

Exercise 1 : (2+4+2+4 Points)

Is $x \geq 0 \wedge y > 0$ the weakest liberal precondition for the integer division program

$$\pi \equiv q := 0; r := x; \text{ while } r \geq y \text{ do } q := q + 1; r := r - y \text{ od}$$

and the postcondition

$$x = q * y + r \wedge 0 \leq r < y \quad ?$$

If not:

1. Give a precondition wlp , $wlp \in \mathbf{Bexpr}$, which is the weakest liberal precondition.

Prove that your formula wlp is indeed the desired weakest liberal precondition, i.e., prove:

$$wlp \iff wlp(\pi, x = q * y + r \wedge 0 \leq r < y) \quad (*)$$

2. To prove equivalence (*), show in a first step that the Hoare assertion

$$\{wlp\} \pi \{x = q * y + r \wedge 0 \leq r < y\}$$

is partially correct, i.e., prove (by providing a linear proof sketch):

$$\models_{pc} \{wlp\} \pi \{x = q * y + r \wedge 0 \leq r < y\}$$

3. What else has to be shown in order to prove (*) and hence the equivalence of wlp and the weakest liberal precondition $wlp(\pi, x = q * y + r \wedge 0 \leq r < y)$?
4. Prove the properties identified in 3.).