USED_{\varepsilon}(v)$: v is a variable that occurs in the statement ι of the edge ε and “is forced to live” by it (e.g. for ι being an output operation).
- ▶ $ASS-USED_{\varepsilon}(v)$: v is a variable that occurs in the right-hand side expression of the assignment ι of the edge ε .

Faint Variable Analysis

The SI-Equation System:

$$\begin{aligned} \text{FAINT}_n(v) = & \\ & \prod_{m \in \text{succ}(n)} \overline{\text{REL-USED}_{(n,m)}(v)} * \\ & (\text{FAINT}_m(v) + \text{MOD}_{(n,m)}(v)) * \\ & (\text{FAINT}_m(\text{LhsVar}_{(n,m)}) + \overline{\text{ASS-USED}_{(n,m)}(v)}) \end{aligned}$$

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Summary

The faint variables problem is an example of a DFA problem, for which a formulation is

- ▶ obvious on (node- and edge-labelled) single instruction flow graphs,
- ▶ not at all obvious, if not impossible at all on (node- and edge-labelled) basic-block flow graphs.

Chapter B.5

Conclusion

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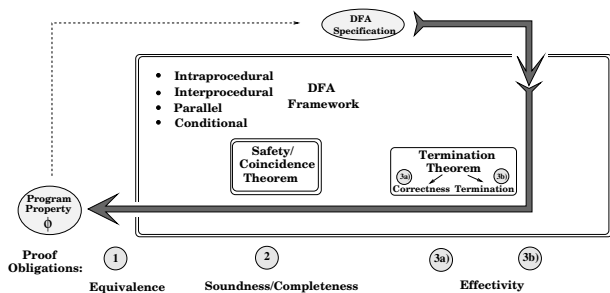
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Conclusion




In principle, all 4 representation variants of flow graphs are **equally powerful**.

Hence, **conceptually** the general framework resp. tool kit view



and knowing that the variants differ in their adequacy and in the **specification**, **implementation** and **proof obligations** they require depending on the task at hand suffices.

Appendix B: Further Reading

-  Alfred V. Aho, Monica S. Lam, Ravi Sethi, Jeffrey D. Ullman. *Compilers: Principles, Techniques, & Tools*. Addison-Wesley, 2nd edition, 2007. (Chapter 9.4, Constant Propagation)
-  Jens Knoop. *From DFA-Frameworks to DFA-Generators: A Unifying Multiparadigm Approach*. In Proceedings of the 5th International Conference on Tools and Algorithms for the Construction and Analysis of Systems (TACAS'99), Springer-V., LNCS 1579, 360-374, 1999.
-  Jens Knoop, Dirk Koschützki, Bernhard Steffen. *Basic-block Graphs: Living Dinosaurs?* In Proceedings of the 7th International Conference on Compiler Construction (CC'98), Springer-V., LNCS 1383, 65 - 79, 1998.