Exercise 1: (4+4 Points)
Give a WHILE program π, for which the Hoare assertion
\[
\{true\} \pi \{false\}
\]
is partially correct, and prove your claim in two ways, by providing

1. a tree-like proof
2. a linear proof sketch

Exercise 2: (8 Points)
Prove that the WHILE program computing the product of two numbers of Assignment 3 is also
totally correct wrt the precondition \(x = n \land y = m \land n > 1\) and the postcondition \(y = n \cdot m\),
i.e., prove the validity of the Hoare assertion:
\[
[x = n \land y = m \land n > 1] \textbf{while } x \neq 1 y := y + m; x := x - 1 \textbf{ od } [y = n \cdot m]
\]

Exercise 3: (10 Points)
Consider the below (pre-) orders given by their Hasse diagrams (resp. Hasse-like diagrams):

Which of these orders are (i) partial orders (dtsch. partielle Ordnungen), which ones are (ii)
chain-complete partial orders CCPOs (dtsch. (ketten-) vollständige partielle Ordnungen (KV-
POs)), (ii) lattices (dtsch. Verbände), which ones are (iii) complete lattices (dtsch. vollständige
Verbände)? Explain your reasoning (proof or counterexample).

Note: Partial orders and Hasse diagrams are introduced in Appendices A.2.1 and A.2.2, respec-
tively, lattices and complete lattices in Appendix A.4.
Exercise 4: (4 Points)
Which of the following pairs are i) partial orders, (ii) (chain) complete partial orders, (iii) lattices, (iv) complete lattices? Explain your reasoning (proof or counterexample).

1. \((ST, \subseteq)\) with
   - \(ST =_{df} \Sigma \rightarrow \Sigma\) the set of state transformations.
   - \(\subseteq \subseteq ST \times ST\) defined by:
     \(\forall g_1, g_2 \in ST. g_1 \subseteq g_2 \iff_{df}\)
     \(\forall \sigma \in \Sigma. g_1(\sigma)\ defined = \sigma' \Rightarrow g_2(\sigma)\ defined = \sigma'\)

2. \((P(S), \subseteq)\) with \(S\) finite set, \(\subseteq\) subset relation, and \(P\) powerset operator, i.e., \(P(S) =_{df} \{S' \mid S' \subseteq S\}\).

3. \((P(S), \subseteq)\) with \(S\) nonfinite set, \(\subseteq\) subset relation and \(P\) powerset operator.

4. \((P_{fin}(S), \subseteq)\) with \(S\) nonfinite set, \(\subseteq\) subset relation and \(P_{fin}\) ‘finite’ powerset operator, i.e., \(P_{fin}(S) =_{df} \{S' \mid S' \subseteq S, S'\ finite\}\).

Exercise 5: (4 Points)
Prove or refute (proof or counterexample):

1. Every CCPO \((S, \subseteq)\) is a lattice: \((S, \subseteq)\ CCPO \Rightarrow (S, \subseteq)\ lattice\).
2. Every CCPO \((S, \subseteq)\) is a complete lattice: \((S, \subseteq)\ CCPO \Rightarrow (S, \subseteq)\ complete\ lattice\).
3. Every lattice \((S, \subseteq)\) is a CCPO: \((S, \subseteq)\ lattice \Rightarrow (S, \subseteq)\ CCPO\).
4. Every complete lattice \((S, \subseteq)\) is a CCPO: \((S, \subseteq)\ complete\ lattice \Rightarrow (S, \subseteq)\ CCPO\).

Exercise 6: (Without Submission)
Install the KeY-Hoare system (cf. Chapter 4.9 for the system’s URL) and conduct correctness proofs for the following three Hoare assertions:

- \(\models_{pk} \{true\} \ while\ true\ do\ skip\ od\ \{false\}\)
- \(\models_{pk} \{x = n \land y = m\} \ while\ x \neq 1\ do\ y := y + m;\ x := x - 1\ od\ \{y = n * m\}\)
- \(\models_{tk} \{x = n \land y = m \land n > 1\} \ while\ x \neq 1\ do\ y := y + m;\ x := x - 1\ od\ \{y = n * m\}\)

The proofs shall be (voluntarily) presented “live” on May 15, 2019.

Submission: Wednesday, 15 May 2019, before the lecture.