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Compilers and Languages
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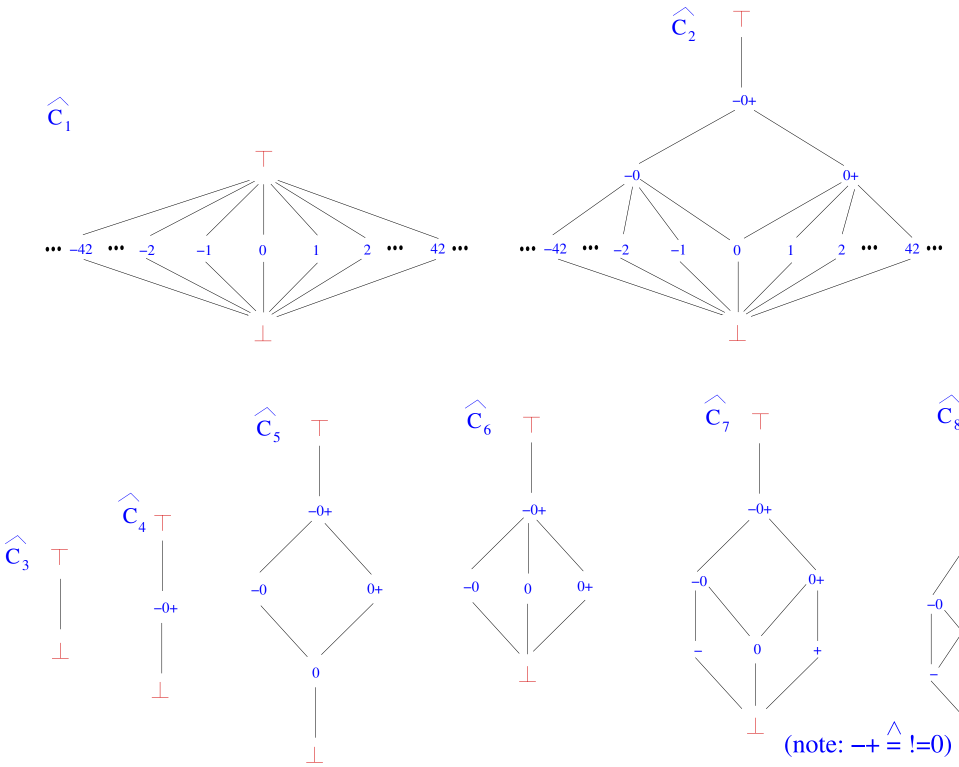
“ Analysis and Verification (185.276, VU 2.0, ECTS 3.0) ” SS 2019

Assignment 8

5 June 2019

Exercise 1 : (10 Points)

Consider the lattices $\widehat{\mathcal{C}_1}, \ldots, \widehat{\mathcal{C}_8}$ for constant and sign analysis for programs over a single variable x. Every element of the (various) lattices represents thus directly the (abstract) value of x. Note that ‘smaller means better’ in $\widehat{\mathcal{C}_1}, \ldots, \widehat{\mathcal{C}_8}$.



Order the lattices $\widehat{\mathcal{C}_1}, \ldots, \widehat{\mathcal{C}_8}$ in one or more chains to a lattice (add artifical top and bottom elements, if necessary) such that no element can be moved to a longer chain. Being greater in a chain shall mean that a ‘more accurate,’ a ‘more informative’ constant and sign analysis is possible. Lattices in this relation are then linked by a Galois connection (Galois-Verbindung).

Exercise 2 : (3*(2+2) Points)

Specify (without proof) the abstraction and concretization functions $\alpha, \alpha', \alpha'', \gamma, \gamma'\gamma''$ of the Galois connections (Galois-Verbindungen) which justify (part of) the order given in exercise 1:

a) $(\widehat{\mathcal{C}_8}, \alpha, \gamma, \widehat{\mathcal{C}_5})$

b) $(\widehat{\mathcal{C}_8}, \alpha', \gamma', \widehat{\mathcal{C}_6})$

c) $(\widehat{\mathcal{C}_8}, \alpha'', \gamma'', \widehat{\mathcal{C}_7})$

Exercise 3 : (2+2+2 Points)

Are

a) $(\widehat{\mathcal{C}_8}, \alpha, \gamma, \widehat{\mathcal{C}_5})$

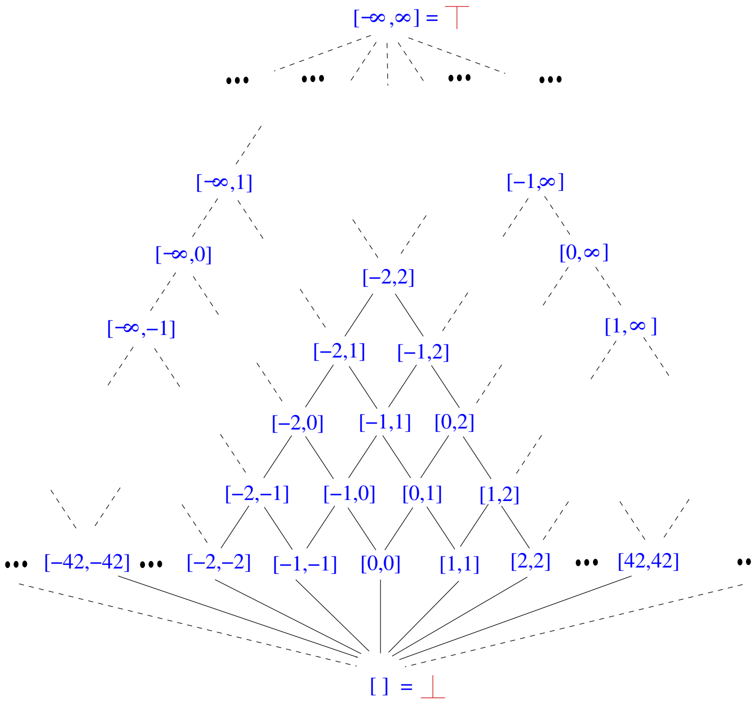
b) $(\widehat{\mathcal{C}_8}, \alpha', \gamma', \widehat{\mathcal{C}_6})$

c) $(\widehat{\mathcal{C}_8}, \alpha'', \gamma'', \widehat{\mathcal{C}_7})$

of exercise 2 even Galois insertions (Galois-Passungen)? Provide a counterexample, if not.

Exercise 4 : (2 Points)

Consider additionally to the lattices of exercise 1 the lattice $\widehat{\mathcal{C}_9}$, where like in exercise 1 'smaller means better:'



Where does $\widehat{\mathcal{C}_9}$ fit into your lattice of exercise 1?

Exercise 5 : (2+2 Points)

Consider additionally to the lattices of exercise 1 and exercise 4 the lattices $\widehat{\mathcal{C}_{10}}$ and $\widehat{\mathcal{C}_{11}}$:

- $\widehat{\mathcal{C}_{10}} =_{\text{df}} (\{\bot\}, \sqsubseteq, \sqcap, \sqcup, \bot, \bot)$ mit $\bot \sqsubseteq \bot$, $\bot \sqcap \bot = \bot \sqcup \bot = \bot$
- $\widehat{\mathcal{C}_{11}} = (C_{11}, \sqsubseteq, \sqcap, \sqcup, \bot, \top) =_{\text{df}} (\mathcal{P}(\mathbb{Z}), \subseteq, \cap, \cup, \emptyset, \mathbb{Z})$ powerset lattice of $\mathbb{Z}$.

Where do $\widehat{\mathcal{C}_{10}}$ and $\widehat{\mathcal{C}_{11}}$ fit into your lattice of exercise 1 resp. exercise 4 assuming that the analysis performed with $\widehat{\mathcal{C}_{11}}$ is given by the collecting semantics, i.e., lattice elements $M = \{z_1, z_2, \ldots, z_i, \ldots\} \in \mathcal{P}(\mathbb{Z})$ carry intuitively the meaning that program variable x 'has value z_1 or z_2 or ... or z_i or ...;' again, 'smaller means better.'

Submission: Wednesday, 19 June 2019, before the lecture (two weeks!).