Data-Flow Analysis for Hot-Spot Program Optimization

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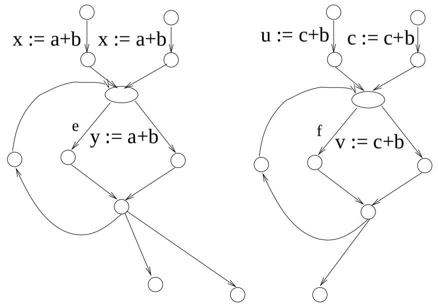
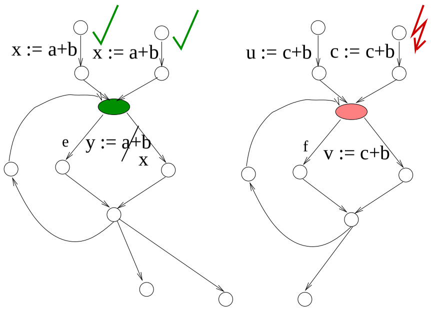
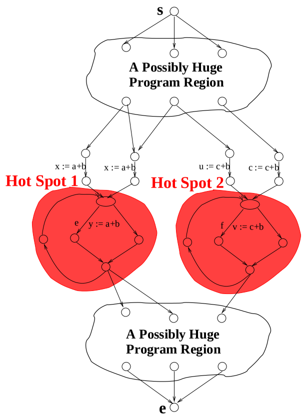
Motivation



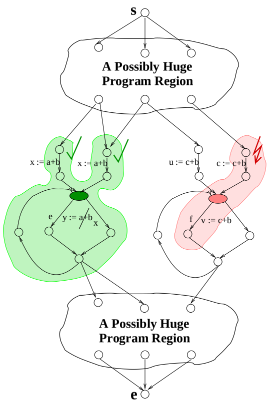
Motivation (Cont'd)



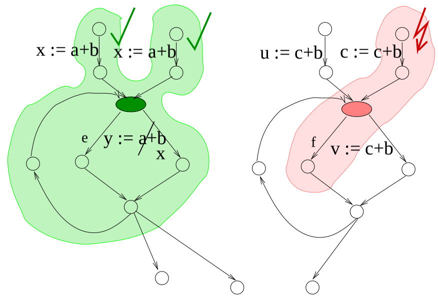
Motivation (Cont'd)



Motivation (Cont'd)



Motivation (Cont'd)



Why Standard Data-Flow Analysis Fails

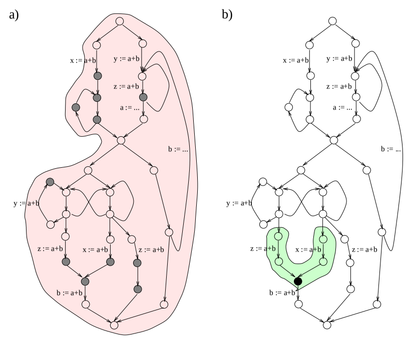
Availability of terms...

$$\text{AVAIL}(n) \quad = \quad \begin{cases} \mathit{false} & \text{if } n = \mathbf{s} \\ \prod_{m \in pred(n)} [\![(m,n)]\!](\text{AVAIL}(n)) & \text{otherwise} \end{cases}$$

where

$$[\![(m,n)]\!](b) = (\textbf{COMP}_{(m,n)} + b) * \textbf{TRANSP}_{(m,n)}$$

Availability at a Single Program Point



Outline of the Talk

Standard vs. Reverse Data-Flow Analysis...

- Background
- Essentials
- The Connecting Link
- The Clou: Why does it work?
- Applications
- Conclusions

Background

Demand-Driven Data-Flow Analysis...

- Agrawal (2000)
- Horwitz, Reps, Sagiv (1994+)
- Duesterwald, Gupta, Soffa (1995+)
- ...
- Knoop (Euro-Par 1999, KPS 2007)

Reverse Data-Flow Analysis: The Basics

(Standard) Data-Flow Analysis...

- *Data-Flow Lattice* $\hat{\mathcal{C}} = (\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \bot, \top)$
- *Data-Flow Functional* $[\![\ ]\!] : E \to (\mathcal{C} \to \mathcal{C})$

Reverse Data-Flow Analysis...

- *Reverse Data-Flow Functional* (Hughes, Launchbury 1992+) $[\![\ ]\!]_R : E \to (\mathcal{C} \to \mathcal{C})$ defined by

$$\forall\, e \in E\ \forall\, c \in \mathcal{C}.\ [\![\, e\,]\!]_R(c) =_{df} \bigsqcap \{\, c' \mid [\![\, e\,]\!](c') \sqsupseteq c\,\}$$

Availability of Terms

- **Abstract semantics for *availability of terms***
  1. *Data-Flow Lattice*:
     $(\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \bot, \top) =_{df} (\mathcal{B}, \wedge, \vee, \leq, false, true)$
  2. *Data-Flow Functional*: $[\![\ ]\!]_{av} : E \to (\mathcal{B} \to \mathcal{B})$ defined by

$$\forall\, e \in E.\ [\![\, e\,]\!]_{av} =_{df} \begin{cases} Cst_{true} & \text{if } Comp_e \wedge Transp_e \\ Id_{\mathcal{B}} & \text{if } \neg Comp_e \wedge Transp_e \\ Cst_{false} & \text{otherwise} \end{cases}$$

On the Relationship of $[\![\ ]\!]$ and $[\![\ ]\!]_R$

Lemma

1. $[\![\, e\,]\!]_R$ is well-defined and monotonic.
2. $[\![\, e\,]\!]_R$ is additive, if $[\![\, e\,]\!]$ is distributive.

Monotonicity, Distributivity, and Additivity

...of data-flow functions.

Definition [Monotonicity, Distributivity, Additivity]
Let $\hat{\mathcal{C}} = (\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \bot, \top)$ be a complete lattice and $f : \mathcal{C} \to \mathcal{C}$ a function on $\mathcal{C}$. Then: f is

1. *monotonic* iff $\forall\, c, c' \in \mathcal{C}.\ c \sqsubseteq c' \Rightarrow f(c) \sqsubseteq f(c')$
   (Preserving the order of elements)
2. distributive iff $\forall\, C' \subseteq \mathcal{C}.\ f(\bigsqcap C') = \bigsqcap \{f(c) \mid c \in C'\}$
   (Preserving greatest lower bounds)
3. *additive* iff $\forall\, C' \subseteq \mathcal{C}.\ f(\bigsqcup C') = \bigsqcup \{f(c) \mid c \in C'\}$
   (Preserving least upper bounds)

Often useful

...the following equivalent characterization of monotonicity:

Lemma

Let $\hat{\mathcal{C}} = (\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \bot, \top)$ be a complete lattice and $f : \mathcal{C} \to \mathcal{C}$ a function on $\mathcal{C}$. Then:

f is monotonic $\iff \forall\, C' \subseteq \mathcal{C}.\ f(\bigsqcap C') \sqsubseteq \bigsqcap \{f(c) \mid c \in C'\}$
($\iff \forall\, C' \subseteq \mathcal{C}.\ f(\bigsqcup C') \sqsupseteq \bigsqcup \{f(c) \mid c \in C'\}$ **)**

On the Relationship of $[\![\ ]\!]$ and $[\![\ ]\!]_R$ (Cont'd)

Lemma

1. $[\![\, e\,]\!]_R \circ [\![\, e\,]\!] \sqsubseteq Id_{\mathcal{C}}$, if $[\![\, e\,]\!]$ is monotonic.
2. $[\![\, e\,]\!] \circ [\![\, e\,]\!]_R \sqsupseteq Id_{\mathcal{C}}$, if $[\![\, e\,]\!]$ is distributive.

In terms of the theory of "abstract interpretation":

- $[\![\, e\,]\!]$ and $[\![\, e\,]\!]_R$ form a Galois-connection.

Reverse DFA: The R-$MinFP$-Approach

The R-$MinFP$-Equation System:

$$\textbf{\textit{reqInf}}\,(n) = \begin{cases} c_q & \text{if } n = \mathbf{q} \\ \bigsqcup \{ [\![\, (n,m) \,]\!]_R(\textbf{\textit{reqInf}}\,(m)) \mid m \in succ(n) \} & \text{otherwise} \end{cases}$$

The R-$MinFP$-Solution:

$$\forall\, c_q \in \mathcal{C}\; \forall\, n \in N.\; R\text{-}MinFP_{c_q}(n) =_{df} \textbf{\textit{reqInf}}^{\,*}_{c_q}(n)$$

where $\textbf{\textit{reqInf}}^{*}_{c_q}$ denotes the least solution of the R-$MinFP$-equation system wrt $c_q \in \mathcal{C}$.

Standard DFA: The $MaxFP$ -Approach

The $MaxFP$-Equation System:

$$\textbf{\textit{inf}}\,(n) \;=\; \begin{cases} c_{\mathbf{s}} & \text{if } n = \mathbf{s} \\ \bigsqcap \{ [\![\, (m,n) \,]\!](\textbf{\textit{inf}}\,(m)) \mid m \in pred(n) \} & \text{otherwise} \end{cases}$$

The $MaxFP$-Solution:

$$\forall\, c_{\mathbf{s}} \in \mathcal{C}\; \forall\, n \in N.\; MaxFP_{([\![\;]\!],c_{\mathbf{s}})}(n) =_{df} \textbf{\textit{inf}}^{\,*}_{c_{\mathbf{s}}}(n)$$

where $\textbf{\textit{inf}}^{\,*}_{c_{\mathbf{s}}}$ denotes the greatest solution of the $MaxFP$ -equation system wrt $c_{\mathbf{s}} \in \mathcal{C}$.

The Connecting Link

Link Theorem

For distributive data-flow functionals $[\![\;]\!]$, $q \in N$, and $c_{\mathbf{s}}, c_q \in \mathcal{C}$, we have:

$$R\text{-}MinFP_{c_q}(\mathbf{s}) \sqsubseteq c_{\mathbf{s}} \iff MaxFP_{c_{\mathbf{s}}}(q) \sqsupseteq c_q$$

Continuing the Analogy

...of Standard and Reverse Data-Flow Analysis regarding

- Soundness & Completeness (in terms of program verification) / Safety & Coincidence (Precision) (in terms of data-flow analysis)

Essential

...the extensibility of data-flow functionals to paths

$$[\![\, p \,]\!] =_{df} \begin{cases} Id_{\mathcal{C}} & \text{if } q < 1 \\ [\![\, \langle e_2, \ldots, e_q \rangle \,]\!] \circ [\![\, e_1 \,]\!] & \text{otherwise} \end{cases}$$

The MOP -Approach

$$\forall\, c_{\mathbf{s}} \in \mathcal{C}\; \forall\, n \in N.\; MOP_{c_{\mathbf{s}}}(n) = \bigsqcap \{ [\![\, p \,]\!](c_{\mathbf{s}}) \mid p \in \mathbf{P}[\mathbf{s}, n] \}$$

Standard DFA: Main Results

Theorem [Soundness / Safety]

$$\forall\, c_{\mathbf{s}} \in \mathcal{C}\; \forall\, n \in N.\; MaxFP_{c_{\mathbf{s}}}(n) \sqsubseteq MOP_{c_{\mathbf{s}}}(n)$$

if the data-flow functional $[\![\;]\!]$ is monotonic.

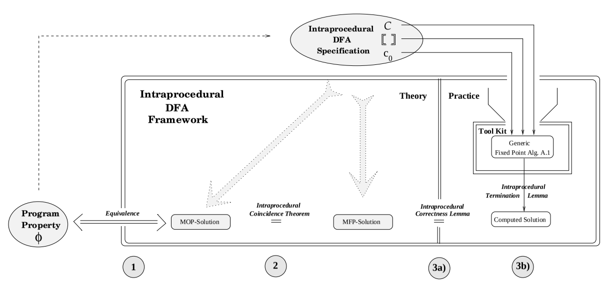
Theorem [Completeness / Coincidence (Precision)]

$$\forall\, c_{\mathbf{s}} \in \mathcal{C}\; \forall\, n \in N.\; MaxFP_{c_{\mathbf{s}}}(n) = MOP_{c_{\mathbf{s}}}(n)$$

if the data-flow functional $[\![\;]\!]$ is distributive.

Standard DFA: The Tool Kit View

...at a glance:



Of course...

Reverse data-flow functionals can be extended to paths, too:

$$[\![\, p \,]\!]_R =_{df} \begin{cases} Id_{\mathcal{C}} & \text{if } q < 1 \\ [\![\, \langle e_1, \ldots, e_{q-1} \rangle \,]\!]_R \circ [\![\, e_q \,]\!]_R & \text{otherwise} \end{cases}$$

The R-JOP-Approach

The R-JOP-Solution:

$$\forall c_q \in \mathcal{C}\ \forall n \in N.\ R\text{-}JOP_{c_q}(n) =_{df} \bigsqcup \{ [\![\, p \,]\!]_R(c_q) \mid p \in \mathbf{P}[n, \mathbf{q}] \}$$

Reverse DFA: Main Results

Theorem [Soundness / Reverse Safety]

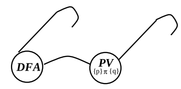
$$\forall c_q \in \mathcal{C}\ \forall n \in N.\ R\text{-}MinFP_{c_q}(n) \sqsupseteq R\text{-}JOP_{c_q}(n)$$

Theorem [Completeness / Reverse Coincidence (Precision)]

$$\forall c_q \in \mathcal{C}\ \forall n \in N.\ R\text{-}MinFP_{c_q}(n) = R\text{-}JOP_{c_q}(n)$$

if $[\![\ ]\!]$ is distributive.

Putting it together...



Data-flow Analysis

MaxFP
Coincidence Theorem ||
$MOP_{c_s}(q) \sqsupseteq c_q$
Link Theorem ⇕
$R\text{-}JOP_{c_q}(s) \sqsubseteq c_s$
Reverse Coincidence Theorem ||
R-MinFP

$[\![e]\!] \in [\mathcal{C} \xrightarrow{\text{distributive}} \mathcal{C}]$

induces

$[\![e]\!]_R(c) =_{df} \bigsqcap \{ c' \mid [\![e]\!](c') \sqsupseteq c \}$

Program Verification

Strongest Postcondition View

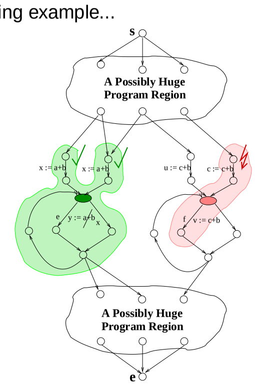
{p} π {?}

{?} π {q}

Weakest Precondition View

Are We Done?

Recall the motivating example...

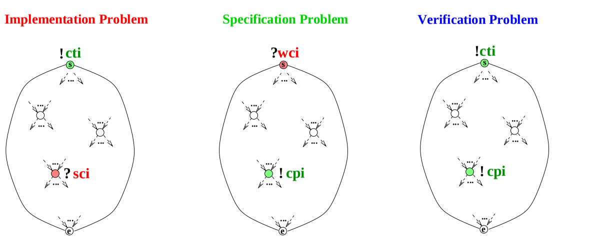


Mastering the Road to Success

...requires more. It requires us to conclude from “weakest pre-conditions” on “strongest post-conditions”.

...essentially, this means to replace the analysis problem by a verification problem.

Changing the Perspective



! **Given:** Context Information **cti**
? **Sought:** Strongest Component Information **sci**

! **Given:** Component Information **cpi**
? **Sought:** Weakest Context Information **wci**

! **Given:** Context Information **cti**
Component Information **cpi**
? **Sought:** Validity of **cpi** with respect of **cti**

Changing the Perspective: The Standard Taxonomy

Conventional
Classification of DFA Techniques

Exhaustive **DFA** — ***Demand-Driven*** **DFA**

Changing the Perspective: Conclusions Derived

The *specification* **problem:**

{?} π {q}

The *verification* **problem:**

{p} π {q} ?

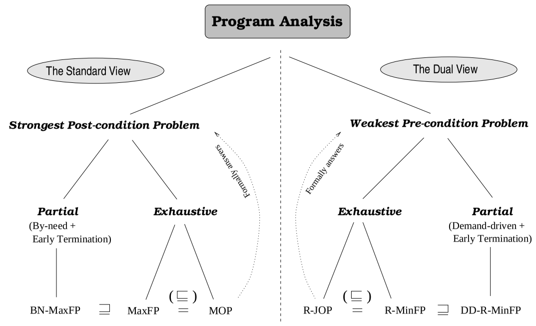
... the domain of *demand-driven DFA*

The *implementation* **problem:**

{p} π {?}

... the domain of *exhaustive DFA*

(R)DFA-Frameworks / (R)DFA-Tool Kits



Gen/Kill-Problems

...allow us to master *the road to success*: The SPC-analysis problem boils down to a WPC-verification problem.

This is important because...

- Redundant Expression/Assignment Elimination
- Dead-Code Elimination
- Strength Reduction
- ...

are based on Gen-Kill-problems.

Concluding the Example: Availability

Abstract semantics for *availability*

1. *Data-flow lattice*:
   $(\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \bot, \top) =_{df} (\mathcal{B}_X, \wedge, \vee, \leq, false, failure)$
   with $\bot = false \sqsubset true \sqsubset failure = \top$
2. *Data-flow functional*: $[\![\ ]\!]_{av} : E \to (\mathcal{B}_X \to \mathcal{B}_X)$ defined by

$$\forall\, e \in E.\ [\![\, e \,]\!]_{av} =_{df} \begin{cases} Cst^X_{true} & \text{if } Comp_e \wedge Transp_e \\ Id_{\mathcal{B}_X} & \text{if } \neg Comp_e \wedge Transp_e \\ Cst^X_{false} & \text{otherwise} \end{cases}$$

Reverse Availability

Reverse abstract semantics for *availability*

1. *Data-flow lattice*:
   $(\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \bot, \top) =_{df} (\mathcal{B}_X, \wedge, \vee, \leq, false, failure)$
2. *Reverse data-flow functional*: $[\![\ ]\!]_{av_R} : E \to (\mathcal{B}_X \to \mathcal{B}_X)$
   defined by

$$\forall\, e \in E.\ [\![\, e \,]\!]_{av_R} =_{df} \begin{cases} R\text{-}Cst^X_{true} & \text{if } [\![\, e \,]\!]_{av} = Cst^X_{true} \\ R\text{-}Id_{\mathcal{B}_X} & \text{if } [\![\, e \,]\!]_{av} = Id_{\mathcal{B}_X} \\ R\text{-}Cst^X_{false} & \text{if } [\![\, e \,]\!]_{av} = Cst^X_{false} \end{cases}$$

Supporting Functions

$$\forall\, b \in \mathcal{B}_X.\ R\text{-}Cst^X_{true}(b) =_{df} \begin{cases} false & \text{if } b \in \mathcal{B} \\ failure & \text{otherwise} \\ & \text{(i.e., if } b = failure) \end{cases}$$

$$\forall\, b \in \mathcal{B}_X.\ R\text{-}Cst^X_{false}(b) =_{df} \begin{cases} false & \text{if } b = false \\ failure & \text{otherwise} \end{cases}$$

$$R\text{-}Id_{\mathcal{B}_X} =_{df} Id_{\mathcal{B}_X}$$

Summing Up / Extensions

In this talk...

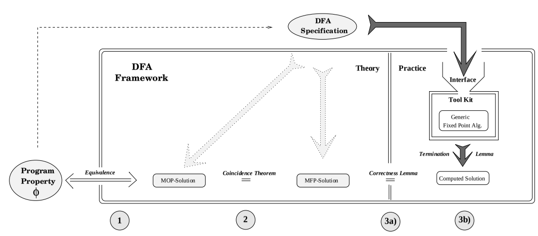
- The intraprocedural basic setting of (R)DFA (Knoop, KPS 2007)

Extensions are possible...

- Interprocedural setting (Knoop, CC 1992, LNCS 1428 (1998))
- Parallel setting (Knoop, Euro-Par 1999)
- ...

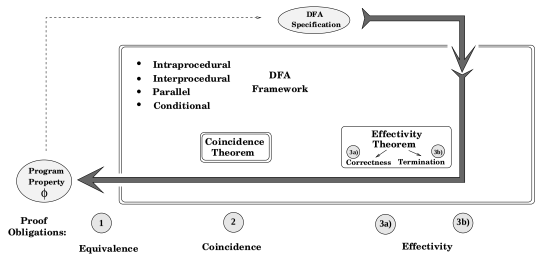
(R)DFA-Frameworks / (R)DFA-Tool Kits (Cont'd)

...the general pattern:

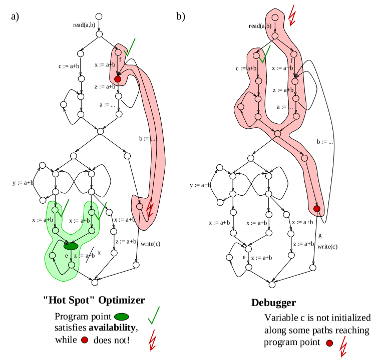


(R)DFA-Frameworks / (R)DFA-Tool Kits

...the general pattern more abstract:



Applications



From Applications towards Conclusions

Reverse Data-Flow Analysis especially well-suited for...

- Hot-Spot Optimization
- Debugging
- Just-in-time Compilation

based on answering data-flow queries.

Hence...

- Data-Flow Analysis for Debugging
- Data-Flow Analysis for Just-in-time Compilation

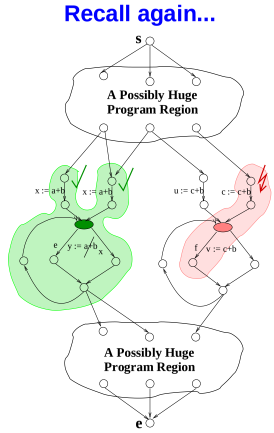
were titles considered optionally.

Conclusions (Cont'd)

As an appealing add-on...

- RDFA is tailored for parallelization!

Recall again...



Conclusions and Perspectives

Data-Flow Analysis for Multi-Core Architectures