Assignment 4
Advanced Functional Programming
Thema: Streams
Issued on: 05/08/2008, due date: 05/19/2008

For this assignment a Haskell script named `AssFFP4.hs` shall be written offering functions which solve the problems described below. This file `AssFFP4.hs` shall be stored in your home directory, as usual on the top most level. Comment your programs meaningfully. Use constants and auxiliary functions, where appropriate.

- Implement a 0-ary Haskell function `pof2s :: [Integer]`, which computes the stream of the powers of 2,i.e., the stream `[1,2,4,8,16,32,...`.

  Try to implement `pof2s` recursively using only `map`, the list constructor `:` and arithmetic functions.

- The following figure shows the initial part of Pascal's triangle:

  ```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5   10  10  5   1
     ...
  ```

  We denote the rows of this triangle as P-tuples.

  Implement a 0-ary Haskell function `pts :: [[Integer]]`, which gives the stream of P-tuples, i.e. the stream `[[1],[1,1],[1,2,1],[1,3,3,1],...`.

  Try to implement `pts` recursively using only `zipWith` (together with the standard list constructors and operators and arithmetic operations).

- Let $n \geq 1$ be a natural number and let $d_1 d_2 \ldots d_k$ be the sequence of decimal numeric characters of n. Moreover, let $p_1, p_2, p_3, \ldots, p_k, \ldots$ be the sequence of prime numbers. Then the number

  $$2^{d_1} 3^{d_2} 5^{d_3} \ldots p_k^{d_k}$$

  is called the *Gödel number* of n. For example, $144 = 2^4 * 3^2$ is the Gödel number of 42 and $400 = 2^4 * 3^0 * 5^2$ is the Gödel number of 402.

  1. Write a Haskell function `g :: Integer -> Integer`, which maps positive arguments to their Gödel number, non-positive arguments to 0.
  2. Write a 0-ary Haskell function `gs :: [Integer]`, which gives the stream of Gödel numbers starting at 1, i.e. the first element of the stream is the Gödel number of 1, the second element is the Gödel number of 2, etc.

- In honour of Lambert Meerten a positive natural number n, which coincides with its Gödel number, is called a *Meerten number*. A number, which differs at most by d with $d \in \mathbb{N}_0$ from its Meerten number, we call a *d-Meerten number*.

  Write a Haskell function `dM :: Integer -> [Integer]`, which gives for non-negative arguments d the stream of d-Meerten numbers, for negative arguments the 0-stream, i.e. the stream which consists solely of zeros. Note that for $d = 0$, `dM` yields the stream of Meerten numbers.

  Remark: Meerten numbers are "rare". For $d = 0$ your algorithm might need very much time.