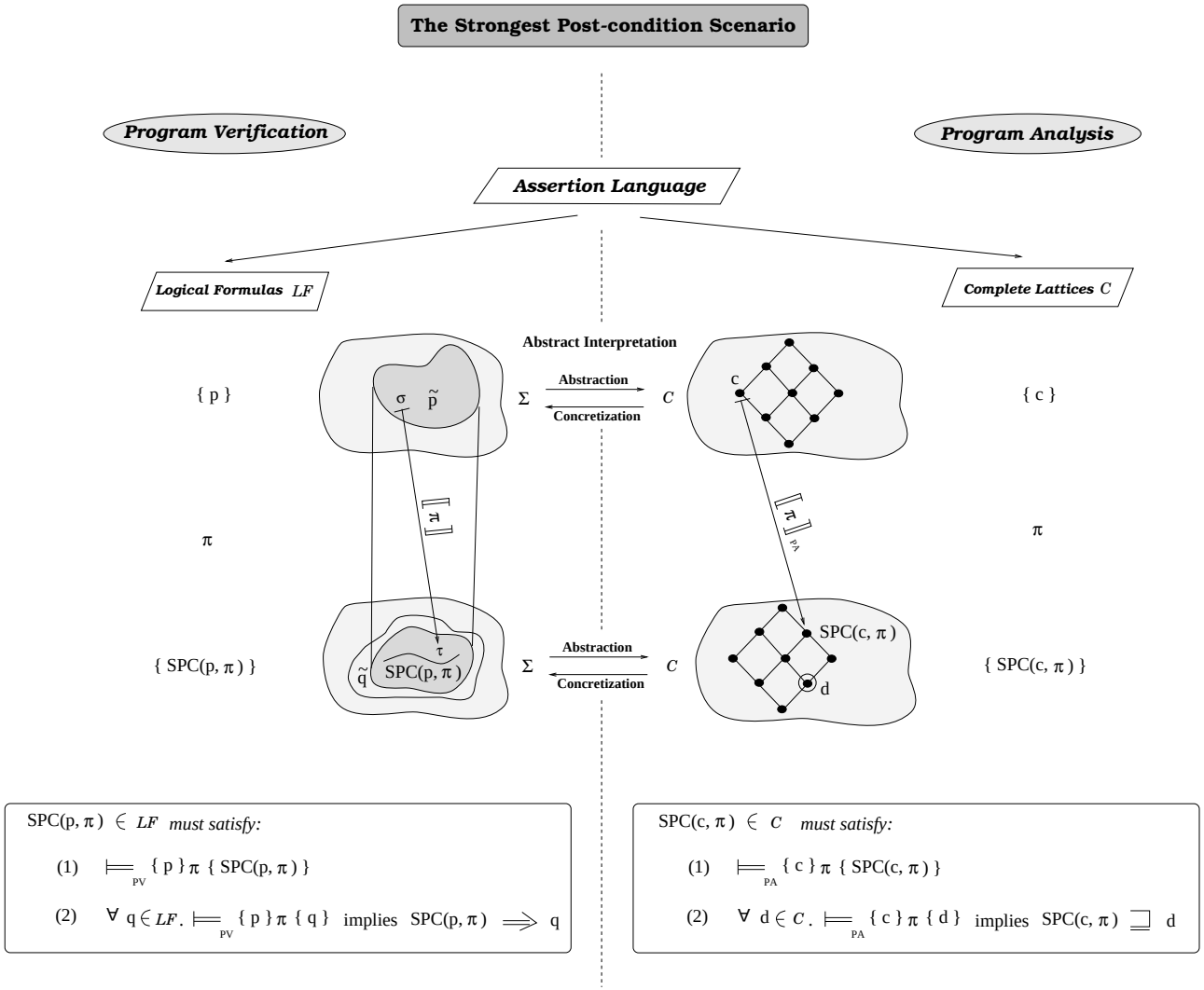
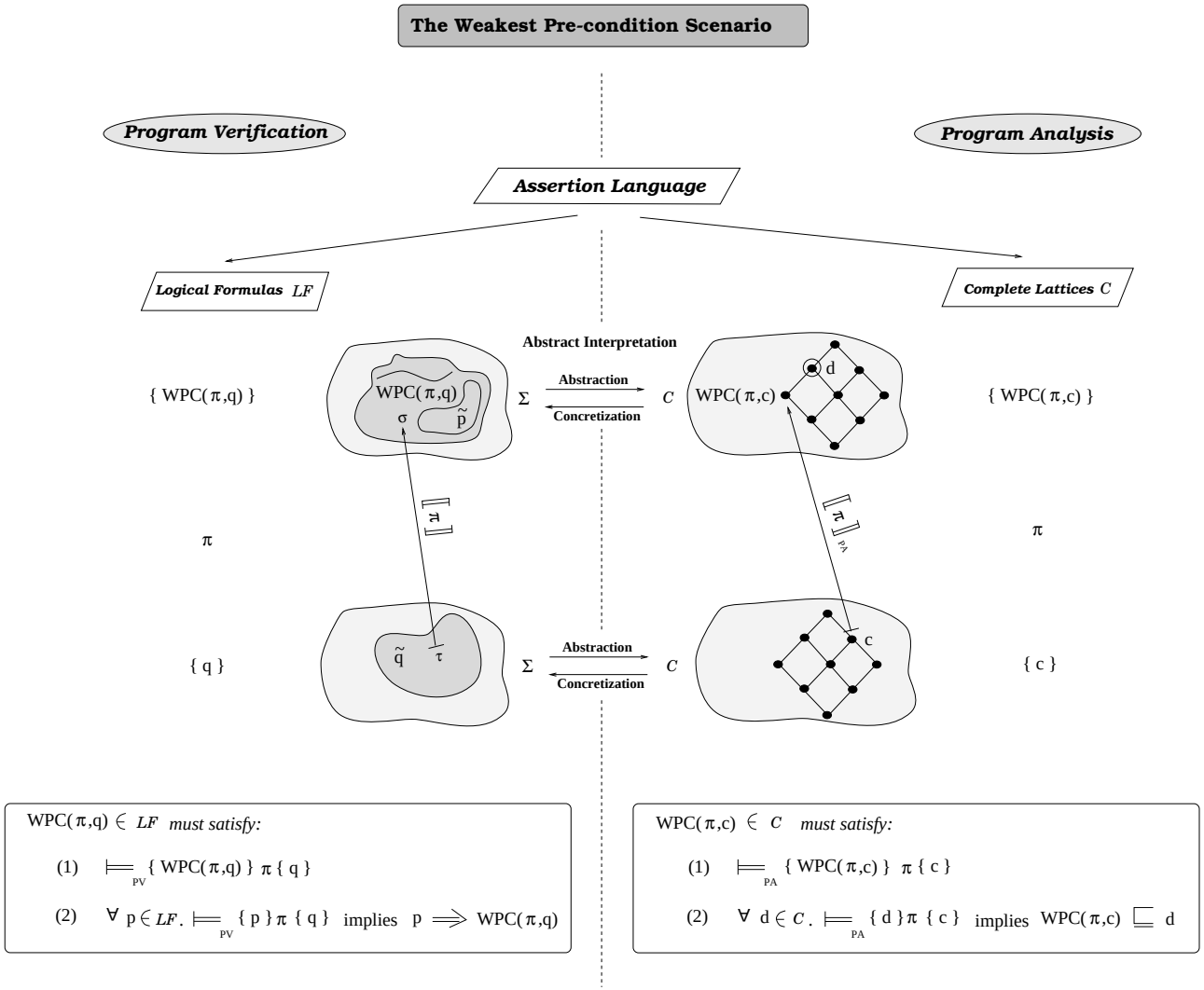
# Programmverifikation vs. -analyse (1)



# Programmverifikation vs. -analyse (2)



# Reverse abstrakte Semantik

## Reverse abstrakte Semantik

1. *Datenflussanalyseverband* $\widehat{\mathcal{C}} = (\mathcal{C}, \sqcap, \sqcup, \sqsubseteq, \bot, \top)$
2. *Reverses Datenflussanalysefunktional*
   $[\![\ ]\!]_R : E \to (\mathcal{C} \to \mathcal{C})$ definiert durch

$$\forall\, e \in E\ \forall\, c \in \mathcal{C}.\ [\![\, e \,]\!]_R(c) =_{df} \bigsqcap \{\, c' \mid [\![\, e \,]\!](c') \sqsupseteq c \,\}$$

wobei $[\![\ ]\!] : E \to (\mathcal{C} \to \mathcal{C})$ eine abstrakte Semantik auf $\mathcal{C}$ ist.

# Zusammenhang von $[\![\ ]\!]$ und $[\![\ ]\!]_R$ (1)

**Lemma**

Sei $[\![\ ]\!]$ ein Datenflussanalysefunktional. Dann gilt für jede Kante $e \in E$:

1. $[\![ e ]\!]_R$ ist wohldefiniert und monoton.
2. $[\![ e ]\!]_R$ ist additiv, falls $[\![ e ]\!]$ distributiv ist.

# Zusammenhang von $[\![\ ]\!]$ und $[\![\ ]\!]_R$ (2)

**Lemma**

Sei $[\![\ ]\!]$ ein Datenflussanalysefunktional. Dann gilt für jede Kante $e \in E$:

1. $[\![ e ]\!]_R \circ [\![ e ]\!] \sqsubseteq Id_{\mathcal{C}}$, falls $[\![ e ]\!]$ monoton ist.
2. $[\![ e ]\!] \circ [\![ e ]\!]_R \sqsupseteq Id_{\mathcal{C}}$, falls $[\![ e ]\!]$ distributiv ist.

Sprechweise in der Theorie "Abstrakter Interpretation":

- $[\![ e ]\!]$ und $[\![ e ]\!]_R$ bilden eine Galois-Verbindung.

# Zusammenhang von $[\![\ ]\!]$ und $[\![\ ]\!]_R$ (3)

## Hilfssatz

1. $\forall\, n \in N' \cap N.\ \mathbf{P}_{G'}[\mathbf{s}, n] = \mathbf{P}_G[\mathbf{s}, n]$
2. $\forall\, q \in N' \backslash \{\mathbf{s}\}.\ \mathbf{P}_{G'}[\mathbf{s}, q] = \mathbf{P}_G[\mathbf{s}, \mathbf{q}]$
3. $\forall\, c_{\mathbf{s}} \in \mathcal{C}\ \forall\, n \in N' \cap N.\ \mathit{MOP}_{(G', c_{\mathbf{s}})}(n) = \mathit{MOP}_{(G, c_{\mathbf{s}})}(n)$
4. $\mathit{MOP}_{(G, c_{\mathbf{s}})}(q) = \mathit{MOP}_{(G, c_{\mathbf{s}})}(\mathbf{q})$

# Der $R$-$JOP$-Ansatz

## Die $R$-$JOP$-Lösung:

$$\forall\, c_q \in \mathcal{C}\ \forall\, n \in N.\ R\text{-}JOP_{c_q}(n) =_{df} \bigsqcup \{\, [\![\, p \,]\!]_R(c_q) \mid p \in \mathbf{P}[n, \mathbf{q}] \,\}$$

# Der $R$-$MinFP$-Ansatz

**Das $R$-$MinFP$-Gleichungssystem:**

$$\textbf{reqInf}\ (n) = \begin{cases} c_q & \text{falls } n = \mathbf{q} \\ \bigsqcup \{\, [\![\, (n,m) \,]\!]_R(\textbf{reqInf}\ (m)) \mid m \in succ(n) \,\} & \text{sonst} \end{cases}$$

Bezeichne $\textbf{reqInf}^*_{c_q}$ die kleinste Lösung dieses Gleichungssystems bzgl. $c_q \in \mathcal{C}$.

**Die $R$-$MinFP$-Lösung:**

$$\forall\, c_q \in \mathcal{C}\ \forall\, n \in N.\ R\text{-}MinFP_{c_q}(n) =_{df} \textbf{reqInf}\ ^*_{c_q}(n)$$

# Der generische $R$-$MinFP$-Alg. (1)

**Input:** (1) A flow graph $G = (N, E, \mathbf{s}, \mathbf{e})$, (2) a program point $q$, (3) a reverse abstract semantics (i.e., a data-flow lattice $\mathcal{C}$, and a reverse data-flow functional $[\![\ ]\!]_R : E \to (\mathcal{C} \to \mathcal{C})$ induced by a functional $[\![\ ]\!] : E \to (\mathcal{C} \to \mathcal{C})$), and (4) a component information $c_q \in \mathcal{C}$.

**Output:** Under the assumption of termination (cf. Theorem **??**), the $R$-$MinFP$-solution. Depending on the properties of the underlying reverse data-flow functional, this has the following interpretation.

(1) $[\![\ ]\!]_R$ *is additive:* Variable *reqInf*[$\mathbf{s}$] stores the weakest context information of $c_q$, i.e., the least data-flow fact which must be ensured at the program entry in order to guarantee $c_q$ at $q$. If this is $\top$, the requested component information cannot be satisfied at all.

(2) $[\![\ ]\!]_R$ *is monotonic:* Variable *reqInf*[$\mathbf{s}$] stores a lower bound of the weakest context candidate of $c_q$. Generally, this is not a sufficient context information itself. Hence, except for the special case *reqInf*[$\mathbf{s}$] $= \top$, which implies that $c_q$ cannot be satisfied by any consistent context information, nothing can be concluded from the value of *reqInf*[$\mathbf{s}$].

**Remark:** The variable *workset* controls the iterative process. Its elements are nodes of $G$, whose informations annotating them have recently been updated.

# Der generische $R$-$MinFP$-Alg. (2)

( Prologue: Initialization of the annotation array *reqInf*, and the variable *workset* )

FORALL $n \in N \backslash \{\mathbf{q}\}$ DO *reqInf*$[n] := \bot$ OD;
*reqInf*$[\mathbf{q}] := c_q$;
*workset* $:= \{\mathbf{q}\}$;

# Der generische $R$-$MinFP$-Alg. (3)

( Main process: Iterative fixed point computation )

WHILE *workset* $\neq \emptyset$ DO
  CHOOSE $m \in$ *workset*;
    *workset* := *workset*$\backslash\{m\}$;
    ( Update the predecessor-environment of node $m$ )
    FORALL $n \in pred(m)$ DO
      *join* := $[\![ (n,m) ]\!]_R$(*reqInf*[$m$]) $\sqcup$ *reqInf*[$n$];
      IF *reqInf*[$n$] $\sqsubset$ *join*
        THEN
          *reqInf*[$n$] := *join*;
          *workset* := *workset* $\cup \{n\}$
      FI
    OD
  ESOOHC
OD.

# Reverses Sicherheitstheorem

## Reverses Sicherheitstheorem

Die $R$-$MinFP$-Lösung ist eine obere (d.h. sichere) Approximation der $R$-$JOP$-Lösung, d.h.,

$$\forall\, c_q \in \mathcal{C}\ \forall\, n \in N.\ R\text{-}MinFP_{c_q}(n) \sqsupseteq R\text{-}JOP_{c_q}(n)$$

# Reverses Koinzidenztheorem

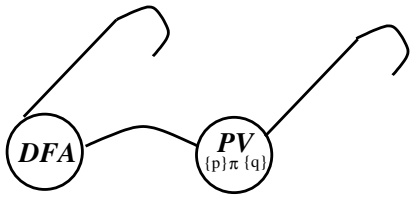
## Reverses Koinzidenztheorem

Diee $R$-$MinFP$-Lösung stimmt mit der $R$-$JOP$-Lösung überein, d.h.,

$$\forall\, c_q \in \mathcal{C}\ \forall\, n \in N.\ R\text{-}MinFP_{c_q}(n) = R\text{-}JOP_{c_q}(n)$$

falls $[\![\ ]\!]$ distributiv ist.

# DFA vs. Verifikation: Überblick




**Data-flow Analysis** | **Program Verification**

MaxFP — $[\![e]\!] \in \lfloor C \xrightarrow{\text{distributive}} C \rfloor$ — *Strongest Postcondition View*

*Coincidence Theorem* ||

$MOP_{c_s}(q) \sqsupseteq c_q$ — $\{p\}\ \pi\ \{?\}$

*induces*

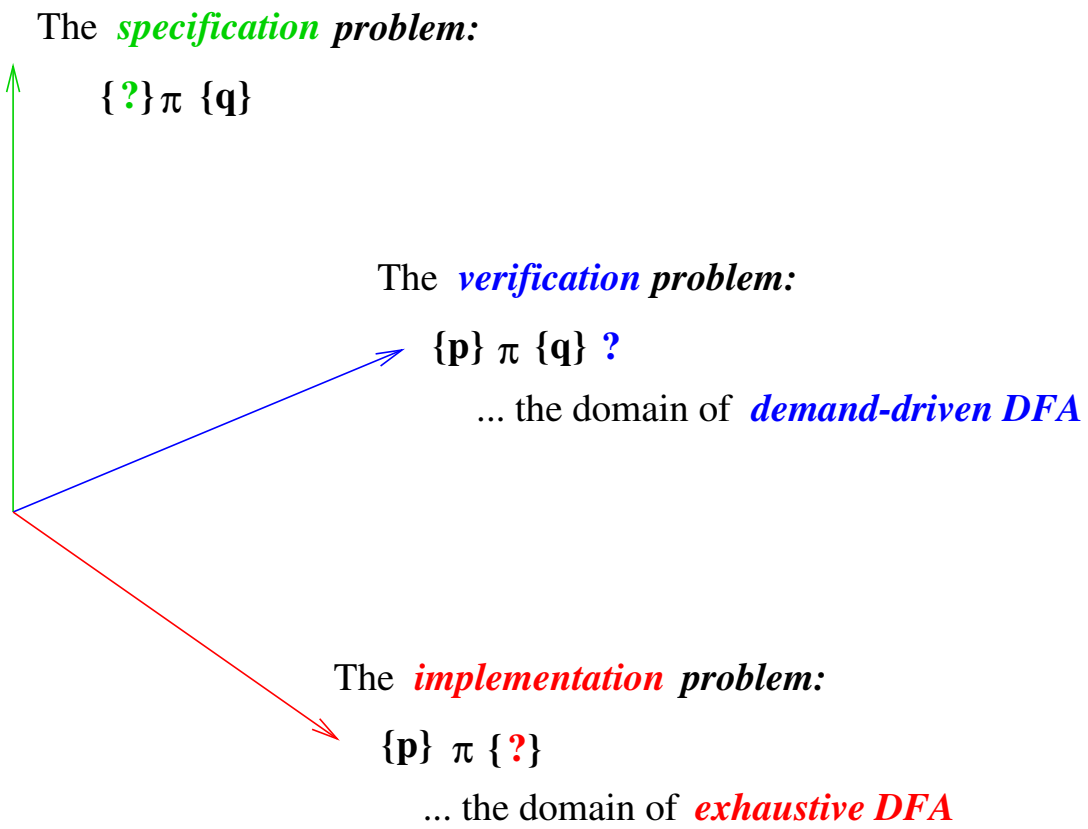
*Link Theorem* ⇕

$R\text{-}JOP_{c_q}(s) \sqsubseteq c_s$ — $[\![e]\!]_R(c) =_{df} \bigsqcap \{c' \mid [\![e]\!](c') \sqsupseteq c\}$ — $\{?\}\ \pi\ \{q\}$

*Reverse Coincidence Theorem* ||

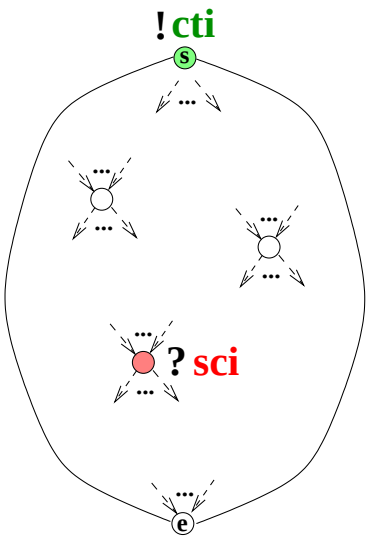
R-MinFP — *Weakest Precondition View*

# Drei unterschiedliche Problemperspektiven (1)



# Drei unterschiedliche Problemperspektiven (2)

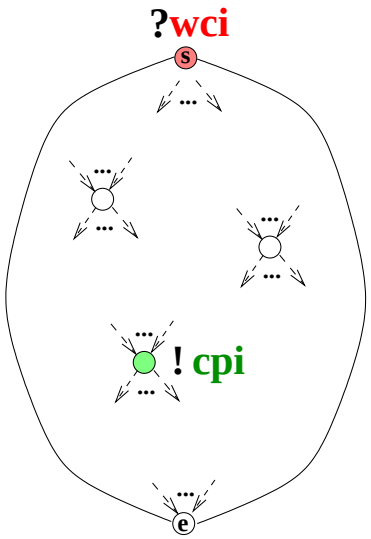
**Implementation Problem**




! **Given:** Context Information **cti**

? **Sought:** Strongest Component Information **sci**

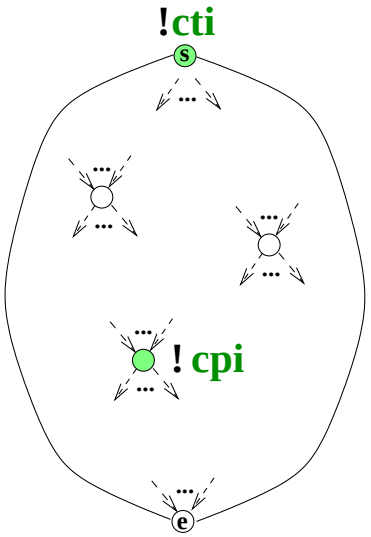
**Specification Problem**




! **Given:** Component Information **cpi**

? **Sought:** Weakest Context Information **wci**

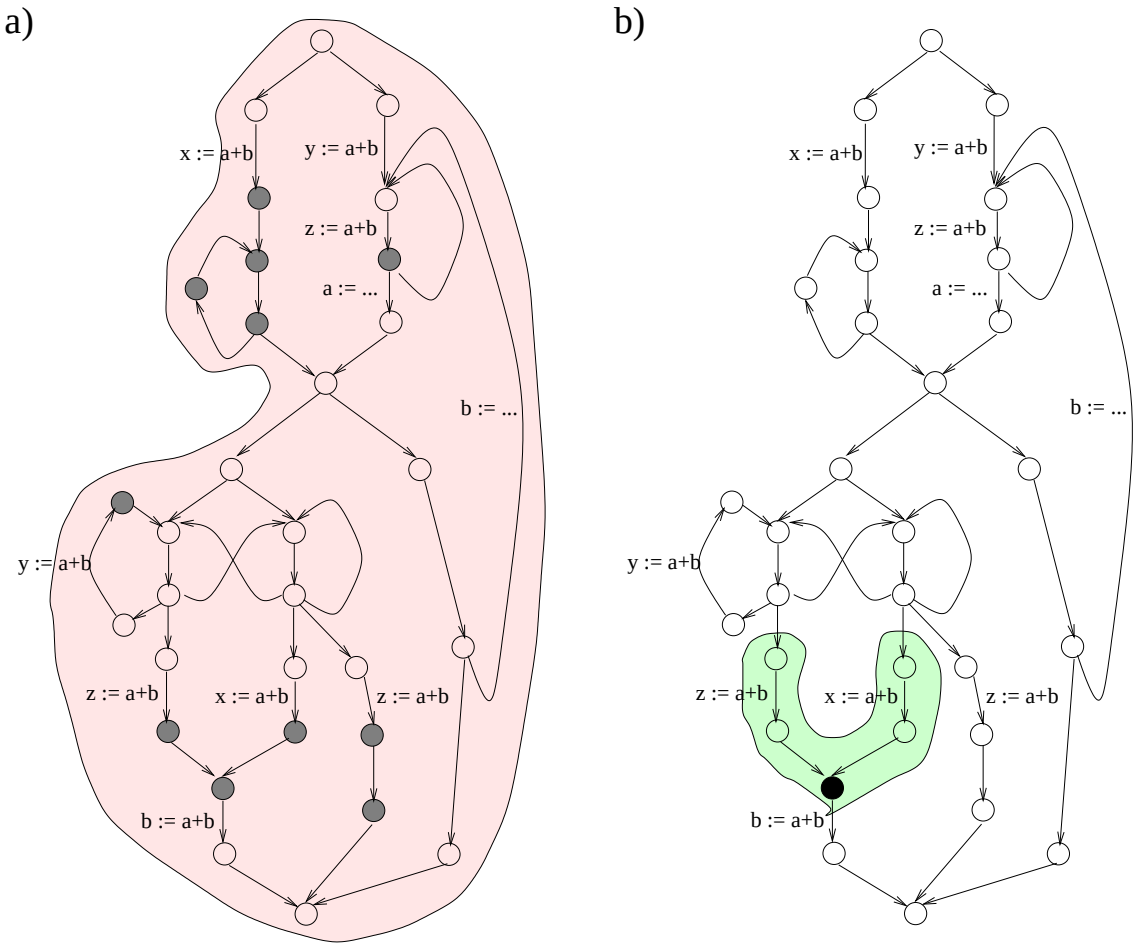
**Verification Problem**



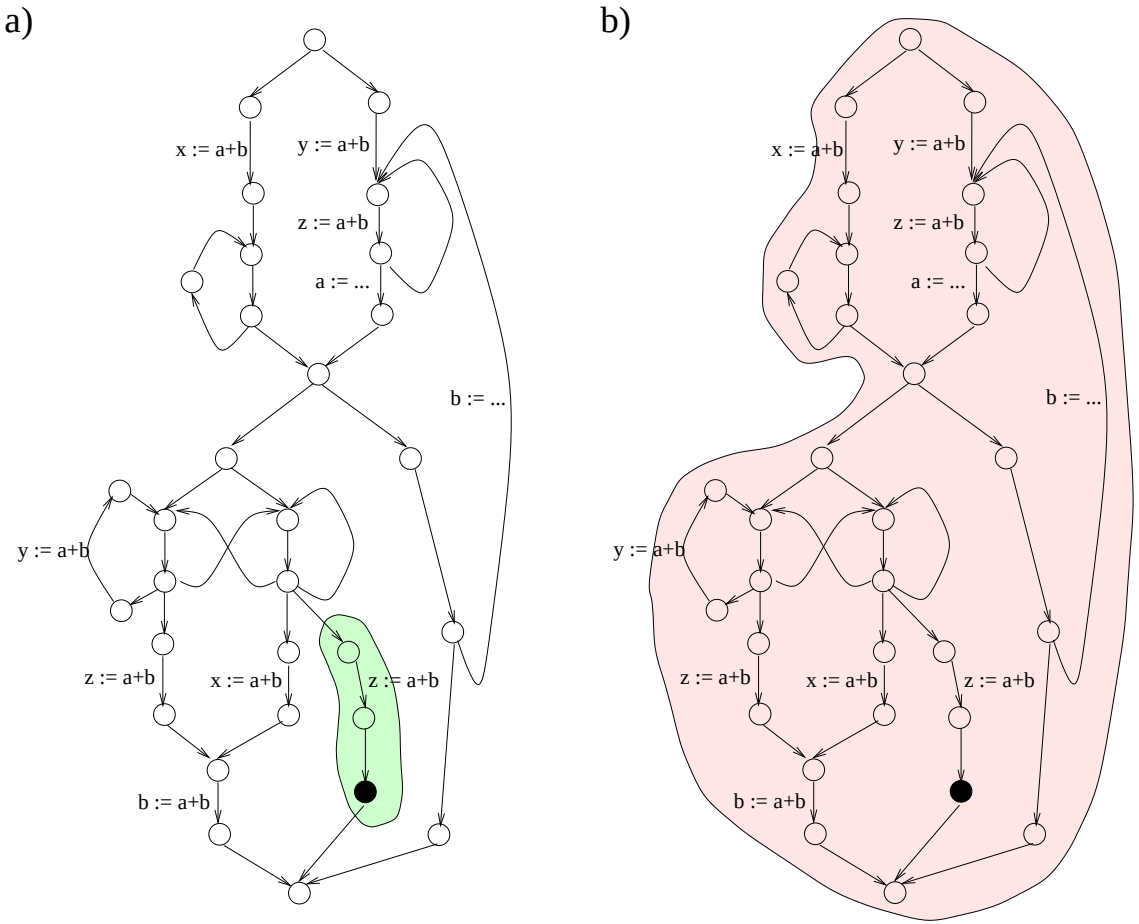

! **Given:** Context Information **cti**
Component Information **cpi**

? **Sought:** Validity of **cpi** with respect of **cti**

# Bsp: Verfügbarkeit an einem Punkt (1)

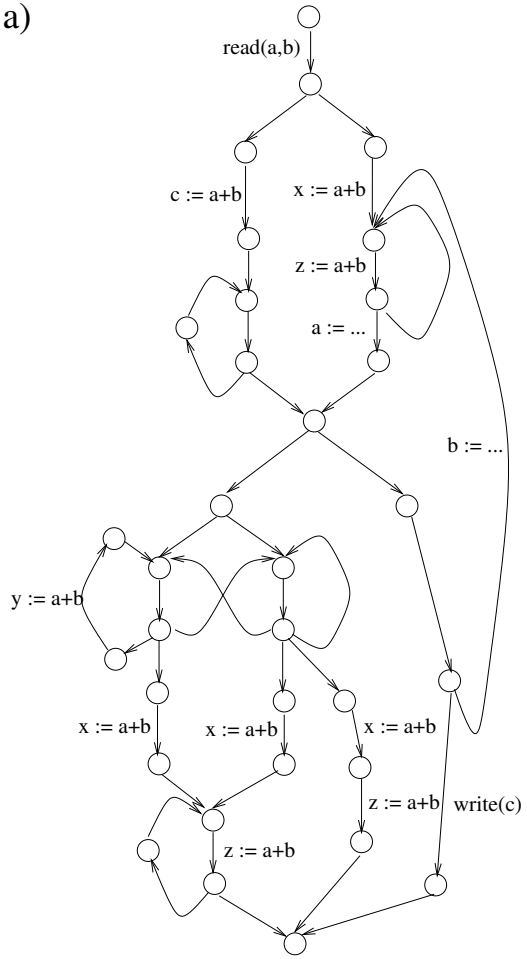


# Bsp: Verfügbarkeit an einem Punkt (2)



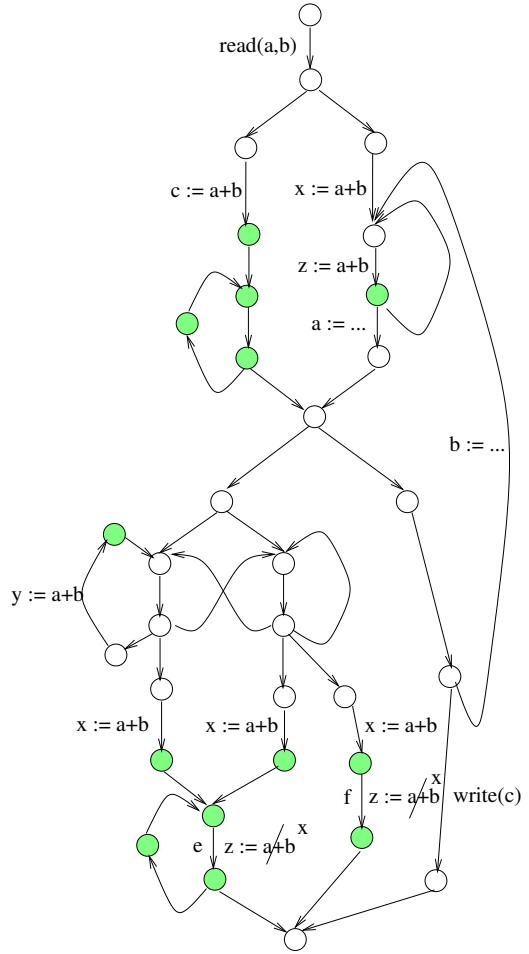
# Anwendung: Einfacher Optimierer (1)

a)




Flow graph

b)

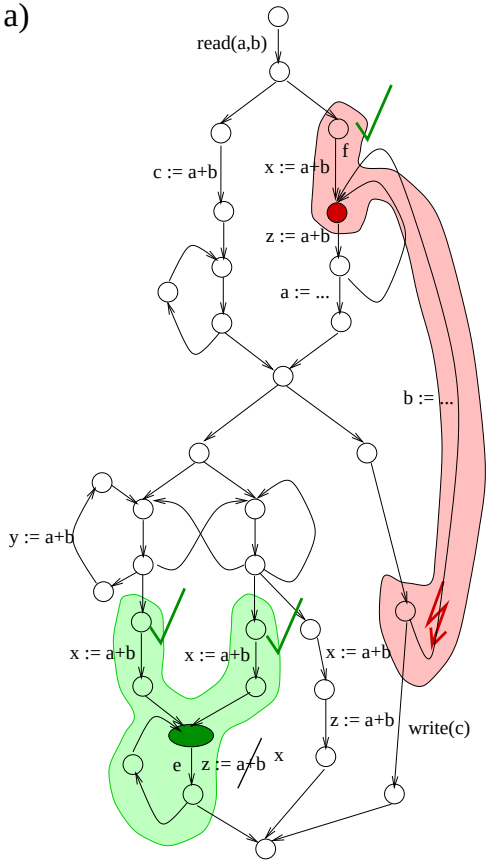



Data-flow information

Program points satisfying **availability** ●

# Anwendung: “Hot Spot” Optimierer und Debugger (2)

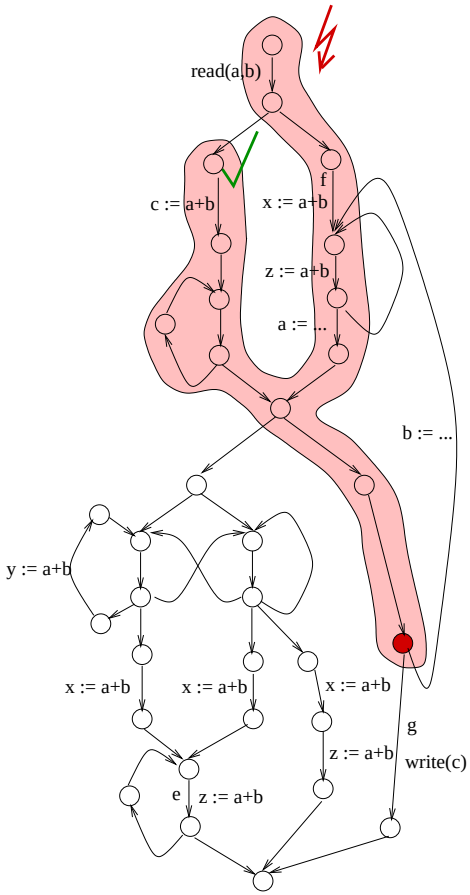
a)




**"Hot Spot" Optimizer**

Program point ● satisfies **availability**, while ● does not!

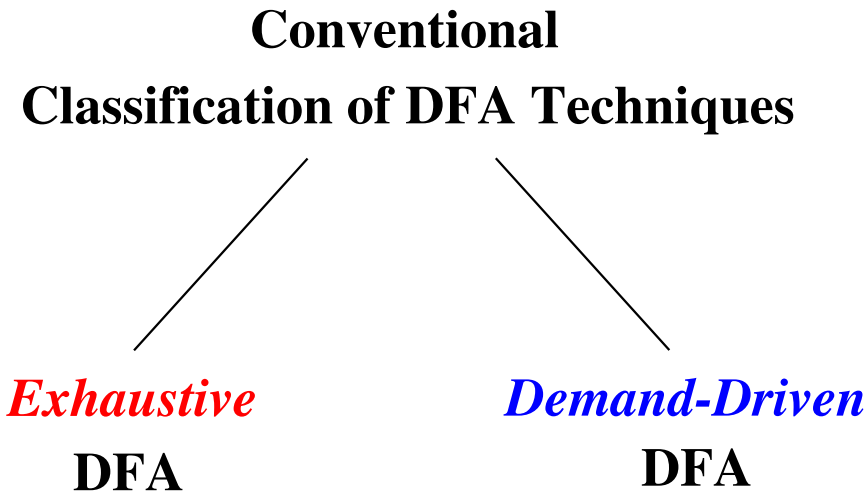
b)



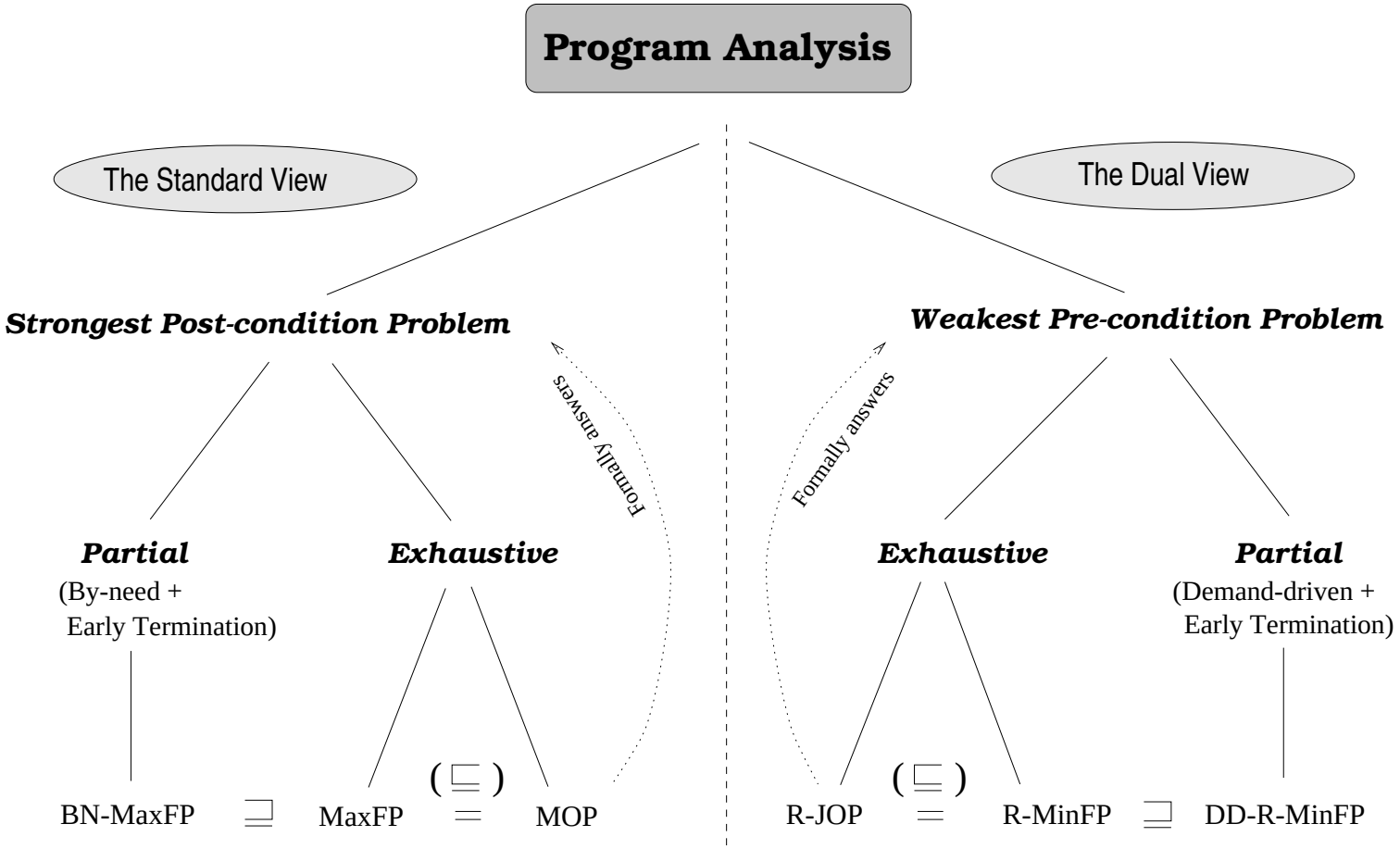

**Debugger**

Variable c is not initialized along some paths reaching program point ●

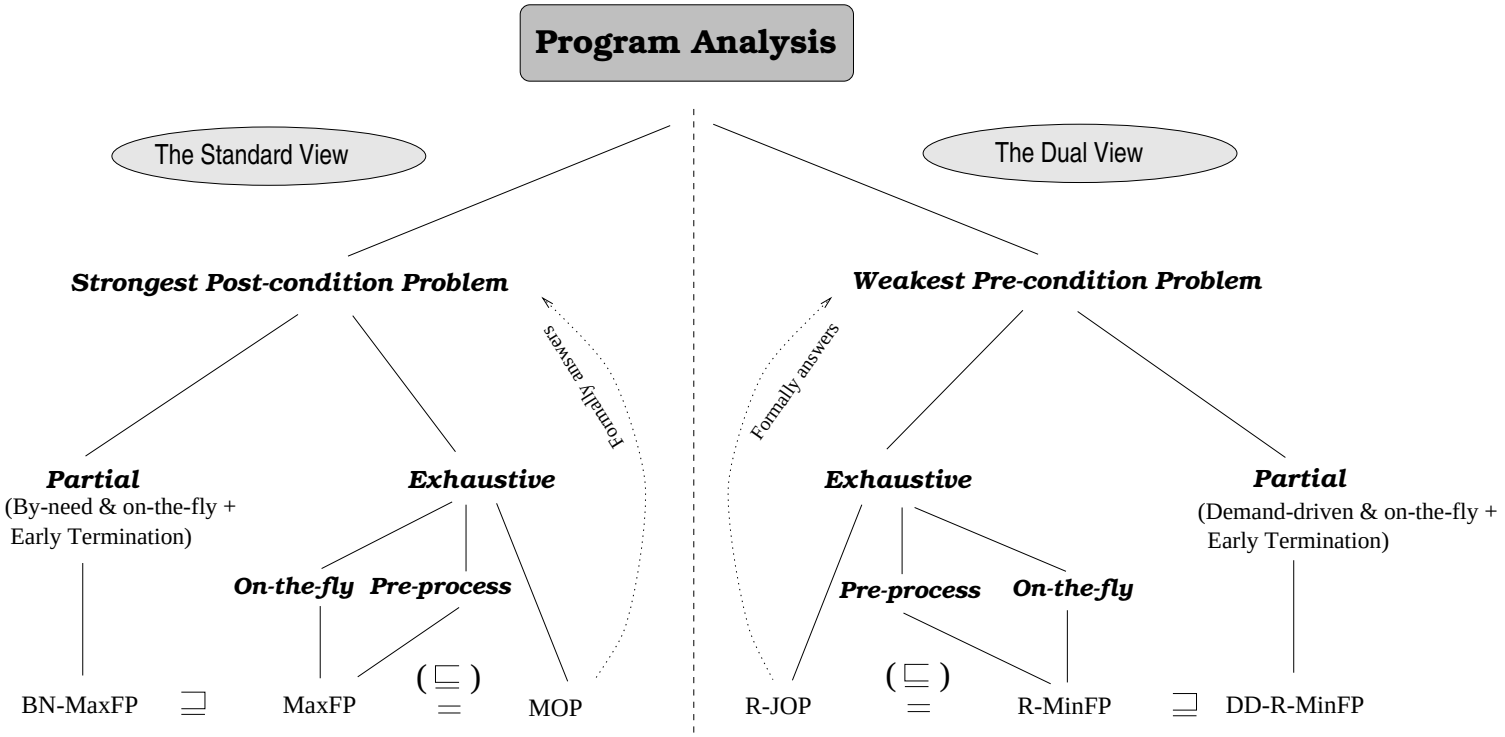
# Erschöpfende vs. anforderungsgetriebene DFA (1)



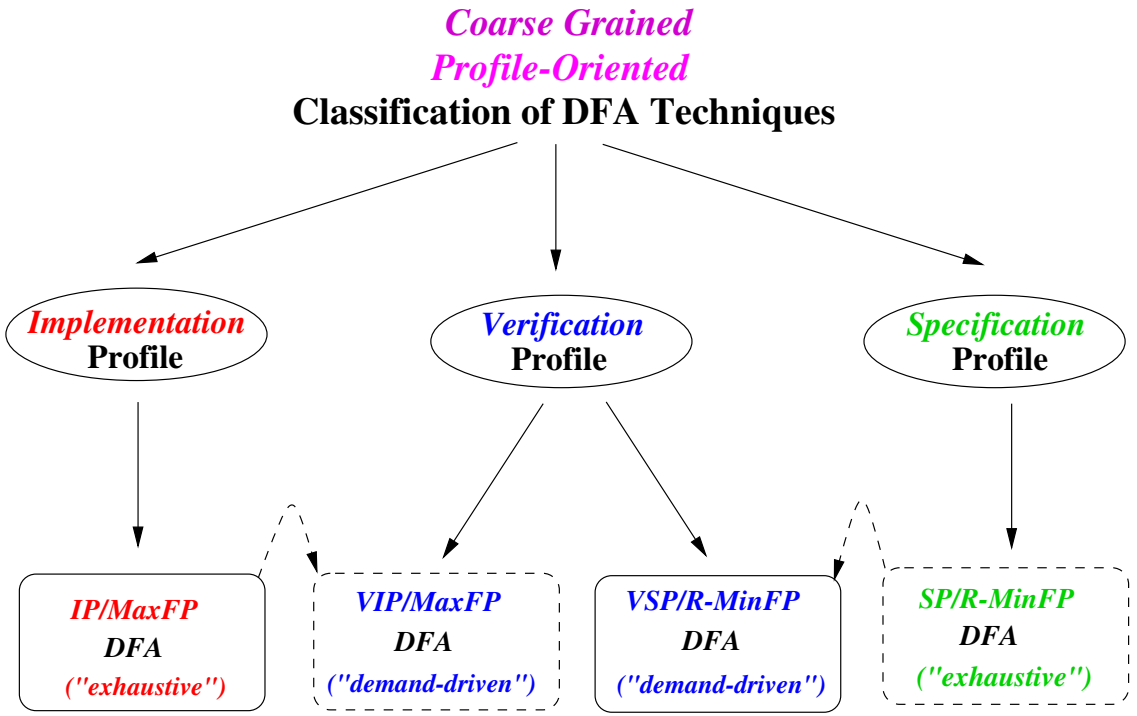
# Erschöpfende vs. anforderungsgetriebene DFA (2)



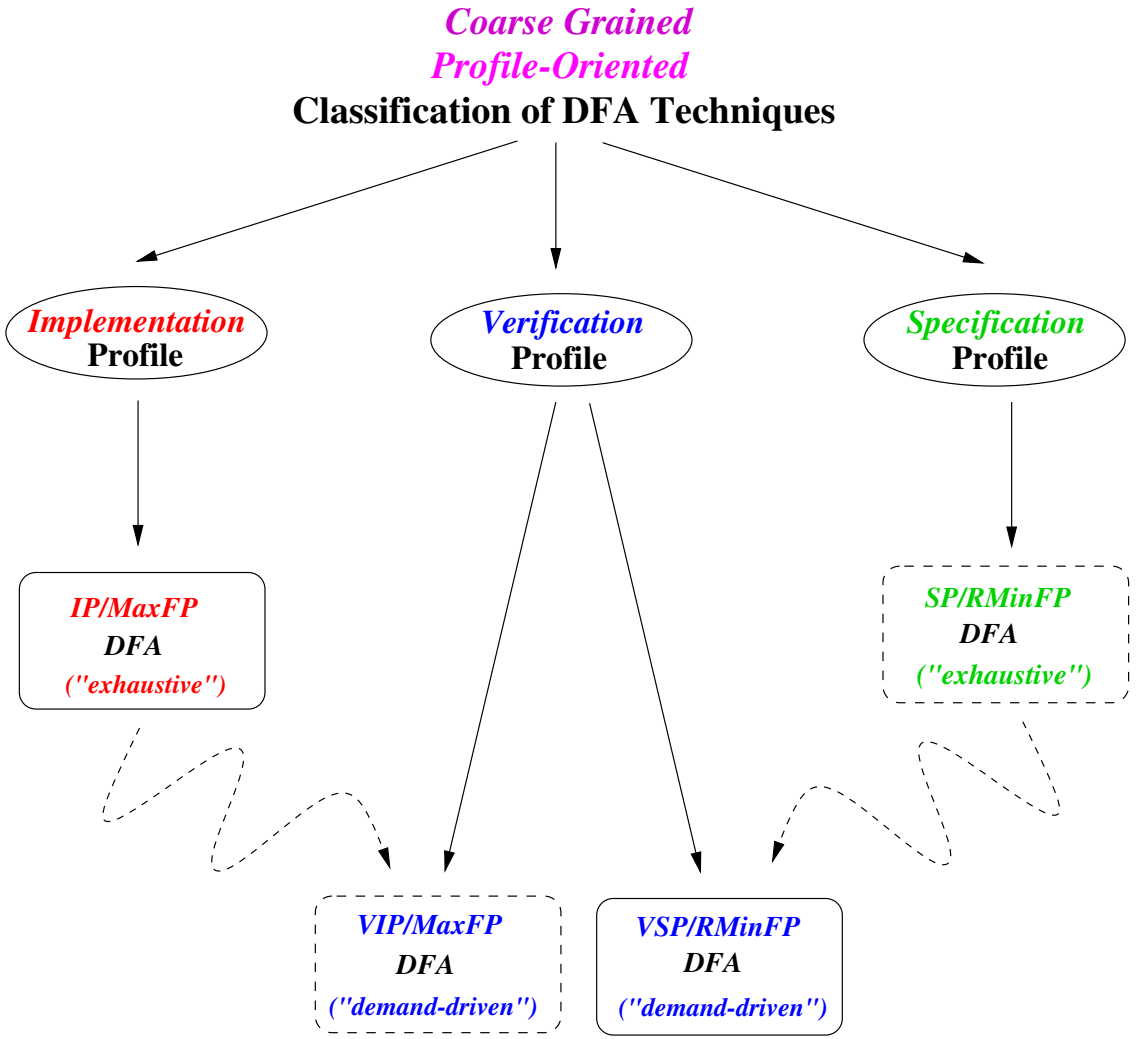
# Erschöpfende vs. anforderungsgetriebene DFA (3)



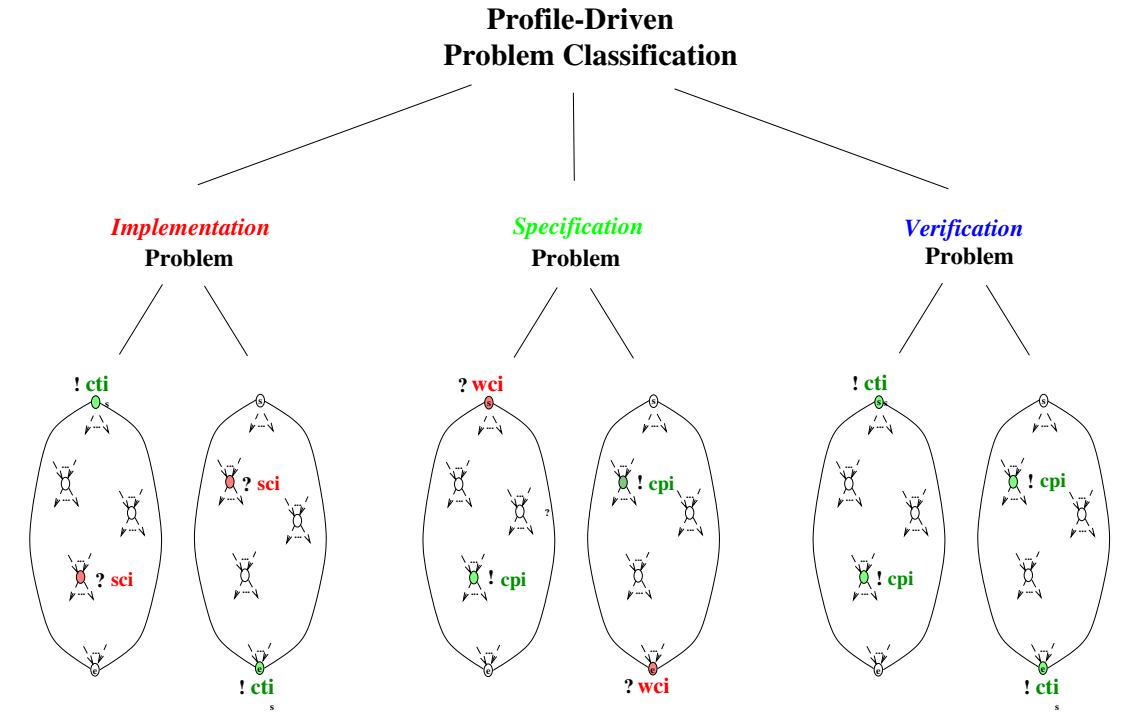
# Eine andere Sicht (1)



# Eine andere Sicht (2)

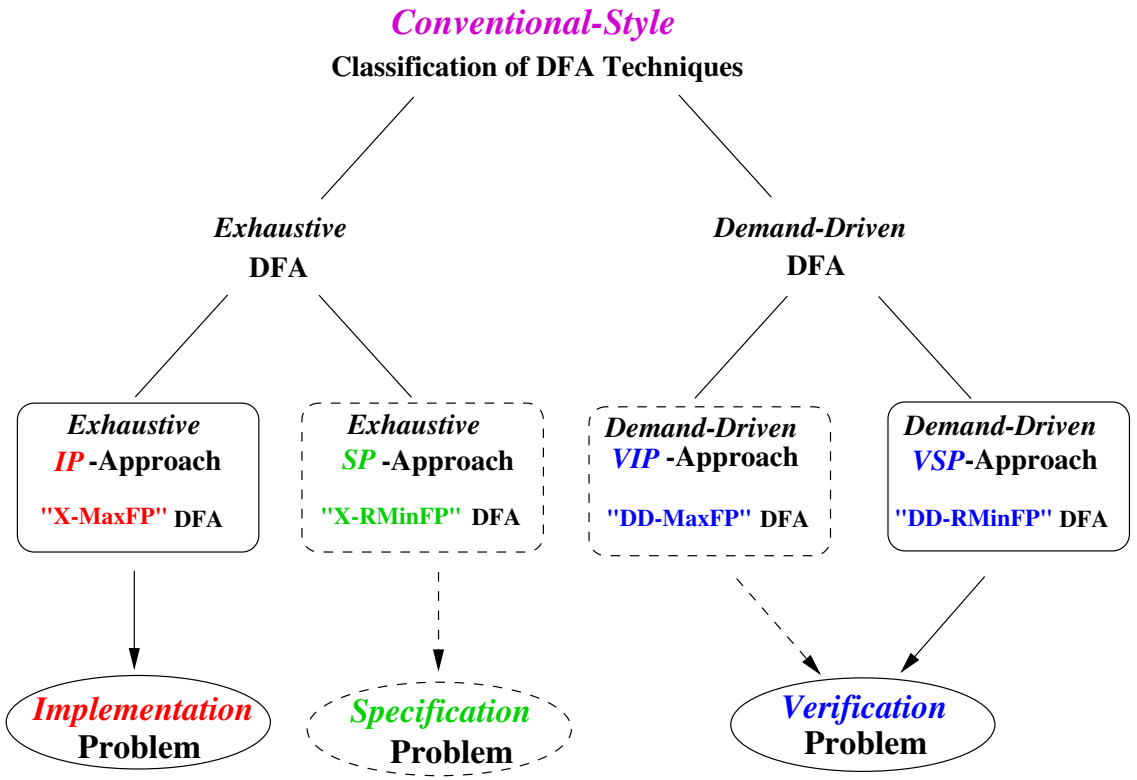


# Im Überblick




**... where "!" and "?"means given and sought, respectively.**

# Zum Abschluss: Algorithmenorientiert (1)



# Zum Abschluss: Problemorientiert (2)